Proposed by Bernardo Ábrego and Silvia Fernández. **November 8-15**

The positive integers p, p_1, p_2,and p_3 are prime numbers satisfying that $p_1 < p_2 < p_3$, and

$$p = p_1^2 + p_2^2 + p_3^2.$$

Prove that $p_1 = 3$.

Solution by. Chuck Goodman

First, we note that P_1 may not be 2.
Since all the other primes are odd, we would have something in the form of $2^2 + (2n+1)^2 + (2m+1)^2$ for some m,n $\in \mathbb{Z}$. This will always be even and P must be odd.

Next, we note that since all the primes on the right hand side are at least 3, P itself must be greater than 3.
So we can say if $P_1 \neq 3$, then none of P_1, P_2, or P_3 is divisible by 3 since $P_1 < P_2 < P$.
Now, if we work mod 6, we see that each $P_i \equiv \pm 1(\text{Mod}6)$
[Since] We know that P_i would be congruent to 2 or 4 iff it was divisible by 2 and it would be congruent to 3 iff it was divisible by 3.
$P_i \equiv \pm 1(\text{Mod}6) \Rightarrow P_i^2 \equiv 1(\text{Mod}6)$
Next, we know that if $P_i^2 \equiv 1(\text{Mod}6)$, then $\sum_1^3 P_i^2 \equiv 3(\text{Mod}6)$
But, this contradicts that fact the P is a prime greater than 3 since P must be divisible by 3!
We conclude the P_i must include 3
and since we have $P_1 < P_2 < P_3$, we must have $P_1 = 3$