Chapter 2 Wave in an Unbounded Medium

Maxwell's Equations

Time-harmonic form (Real quantity) | Phasor form (complex quantity)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \hat{\boldsymbol{E}} = -j\omega \hat{\boldsymbol{B}} \qquad (1.11a)$$

$$\nabla \cdot \vec{D} = \tilde{\rho} \qquad \nabla \cdot \hat{\boldsymbol{D}} = \rho \qquad (1.11b)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \nabla \times \hat{\boldsymbol{H}} = \hat{\boldsymbol{J}} + j\omega \hat{\boldsymbol{D}} \qquad (1.11c)$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \cdot \hat{\mathbf{B}} = 0 \qquad (1.11d)$$

with $\vec{\mathrm{E}}(x,y,z,t) = \mathrm{Re}\left\{ \hat{\mathbf{E}}(x,y,z) e^{j\omega t} \right\}$

Chapter 2.1 Plane Waves in a Simple, Source-Free, and Lossless Medium

Maxwell's Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (2.1a)$$

$$\nabla \cdot \vec{D} = \tilde{\rho} \qquad (2.1b)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (2.1c)$$

$$\nabla \cdot \vec{B} = 0 \qquad (2.1d)$$

2.1 Plane Wave in a Simple, Source-free, and Lossless Medium

The general equations for electric and magnetic fields:

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad (2.2)$$

$$\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \qquad (2.3)$$

If we restric the electric field as the function z, and the electric field to one component E_x only, equation (2.2) becomes:

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0 \qquad (2.5)$$

the general solution of (2.5) is in the form of:

$$E_x = p_1(z - v_p t) + p_2(z + v_p t) \qquad (2.6)$$

2.1.1 The Relation between $\vec{E}$ and $\vec{H}$

Assume we have a electric field of uniform plane wave:

$$E_x = p_1(z - v_p t)$$

The corresponding $\vec{H}(z)$ is not independent, and must be resolved from Maxwell equations (2.1a) and /or (2.1c):

$$H_y = \left(\frac{\varepsilon}{\mu}\right) p_1(z - v_p t) = \left(\frac{1}{\eta}\right) p_1(z - v_p t)$$

where $\eta = \sqrt{\mu / \varepsilon}$ is called "intrinsic impedance" of the medium (in unit ohms).

2.2 Time-Harmonic Uniform Plane Waves in a Lossless Medium

The electric field of a uniform plane wave in a lossless medium is

$$E_x(z,t) = \mathrm{Re}\left\{(C_1 e^{-j\beta t} + C_1 e^{+j\beta t})e^{j\omega t})\right\} \qquad (2.11)$$

or

$$E_x(z,t) = C_1 \cos(\omega t - \beta z) + C_2 \cos(\omega t + \beta z) \qquad (2.12)$$

The electric and magnetic fields of a time-harmonic wave propagating in the + Z direction are

$$E_x(z,t) = C_1 \cos(wt - \beta t)$$

$$H_y(z,t) = \frac{1}{\eta} C_1 \cos(wt - \beta t) \qquad (2.13)$$

where the "intrinsic impedance" (in unit ohms)

$$\eta = \sqrt{\mu / \varepsilon} = \omega\mu/\beta$$

(note: $\beta = \omega\sqrt{\mu\varepsilon}$) .

Example 2-1: AM broadcast Singnal

The instantaneous expression for the electric field component of an AM broadcast signal propagating in air is given by

$$\vec{E}(x,t)=\vec{z}10\cos(1.5\pi\times10^{6}t+\beta x) \qquad V-m^{-1}$$

(a) Determine the direction of propagation and frequency f.

(b) Determine the phase constant β and wavelength.

(c) Find the instantaneous expression for the corresponding $\vec{H}(x,t)$.

Example 2-2: FM broadcast Singnal

An FM broadcast signal traveling in y direction in air has a magnetic field given by the phasor:

$$\hat{H}(y) = 2.92\times10^{-3}e^{-j0.68\pi y}(-\vec{x}+\vec{z}j) \qquad A-m^{-1}$$

(a) Determine the frequency .

(b) Find the corresponding $\hat{E}(y)$.

(c) Find the instantaneous expression for the $\vec{E}(x,t)$ and $\vec{H}(x,t)$.

2.3 Plane Waves in Lossy Media

The electric field of a uniform plane wave in a lossy medium is

$$E_x(z,t) = \mathrm{Re}\left\{(C_1 e^{-\alpha z} e^{-j\beta t} + C_2 e^{\alpha z} e^{+j\beta t}) e^{j\omega t}) \right\}$$

or

$$E_x(z,t) = C_1 e^{-\alpha z}\cos(\omega t - \beta z) + C_2 e^{\alpha z}\cos(\omega t + \beta z) \qquad (2.18)$$

The propagation constant $\gamma = \alpha + i\beta$:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2} - 1]^{1/2} \qquad np - m^{-1}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2} + 1]^{1/2} \qquad rad - m^{-1} \qquad (2-19)$$

The waves propagate in the +Z directions are

$$E_x^+(z,t) = C_1 e^{-\alpha z}\cos(\omega t - \beta z) \qquad (2.21)$$

$$H_x^+(z,t) = \frac{1}{|\eta_c|} C_1 e^{-\alpha z}\cos(\omega t - \beta z + \phi_n) \qquad (2.23)$$

where intrinsic impedance η_c is:

$$\eta_\mathrm{c} = |\eta_\mathrm{c}| e^{j\phi_n} = \sqrt{\frac{\mu}{\varepsilon_{eff}}} = \sqrt{\frac{\mu}{\varepsilon - j\frac{\sigma}{\omega}}}$$

$$= \frac{\sqrt{\frac{\mu}{\varepsilon}}}{[1 + (\frac{\sigma}{\omega\varepsilon})^2]^{1/4}} e^{j(1/2)\tan^{-1}[\sigma/(\omega\varepsilon)]} \qquad (2-22)$$

Example 2-5

Find the complex propagation constant γ and the intrisic impedance η_c of a microwave signal in muscle tissue at 915 MHz (σ=1.6 s-m^{-1} $\varepsilon_r = 51$)

Example 2-6

Consier distilled water at 25 GHz ($\varepsilon_{cr} = 34 - j9.01$).
Calculate the attenuation constant α, phase constant β, penetration depth d, and the wavelength λ.

2.4 Electromagnetic Energy Flow and the Poynting Vector

Poynting Theorem:

$$\int_V \vec{E}\cdot\vec{J}dv = -\frac{\partial}{\partial t}\int_V \left(\frac{1}{2}\mu\left|\vec{H}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{E}\right|^2\right)dv - \oint_S (\vec{E}\times\vec{H})\cdot d\vec{s} \qquad (2.31)$$

Poynting Vecor:

$$\vec{S} = \vec{E}\times\vec{H}$$

Example 2-12: Wire carrying direct current.

Consider a long cylindrical conductor of conductivity σ and radius a, carrying a direct current I as shown in the figure. Find the power dissipated in a portion of the wire of length l, using

(a) the left-hand side of (2.31); (b) he right-hand side of (2.31)

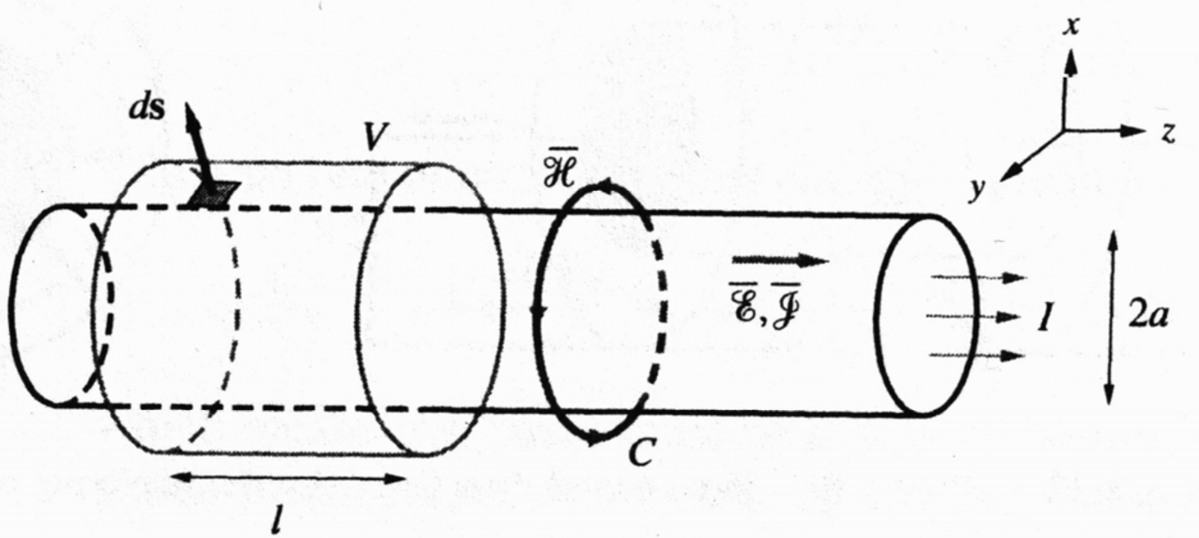

FIGURE 2.11. Long straight wire. The power dissipated in a cylindrical conductor carrying a direct current I is given by I^2R.

Example 2-13 Power flow in a coaxial line

Consider a coaxial line delivering power to a resistor as shown in the figure. Assume the wire to be perfect conductor, so that there is no power dissipation in the wires, and the electric field inside them is zero. The configurations of the electric and magnetic field lines in the coaxial line are shown. Find the power delivered to the resistor R.

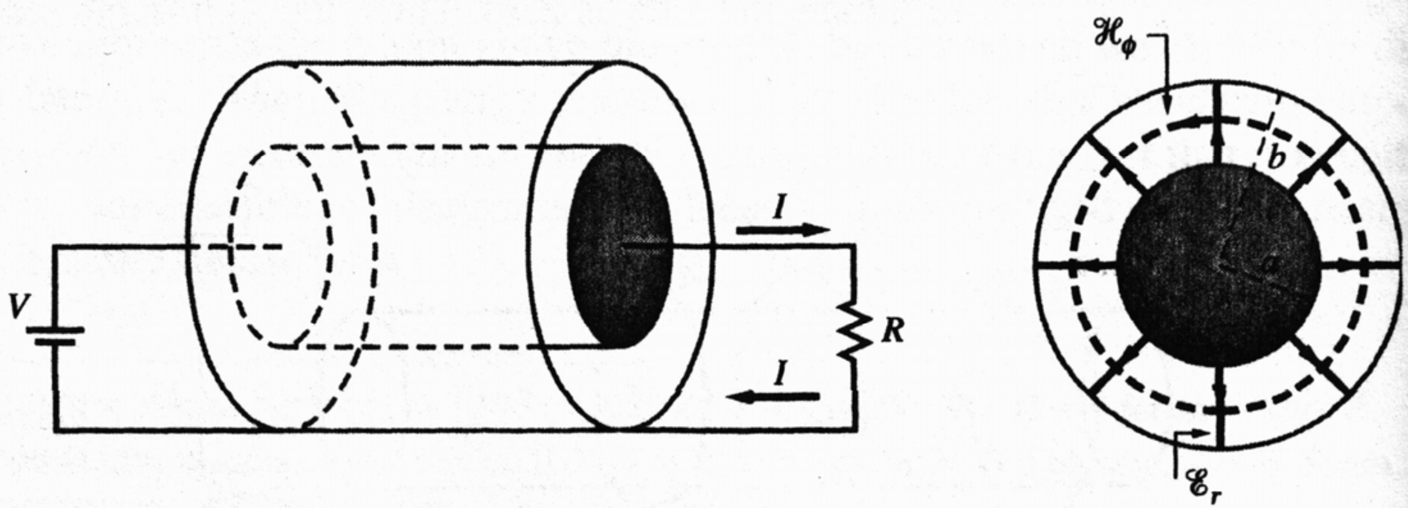

FIGURE 2.12. **Power flow in a coaxial line.** Coaxial wire delivering power to a resistor *R*.

Example 2-18 VHF/UHF broadcast radiation

A survey conducted in the United State indicated that ~50% of the population is exposed to average power densities of 0.005 μW-(cm)$^{-2}$ due to VHF and UHF broadast radiaction. Find the correcponding amplitudes of the electric and magnetic fields.

Example 2-19 VLF waves in the ocean.

Consider VLF wave progation in the ocean. Find the time-average Poynting flux at the sea surface (z=0) and at the depth of z=100m. Assume:

$\hat{E}(z)=\vec{x}2.84e^{-0.218z}e^{-j0.218z}$ k V-m^{-1}

$\hat{H}(z)=\vec{x}2.84e^{-0.218z}e^{-j0.218z}e^{-j\pi/4}$ k A-m^{-1}