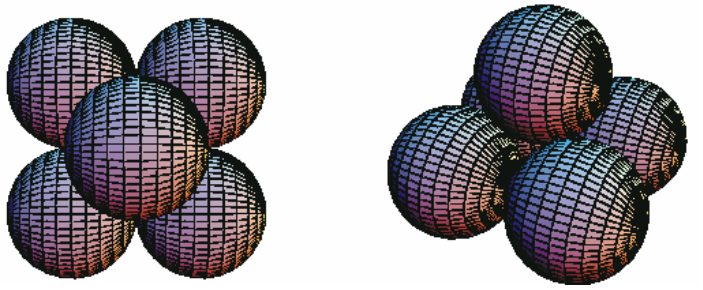
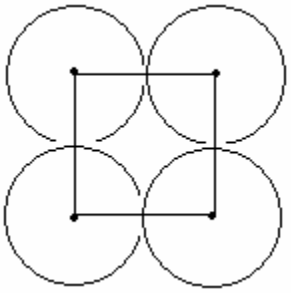
Problem of the Week. **August 23-30**

Proposed by Bernardo Ábrego and Silvia Fernández.

Four basketballs, 12 inches in diameter, are placed on the floor forming a square. Any two balls forming a side of the square are touching (see figure below). A fifth basketball (same size) is placed on top of the previous four. It is perfectly centered and touches all other balls. What is the distance from the center of the fifth ball to the floor?



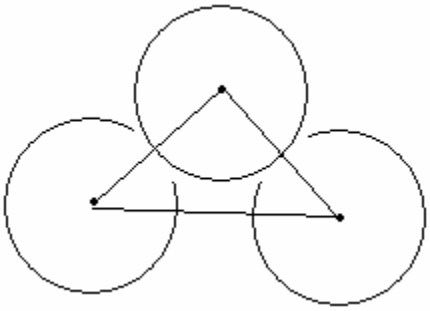
Solution by Joseph Viner. The four bottom spheres are touching and form a square. This means their edges must touch as so:



(top view)
forming a square with their centers.

Since the radius is 6in., the sides of that square would be 12in. and that'd make the diagonal of that square $12\sqrt{2}$ as it forms a 45-45-90 right triangle.

The sphere sitting on top actually forms the same triangle with two of the spheres below it:



(side view, only showing two opposite-corner bottom spheres supporting the top—other two are deleted so you can see the triangle formed)

because if you put a 6^{th} sphere on the bottom, you could complete the square…

they're the same distance apart as the square shown at the top of the page, just rotated 90 degrees up… this is known because the only thing separating the spheres are other identically sized/shaped spheres… so they'd have to form neat 90 degree angles and such…

the altitude of this triangle would be $\sqrt{12^2 - \left(\frac{12\sqrt{2}}{2}\right)^2}$ …. i.e. (Hypoteneuse)^2 – (half of the bottom)^2 … which turns out to be $6\sqrt{2}$.

then you add that height to the 6in. (radius) left over until the ground and you get:

$6 + 6\sqrt{2}$ inches.