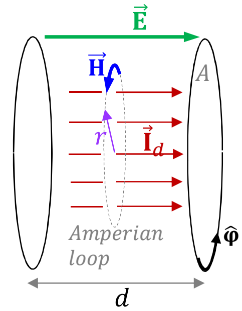
Physics 410 HW Chapter 21 RON RAHAV

21-1 A parallel plate capacitor consists of two circular plates of area S with vacuum between them. It is connected to a battery of constant emf $\mathscr{E}$. The plates are then *slowly* oscillated so that they remain parallel but the separation d between them is varied as $d = d_0 + d_1 \sin \omega t$. Find the magnetic field **H** between the plates produced by the displacement current. Similarly, find **H** if the capacitor is first disconnected from the battery and then the plates are oscillated in the same manner.



The relationship between $\mathcal{E}$ *and* E *in a parallele plate capacitor is given by* $\mathcal{E} = Ed$

$$\mathbf{E} = \frac{\mathcal{E}}{d} = \frac{\mathcal{E}}{d_0 + d_1 sin\omega t}$$

$$\frac{\partial \mathbf{E}}{\partial t} = -\frac{\mathcal{E} d_1 \omega\ cos\omega t}{(d_0 + d_1 sin\omega t)^2}$$

Here, The $\square_e$ *suffix stands for* $\square_{enclosed}$

$$I_{de} = \int_{A_e} \vec{\boldsymbol{J}}_{de} \cdot d\vec{\mathbf{a}} = \int_{A_e} \epsilon \frac{\partial \mathbf{E}}{\partial t} \cdot d\vec{\mathbf{a}} = \epsilon \frac{\partial \mathbf{E}}{\partial t} \pi r^2 = \epsilon \left(-\frac{\mathcal{E} d_1 \omega\ cos\omega t}{(d_0 + d_1 sin\omega t)^2} \right) \pi r^2 = -\frac{\epsilon\, \mathcal{E} d_1 \pi r^2 \omega\ cos\omega t}{(d_0 + d_1 sin\omega t)^2}$$

$$I_{de} = -\frac{\epsilon\, \mathcal{E} d_1 \pi r^2 \omega\ cos\omega t}{(d_0 + d_1 sin\omega t)^2}$$

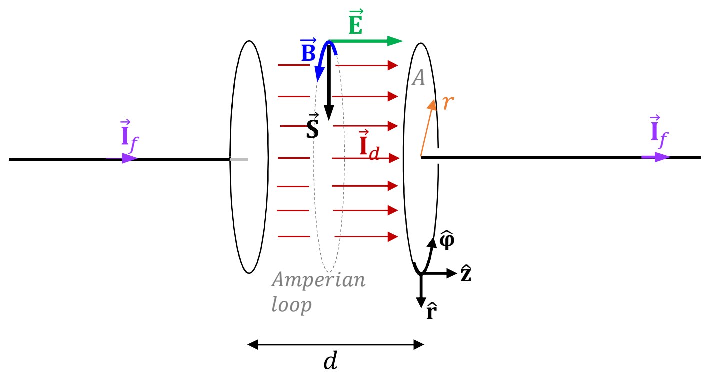
Magetic field around a current

$$\vec{\mathbf{H}} = \frac{I_e}{2\pi r} \hat{\boldsymbol{\varphi}} = \frac{I_{de}}{2\pi r} \hat{\boldsymbol{\varphi}} = \frac{\left(-\frac{\epsilon\, \mathcal{E} d_1 \pi r^2 \omega\ cos\omega t}{(d_0 + d_1 sin\omega t)^2} \right)}{2\pi r} \hat{\boldsymbol{\varphi}} = -\frac{\epsilon\, \mathcal{E} d_1 r \omega\ cos\omega t}{2(d_0 + d_1 sin\omega t)^2} \hat{\boldsymbol{\varphi}}$$

$$\boxed{\vec{\mathbf{H}} = -\frac{\epsilon\, \mathcal{E} d_1 r \omega\ cos\omega t}{2(d_0 + d_1 sin\omega t)^2} \hat{\boldsymbol{\varphi}}}$$

Physics 410 HW Chapter 21 RON RAHAV

21-9 Find the Poynting vector on the bounding surface of region 1 of Figure 21-3. Find the total rate at which energy is entering region 1 and then show that it equals the rate at which the energy of the capacitor is changing.



$$\vec{\mathbf{S}} = \tfrac{1}{\mu}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

In a parallel plater capacitor

$$\vec{\mathbf{E}} = \frac{\sigma}{\epsilon}\hat{\mathbf{z}} = \frac{q}{A\epsilon}\hat{\mathbf{z}} = \frac{q}{\pi r^2\epsilon}\hat{\mathbf{z}}$$

Magetic field around a current

$$\vec{\mathbf{B}} = \frac{\mu I_{enclosed}}{2\pi r}\hat{\boldsymbol{\varphi}} = \frac{\mu I_d}{2\pi r}\hat{\boldsymbol{\varphi}} = \frac{\mu I_f}{2\pi r}\hat{\boldsymbol{\varphi}}$$

in a charging parallel plate capacitor $\vec{\mathbf{I}}_d = \vec{\mathbf{I}}_f$
(see derivation at the end)

$$\vec{\mathbf{S}} = \tfrac{1}{\mu}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = \tfrac{1}{\mu}\frac{q}{\pi r^2\epsilon}\hat{\mathbf{z}} \times \frac{\mu I_f}{2\pi r}\hat{\boldsymbol{\varphi}} = \tfrac{1}{\mu}\frac{q}{\pi r^2\epsilon}\frac{\mu I_f}{2\pi r}(-\hat{\mathbf{r}}) = -\frac{qI_f}{2\pi^2 r^3\epsilon}\hat{\mathbf{r}}$$

$$\boxed{\vec{\mathbf{S}} - \frac{qI_f}{2\pi^2 r^3\epsilon}\hat{\mathbf{r}}}$$

$\hat{\mathbf{r}}$ *points* ***out*** *of the volume.*
So the negative sign means power is going ***in.***

Total Poynting vector power through the area is

$$W_P = \int_A \vec{\mathbf{S}} \cdot d\vec{\mathbf{a}} = \int_A -\frac{qI_f}{2\pi^2 r^3\epsilon}\hat{\mathbf{r}} \cdot da\hat{\mathbf{r}} = -\frac{qI_f}{2\pi^2 r^3\epsilon}\int_A da = -\frac{qI_f}{2\pi^2 r^3\epsilon}(2\pi r d) = \frac{qI_f d}{\pi r^2\epsilon}$$

$$\boxed{W_P = \frac{qI_f d}{\pi r^2\epsilon}}$$

Power of charging capacitor

$$W_C = \frac{dU_C}{dt} = \frac{d}{dt}\left(\frac{1}{2}C\phi^2\right) = \frac{d}{dt}\left(\frac{q^2}{2C}\right) = 2\left(\frac{q}{2C}\right)\frac{dq}{dt} = \frac{q}{C}I_f = \frac{I_f q}{C} = \frac{I_f q d}{\epsilon A} = \frac{I_f q d}{\epsilon\pi r^2}$$

Chain Rule $q = f(t)$; $C = \frac{\epsilon A}{d}$

$$\boxed{W_C = \frac{I_f q d}{\epsilon\pi r^2}}$$

$$\boxed{W_P = W_C}$$

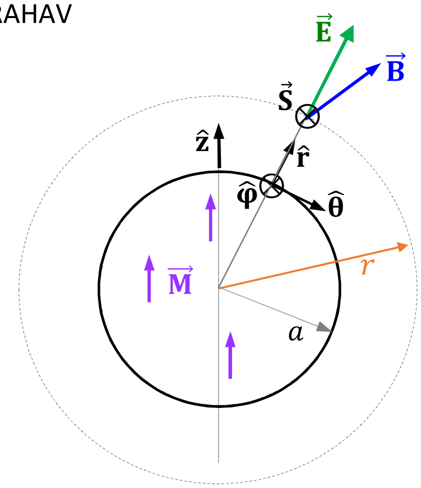
Derivation of $\vec{\mathbf{I}}_d = \vec{\mathbf{I}}_f$

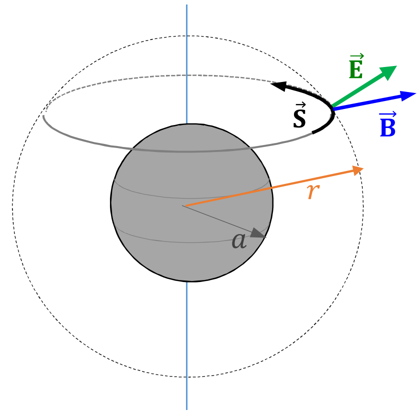
$$\vec{\mathbf{I}}_d = \int_A \vec{\boldsymbol{J}}_\mathrm{d} \cdot d\vec{\mathbf{a}} = \int_A \epsilon\frac{\partial\vec{\mathbf{E}}}{\partial t} \cdot d\vec{\mathbf{a}} = \epsilon\int_A \frac{\partial}{\partial t}\left(\frac{\sigma_f}{\epsilon}\right) \cdot d\vec{\mathbf{a}} = \epsilon\int_A \frac{\partial}{\partial t}\left(\frac{q_f}{A\epsilon}\right) \cdot d\vec{\mathbf{a}} = \epsilon\int_A \frac{\partial q_f}{\partial t}\left(\frac{1}{A\epsilon}\right) \cdot d\vec{\mathbf{a}} = \int_A \vec{\mathbf{I}}_f\left(\frac{1}{A}\right) \cdot d\vec{\mathbf{a}} = \vec{\mathbf{I}}_f\int_A \left(\frac{1}{A}\right) \cdot d\vec{\mathbf{a}} = \vec{\mathbf{I}}_f$$

Physics 410 HW Chapter 21 RON RAHAV

21-11 A conducting sphere of radius a is uniformly magnetized with a magnetization of magnitude M. It also has a net charge Q. ~~Find the total angular momentum of the electromagnetic field of this system.~~

Find the Poynting vector





I choose the z axis and $\hat{\mathbf{z}}$ to coincide with direction of $\vec{\mathbf{M}}$

Inside the sphere

$$\vec{\mathbf{E}} = 0$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = \frac{1}{\mu_0}\vec{\mathbf{0}} \times \vec{\mathbf{B}} = \mathbf{0}$$

$$\boxed{\vec{\mathbf{S}} = \mathbf{0}}$$

Outside the sphere

$$\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

The magnetic field everywhere outside the sphere corresponds to the total dipole moment of the sphere (*page 321 of the book*)

The total diploe moment of the sphere is

$$\vec{\mathbf{m}} = \frac{4}{3}\pi a^3\vec{\mathbf{M}} \qquad (\textit{Equation } 20\text{–}23 \textit{ in the book})$$

The magnetic field from a dipole moment is given by:

$$B_r = \left(\frac{\mu_0 \boldsymbol{m}}{4\pi}\right)\frac{2cos\theta}{r^3} \qquad B_\theta = \left(\frac{\mu_0 \boldsymbol{m}}{4\pi}\right)\frac{sin\theta}{r^3} \qquad (\textit{Equations } 19\text{–}24 \textit{ in the book})$$

$$\vec{\mathbf{B}} = \left(\frac{\mu_0 \boldsymbol{m}}{4\pi}\right)\frac{2cos\theta}{r^3}\hat{\mathbf{r}} + \left(\frac{\mu_0 \boldsymbol{m}}{4\pi}\right)\frac{sin\theta}{r^3}\hat{\boldsymbol{\theta}} = \left(\frac{\mu_0\left(\frac{4}{3}\pi a^3 M\right)}{4\pi}\right)\frac{2cos\theta}{r^3}\hat{\boldsymbol{r}} + \left(\frac{\mu_0\left(\frac{4}{3}\pi a^3 M\right)}{4\pi}\right)\frac{sin\theta}{r^3}\hat{\boldsymbol{\theta}}$$

$$\vec{\mathbf{B}} = \frac{2\mu_0 a^3 Mcos\theta}{3r^3}\hat{\boldsymbol{r}} + \frac{\mu_0 a^3 Msin\theta}{3r^3}\hat{\boldsymbol{\theta}}$$

Note that when θ=0 this reduces to $\vec{\mathbf{B}} = \frac{2\mu_0 Ma^3}{3z^3}\hat{\mathbf{z}}$ which is the book's 20 – 22. For B along the Z axis only.

$$\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \vec{\mathbf{B}} = \left(\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}\right) \times \left(\frac{2\mu_0 a^3 Mcos\theta}{3r^3}\hat{\boldsymbol{r}} + \frac{\mu_0 a^3 Msin\theta}{3r^3}\hat{\boldsymbol{\theta}}\right) = \frac{Q\mu_0 a^3 Msin\theta}{12\pi\epsilon_0 r^5}\hat{\boldsymbol{\varphi}}$$

$$\boxed{\vec{\mathbf{S}} = \frac{Q\mu_0 a^3 Msin\theta}{12\pi\epsilon_0 r^5}\hat{\boldsymbol{\varphi}}}$$

At the surface

At the surface $r = a$ so

$$\boxed{\vec{\mathbf{S}} = \frac{Q\mu_0 a^3 Msin\theta}{12\pi\epsilon_0 a^5}\hat{\boldsymbol{\varphi}} = \frac{Q\mu_0 Msin\theta}{12\pi\epsilon_0 a^2}\hat{\boldsymbol{\varphi}}}$$