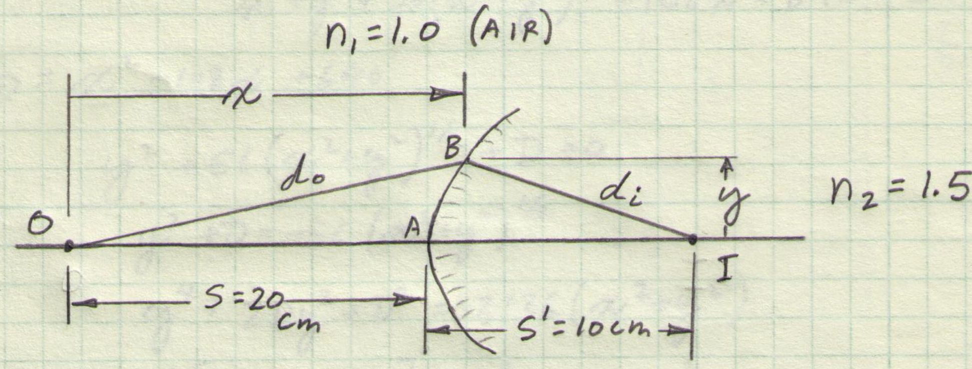
3-2



FIND THE EQUATION FOR THE CARTESIAN OVAL IN THE x-y PLANE.

FROM FERMAT'S PRINCIPLE:

TIME OF TRAVEL OAI = TIME OF TRAVEL OBI

$$n_1 S + n_2 S' = n_1 d_o + n_2 d_i$$

$$20 + 1.5(10) = d_o + 1.5 d_i = 35$$

$$d_o = (x^2+y^2)^{1/2} \qquad d_i = \left[(20+10-x)^2+y^2\right]^{1/2} = \left[(30-x)^2+y^2\right]^{1/2}$$

THE OVAL EQUATION IS

$$\boxed{(x^2+y^2)^{1/2} + 1.5\left[(30-x)^2+y^2\right]^{1/2} = 35}$$

TO PUT IT INTO ANOTHER FORM:

$$(x^2+y^2)^{1/2} - 35 = -1.5\left[(30-x)^2+y^2\right]^{1/2}$$

SQUARE BOTH SIDES

$$(x^2+y^2) - 70(x^2+y^2)^{1/2} + 1225 = 2.25\left[900 - 60x + x^2 + y^2\right]$$

$$(x^2+y^2) - 70(x^2+y^2)^{1/2} + 1225 = 2025 - 135x + 2.25(x^2+y^2)$$

$$0 = 800 - 135x + 70(x^2+y^2)^{1/2} + 1.25(x^2+y^2)$$

OR $$\boxed{1.25(x^2+y^2) + 70(x^2+y^2)^{1/2} - 135x + 800 = 0}$$

(SAME)

TO PLOT THIS MUST SOLVE FOR y

DIVIDE BY 1.25:

$$x^2+y^2+56(x^2+y^2)^{1/2}-108x+640=0$$

let $D=x^2-108x+640$

THEN

$$y^2+56(x^2+y^2)^{1/2}+D=0$$

$$y^2+D=-56(x^2+y^2)^{1/2}$$

SQUARE

$$y^4+2Dy^2+D^2=3136(x^2+y^2)$$

$$y^4+y^2[2D-3136]+D^2-3136x^2=0$$

IN THE FORM OF QUADRATIC EQUATION IN y^2

$a=1 \qquad b=2D-3136 \qquad c=D^2-3136x^2$

$$y^2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

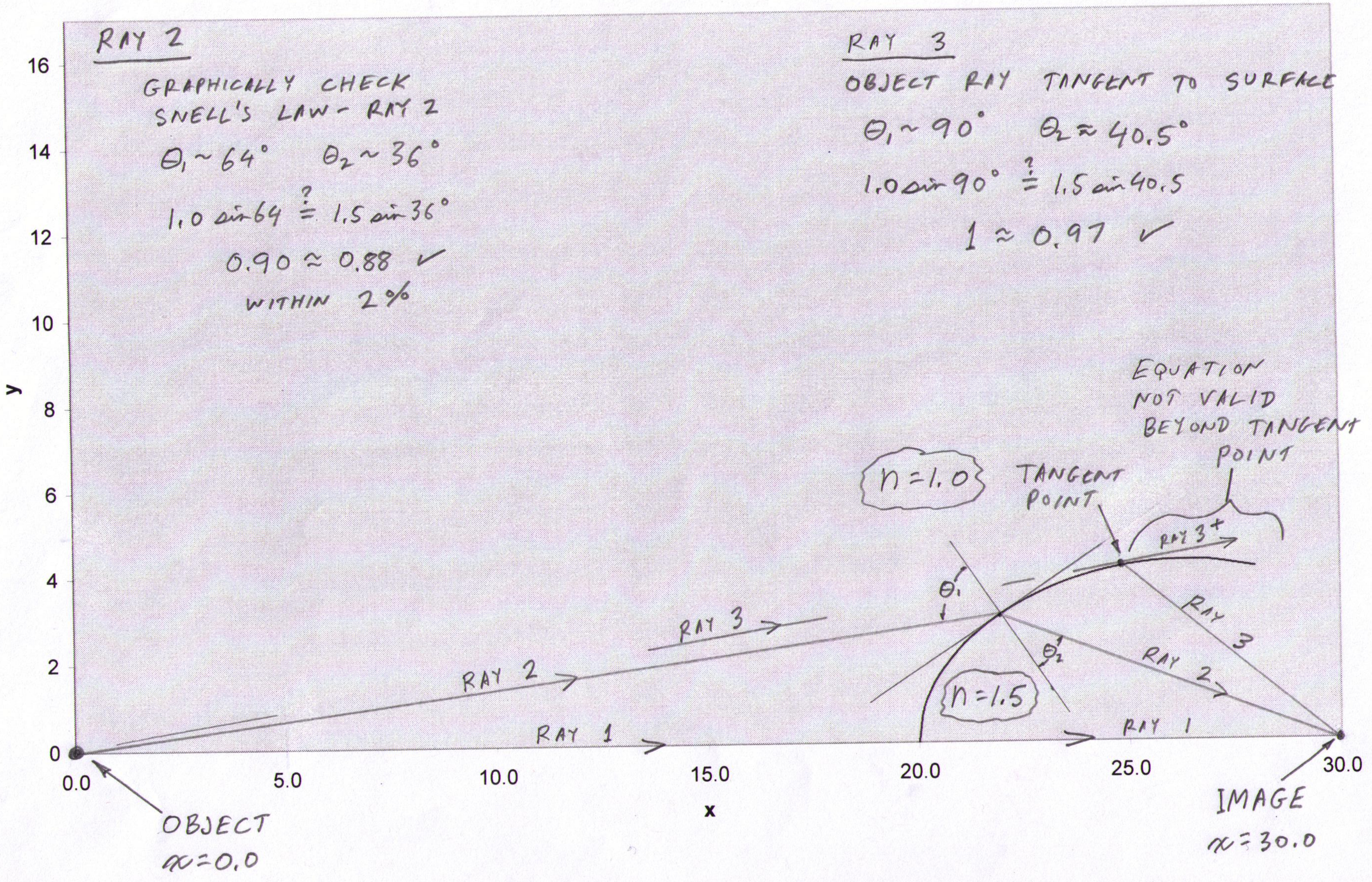
PLOT WITH AN EXCEL SPREADSHEET

Problem 3-2
Cartesian Oval

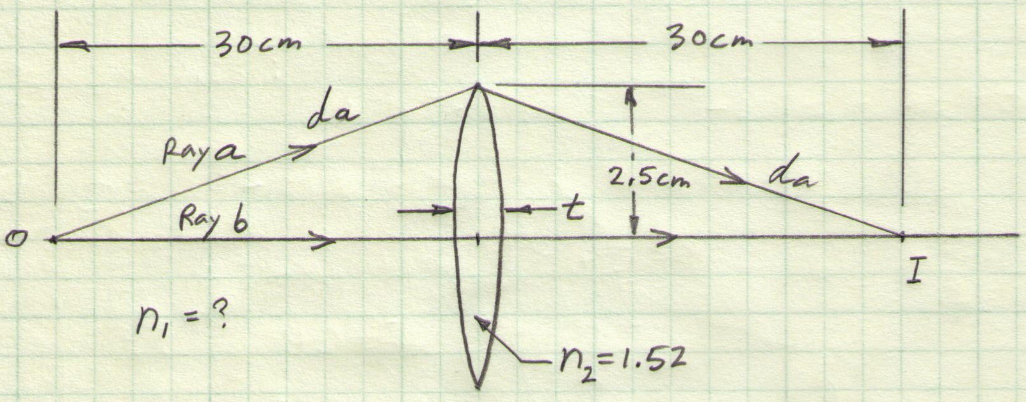
x	D	b	c	y2	y	Delta
20.0	-1120	-5376	0	0	0	0
20.1	-1126.79	-5389.58	2680.344	0.497366	0.705241	0
20.3	-1136.938	-5409.875	6670.879	1.233374	1.110574	0
20.5	-1153.75	-5443.5	13235.06	2.432439	1.559628	0
21.0	-1187	-5510	25993	4.721469	2.172894	0
21.5	-1219.75	-5575.5	38174.06	6.85518	2.61824	0
22.0	-1252	-5640	49680	8.822311	2.970238	0
22.5	-1283.75	-5703.5	60414.06	10.6122	3.257637	0
23.0	-1315	-5766	70281	12.21474	3.494959	0
23.5	-1345.75	-5827.5	79187.06	13.62035	3.690576	0
24.0	-1376	-5888	87040	14.81991	3.849664	0
24.5	-1405.75	-5947.5	93749.06	15.80477	3.975521	0
25.0	-1435	-6006	99225	16.56668	4.070218	0
25.5	-1463.75	-6063.5	103380.1	17.09778	4.134946	0
26.0	-1492	-6120	106128	17.39059	4.170203	0
26.5	-1519.75	-6175.5	107384.1	17.43796	4.175879	0
27.0	-1547	-6230	107065	17.23306	4.151272	0
28.0	-1600	-6336	101376	16.04061	4.005073	0

WHERE DELTA IS A CHECK OF THE ORIGINAL EQUATION AND SHOULD BE ZERO ✓

Cartesion Oval Problem 3-2



3-3



USING THE EQUIVALENCE OF OPTICAL PATHS a & b, WHAT IS THE LENS THICKNESS AT THE CENTER?

$$2n_1\sqrt{2.5^2+30^2} = n_1(30+30-t)+n_2 t$$

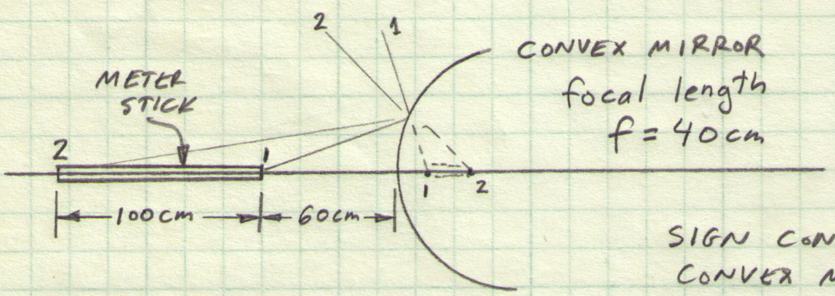
ASSUME IN AIR, $n_1 = 1.0$

$$2\sqrt{2.5^2+30^2} = 60-t+1.52\,t$$

$$2\sqrt{2.5^2+30^2}-60 = 0.52\,t$$

$$\boxed{\begin{array}{l} t = 0.40\text{ cm} \\ t = 4.0\text{ mm} \end{array}}$$

3-9



CONVEX MIRROR
focal length
$f = 40$cm

SIGN convention
CONVEX MIRROR
$f = -40$

HOW LONG IS THE IMAGE?

CLOSEST END OF THE METER STICK:
MIRROR EQUATION: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$

POINT 1

$s_1 = +60 \quad \frac{1}{60}+\frac{1}{s'} = -\frac{1}{40} \qquad s' = -24$ cm
VIRTUAL IMAGE

FARTHEST END OF METER STICK

$s_2 = +160 \quad \frac{1}{160}+\frac{1}{s'} = -\frac{1}{40} \qquad s' = -32$ cm VIRTUAL IMAGE

LENGTH OF VIRTUAL IMAGE = 32-24 cm = 8cm = VIRTUAL IMAGE LENGTH

$\boxed{3-11}$

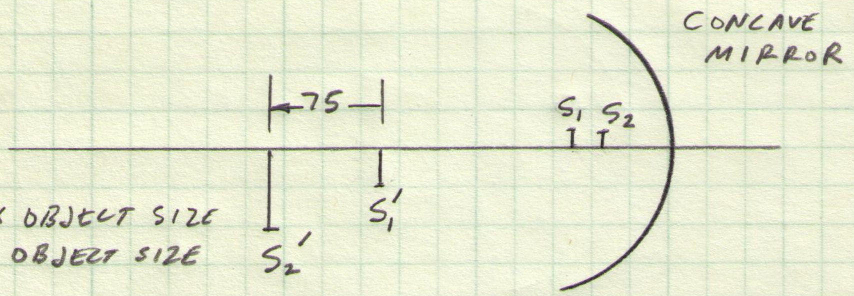


IMAGE 1 = 2 × OBJECT SIZE
IMAGE 2 = 3 × OBJECT SIZE

SIGN CONVENTION
AN IMAGE ON A SCREEN MEANS A REAL IMAGE, → S_1' & S_2' POSITIVE
REAL OBJECT: S_1 & S_2 POSITIVE
CONCAVE MIRROR: f IS POSITIVE

MAGNIFICATION $|m| = s'/s$

POSITION 1 $\quad 2 = \frac{S_1'}{S_1}$ $\qquad$ POSITION 2 $\quad 3 = \frac{S_2'}{S_2}$

MIRROR EQUATION

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \qquad \frac{1}{S_1} + \frac{1}{S_1'} = \frac{1}{f} = \frac{1}{S_2} + \frac{1}{S_2'}$$

$$\frac{2}{S_1'} + \frac{1}{S_1'} = \frac{3}{S_2'} + \frac{1}{S_2'} \qquad \frac{3}{S_1'} = \frac{4}{S_2'} \qquad S_2' = \frac{4}{3} S_1'$$

THE SCREEN (IMAGE) MOVES 75 cm

$$|S_2' - S_1'| = 75 \qquad \left|\frac{4}{3}S_1' - S_1'\right| = 75 \qquad \frac{S_1'}{3} = 75 \qquad S_1' = 225 \text{ cm}$$

$$S_2' = \frac{4}{3}(225) \quad S_2' = 300 \text{ cm} \qquad S_1 = \frac{S_1'}{2} = \frac{225}{2} = 112.5$$

$$S_2 = \frac{S_2'}{3} = \frac{300}{3} \quad S_2 = 100 \text{ cm}$$

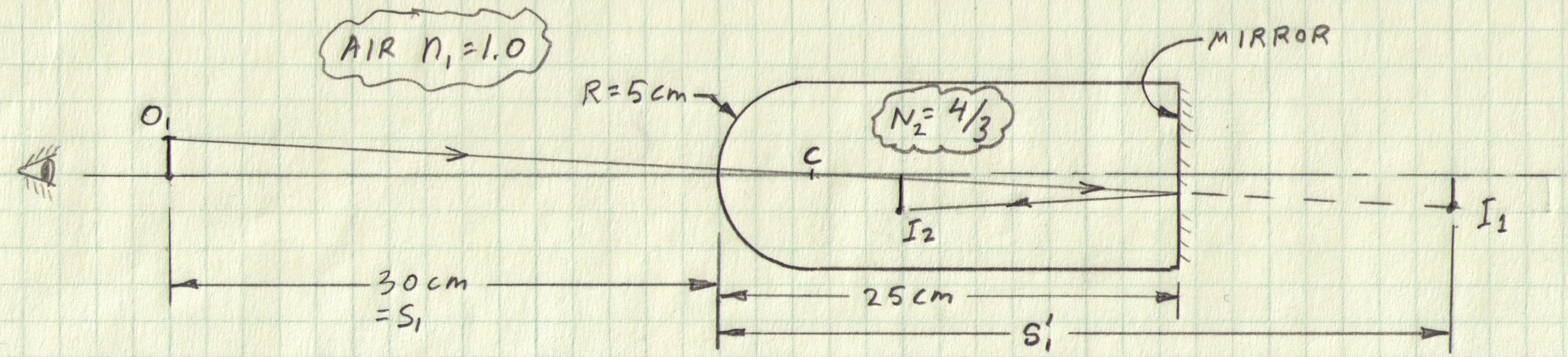
THE OBJECT MOVED $S_1 - S_2 = 112.5 - 100 =$ THE OBJECT MOVED 12.5 cm TOWARDS THE MIRROR

FOCAL LENGTH

POSITION 1 $\left.\right\}$ $\frac{1}{225} + \frac{1}{112.5} = \frac{1}{f}$ $\qquad f = 75 \text{ cm}$

POSITION 2 $\left.\right\}$ $\frac{1}{100} + \frac{1}{300} = \frac{1}{f}$ $\qquad f = 75 \text{ cm}$

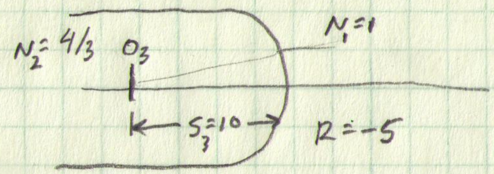
3-15



O_1 = OBJECT. 1) FIRST REFRACTION $S_1 = 30$ $\frac{n_1}{S_1} + \frac{n_2}{S_1'} = \frac{n_2 - n_1}{R}$ $\frac{1}{30} + \frac{1.33}{S_1'} = \frac{1.33 - 1.0}{5}$ $S_1' = 40\text{cm}$

2) REFLECTION OFF MIRROR: SECOND IMAGE I_2 IS $40 - 25 = 15\text{cm}$ TO THE LEFT OF MIRROR

3) REFRACTION THRU GLASS AGAIN: IMAGE AT I_2 IS OBJECT FOR FINAL REFRACTION, $I_2 = O_3$



$S_3 = 25 - 15 = 10\text{cm}$

EQUATION $\frac{n_1}{S} + \frac{n_2}{S'} = \frac{n_2 - n_1}{R} \rightarrow \frac{4/3}{10} + \frac{1.0}{S_3'} = \frac{1.0 - 1.333}{-5}$

$\boxed{S_3' = -15}$

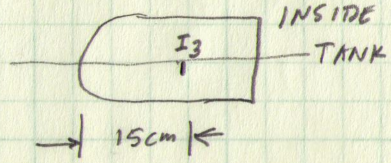
MAGNIFICATION $m_1 = \frac{-n_1 S_1'}{n_2 S} = -\frac{1.0(40)}{(4/3)30} = -1.0$

REFLECTION OFF THE MIRROR $m_2 = 1.0$

m from second refraction

$m_3 = -\frac{(4/3)(-15)}{(1.0)(10)} = 2.0$

FINAL IMAGE IS 15cm INSIDE TANK



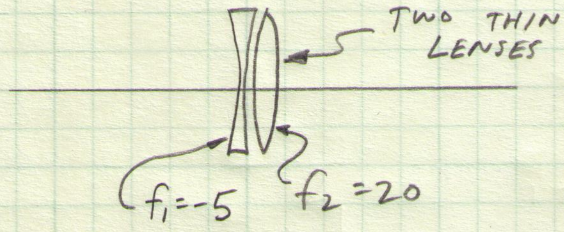
FINAL MAGNIFICATION

$m = m_1 m_2 m_3 = (-1.0)(1.0)(2.0)$

$\boxed{m = -2.0}$

INVERTED IMAGE AT TWICE THE SIZE
VIRTUAL IMAGE (S_3' NEGATIVE)
real image.

$\boxed{3\text{-}19}$

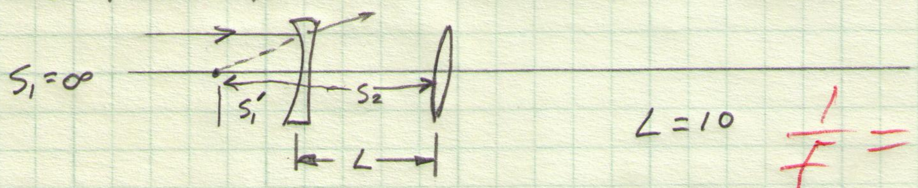


(a) EQUIVALENT FOCAL POINT IF GLUED TOGETHER

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{-5} + \frac{1}{20} \qquad \boxed{f = -6.67}$$

✓

(b) FIND FOCAL POINT IF SEPARATED BY 10 cm



$L = 10$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$$

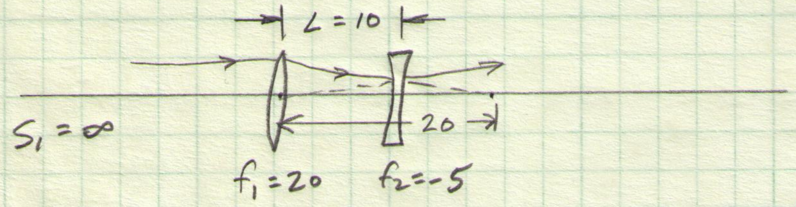
S_1' IS AT f_1 $\quad S_1' = -5$ $\quad$ FOR SECOND LENS $\quad S_2 = 5 + L$

$$\frac{1}{S_2'} + \frac{1}{S_2} = \frac{1}{f_2} \qquad \frac{1}{S_2'} + \frac{1}{5+L} = \frac{1}{20}$$

IF $L = 10$ $\qquad \dfrac{1}{S_2'} = \dfrac{1}{20} - \dfrac{1}{15}$ $\qquad \boxed{S_2' = -60 \text{ cm} = \text{focal point}}$

IF $L = 0$ $\qquad S_2' = f = -6.67$, SAME AS GLUED TOGETHER

IF LIGHT RAYS COMING FROM OTHER DIRECTION

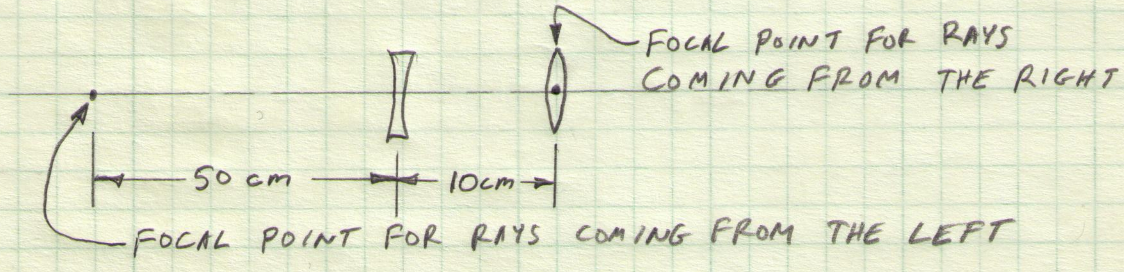


FIRST LENS $\quad S_1' = 20$

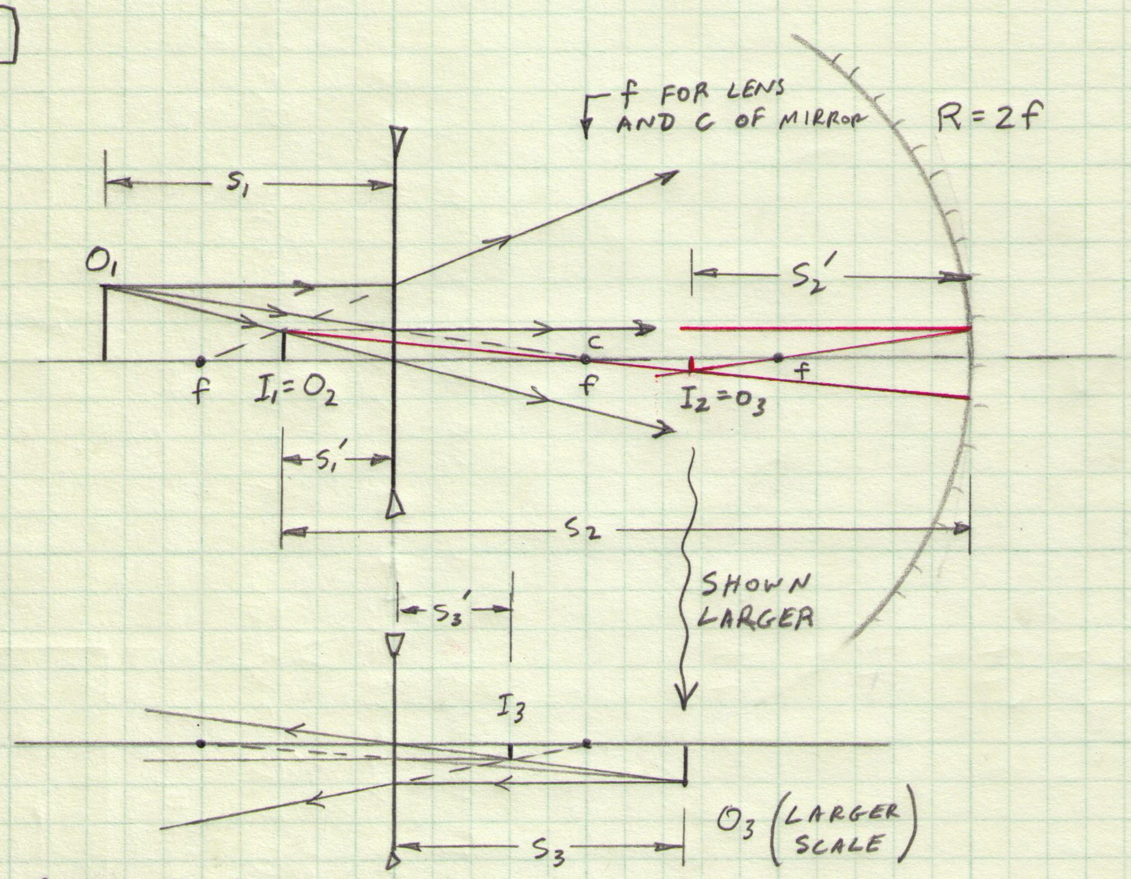
SECOND LENS $\quad S_2 = -10 \qquad \dfrac{1}{S_2} + \dfrac{1}{S_2'} = \dfrac{1}{f_2} \qquad \dfrac{1}{-10} + \dfrac{1}{S_2'} = \dfrac{1}{-5}$

$\boxed{S_2' = -10 \text{cm}}$ → AT THE CENTER OF THE FIRST LENS

LOCATION OF FOCAL POINTS



3-22



THIN LENS EQUATION

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

FIRST REFRACTION – LEFT TO RIGHT

$f \to -f \qquad s_1 = \frac{3}{2}f \qquad \frac{2}{3f} + \frac{1}{s_1'} = -\frac{1}{f}$

$\frac{1}{s_1'} = -\frac{5}{3f} \qquad \boxed{s_1' = -\frac{3f}{5}}$ SHOWN AS $I_1 = O_2$

MAGNIFICATION $\quad m_1 = -\frac{s'}{s} = -\frac{\left(-\frac{3}{5}f\right)}{\frac{3}{2}f} = \boxed{\frac{2}{5} = m_1}$

REFLECTION

$s_2 = 3f + |s_1'| = 3f + \left|\frac{3}{5}f\right| \qquad \boxed{s_2 = \frac{18}{5}f}$

$3\frac{3}{5}$

$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f}$ CONCAVE MIRROR, f is (+)

$\frac{5}{18f} + \frac{1}{s_2'} = \frac{1}{f} \qquad \frac{1}{s_2'} = \frac{13}{18f} \qquad \boxed{s_2' = \frac{18}{13}f}$

$m_2 = -\frac{s_2'}{s_2} = -\frac{\left(\frac{18}{13}f\right)}{\left(\frac{18}{5}f\right)} \qquad \boxed{m_2 = -\frac{5}{13}}$

FINAL REFRACTION TO THE LEFT

$$s_3 = 3f - s_2' = 3f - \frac{18}{13}f = \frac{21}{13}f$$ (USE POSITIVE TO THE LEFT)

$$\frac{1}{s_3} + \frac{1}{s_3'} = -\frac{1}{f} \qquad \frac{1}{s_3'} + \frac{13}{21f} = -\frac{1}{f} \qquad \frac{1}{s_3'} = -\frac{34}{21f}$$

$$s_3' = -\frac{21}{34}f$$ (TO RIGHT OF LENS)

THE SCALE OF MY FIGURE IS PRETTY CLOSE!

$0.6f \approx 0.62f$

THIRD MAGNIFICATION $\quad m_3 = -\frac{s_3'}{s_3} = -\frac{\left(-\frac{21}{34}\right)f}{\left(\frac{21}{13}\right)f} \quad \boxed{m_3 = \frac{13}{34}}$

FINAL MAGNIFICATION

$$m = m_1 m_2 m_3 = \left(\frac{2}{\cancel{5}}\right)\left(-\frac{\cancel{5}}{\cancel{13}}\right)\left(\frac{\cancel{13}}{34}\right) \quad \boxed{m = -\frac{1}{17}}$$

FINAL IMAGE

$\frac{21}{34}f$ TO THE RIGHT OF THE LENS

magnification $-\frac{1}{17}$ You are right!

INVERTED, VIRTUAL IMAGE

✓