Problem of the Week 4, Fall 2006

Solution by organizers. Let x be the time (in hours) between 8:00 am and the first god's arrival, y be the time between the first and second gods' arrivals, and z be the time between the second and third gods' arrivals. Then $x, y,$ and z are real numbers between 0 and 9 so that $x + y + z \leq 9$. These triples can be represented by the points on the boundary and interior of the pyramid with vertices $(0, 0, 0), (9, 0, 0), (0, 9, 0),$ and $(9, 0, 0)$. Note that no pair of gods meets if and only if $y \geq 2$ and $z \geq 2$. These restricted set of triples (x, y, z) is represented by the pyramid with vertices $(0, 2, 2), (5, 2, 2), (0, 7, 2),$ and $(0, 2, 7)$. These two pyramids are similar. Their base is a right isosceles triangle whose legs are equal in length to the height of the corresponding pyramid. The first pyramid has height 9 and the second has height 5. The probability that no pair of gods meet in a day is the ratio of the volume of the small pyramid to the volume of the big pyramid, that is $\frac{5^3/6}{9^3/6} = \frac{5^3}{9^3}$. Thus the probability of a good day is $1 - \frac{5^3}{9^3}$.