Problem of the Week #2 **March 15-22, 2004**

Original Problem: Find with proof the exact value of the sum

$$\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + ... + \frac{n}{2^n} + ...$$

Note: If you are familiar with Σ-notation, the sum above can be written as

$$\sum_{n=1}^{\infty} \frac{n}{2^n}.$$

Further questions for future research projects.

1. You can modify the problem replacing 2 by 3, namely, find the value of

$$\sum_{n=1}^{\infty} \frac{n}{3^n}.$$

   In general, given a positive integer k, find the value of

$$\sum_{n=1}^{\infty} \frac{n}{k^n}.$$

2. In (1) we vary the denominator of the terms in the series. This time we vary the numerator. Find the value of

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}.$$

   In general, given a positive integer j, find the value of

$$\sum_{n=1}^{\infty} \frac{n^j}{2^n}.$$

3. Put together the previous two questions. Given positive integers j and k, find the value of

$$\sum_{n=1}^{\infty} \frac{n^j}{k^n}.$$

Note: Before trying to find the value of any of these series, it is important to prove that they actually converge.