31 Atomic Physics

Outline

31-3 Bohr's Model of the Hydrogen Atom

Bohr′s model of Hydrogen Atom

n = 1, 2, 3, …..

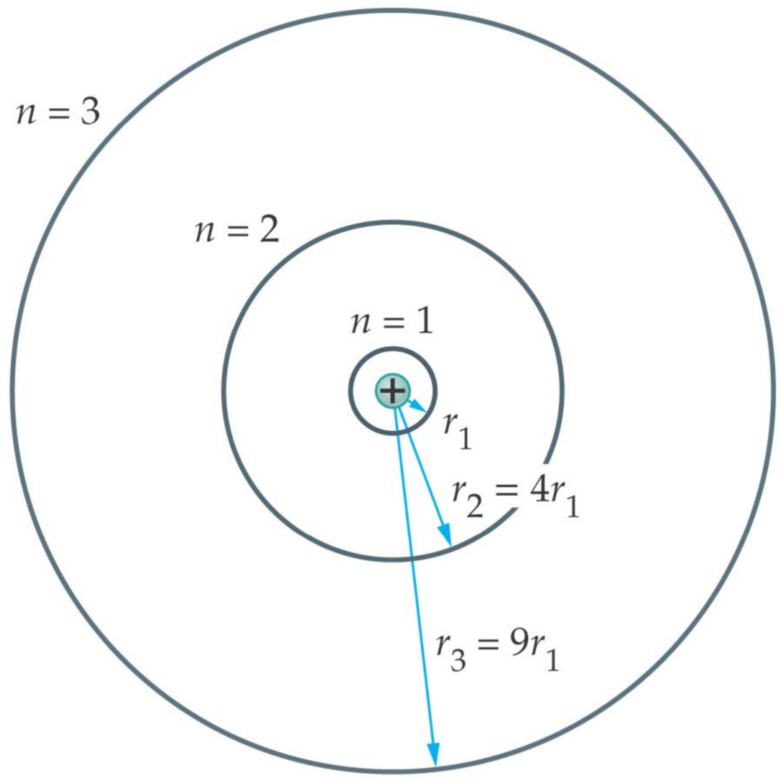

Figure 31-7. The First Three Bohr Orbits

31-3 Bohr's Model of the Hydrogen Atom

The spectrum of Hydrogen

How to create spectrum?

- Electron jumps from one orbit to the another: it gives up or absorbs photons (as a result of energy difference).
- Different orbit has different energy level.
- The photon wavelength is determined by hf.

Electron Energies include:

- Electric Potential energy.
- Kinetic energy.

31-3 Bohr's Model of the Hydrogen Atom

Assumptions of the Bohr model:

- The electron in a hydrogen atom moves in a circular orbit around the nucleus.
- Only certain orbits are allowed, where the angular momentum in the *n*th allowed orbit is

$$L_n = nh/2\pi$$

- Electrons in allowed orbits do not radiate photons. Radiation is emitted when an electron changes from one orbit to another, with frequency given by the energy difference:

$$|\Delta E| = hf$$

31-3 Bohr's Model of the Hydrogen Atom

In order for an electron to move in a circle of radius *r* at speed *v*, the electrostatic force must provide the required centripetal force. Hence: $(\frac{mv^2}{r} = k\frac{e^2}{r^2})$

$$mv^2 = k\frac{e^2}{r_n} \qquad 31-3$$

Adding the angular momentum requirement gives:

$$According \quad Eq.11-12 \quad L = rmv,$$

$$mr_n v_n = nh/2\pi,$$

$$v_n = \frac{nh}{2\pi m r_n}, \quad n = 1, 2, 3, \ldots \qquad 31-4$$

31-3 Bohr's Model of the Hydrogen Atom

Solving for r_n and v_n from these two equations, the allowed radii are:

$$r_n = \left(\frac{h^2}{4\pi^2 mke^2}\right)n^2, \quad n = 1, 2, 3, \ldots \qquad 31-5$$

The speeds at the allowed radii are:

$$v_n = \frac{2\pi ke^2}{nh}, \quad n = 1, 2, 3, \ldots \qquad 31-6$$

Example 1

Find the radius of the Hydrogen at the smallest orbit (Bohr Orbit).

Solution:

$$r_n = \left(\frac{h^2}{4\pi^2 mke^2}\right)n^2, \qquad with \quad n = 1$$

$$= \left(\frac{(6.626\times10^{-34})^2}{4\pi^2(9.109\times10^{-31}kg)(8.988\times10^{9}N.m^2/C^2)(1.602\times10^{-19})^2}\right)\times1^2$$

$$= 5.29\times10^{-11}m$$

Examples

Find the speed and kinetic energy of the electron in the Second Bohr orbit (n=2).

Solution:

$$v_n = \frac{2\pi k e^2}{nh}, \qquad with \quad n = 2$$

$$v_2 = \frac{2\pi(8.99\times10^9\, N\cdot m^2/C^2)(1.60\times10^{-19})^2}{2\times(6.63\times10^{-34}\, J.s)}$$

$$= 1.09\times10^{-6}\, m/s$$

1) Speed

$$v_n = \frac{2\pi k e^2}{nh}, \qquad with \qquad n = 2$$

$$v_2 = \frac{2\pi(8.99\times10^9\, N\cdot m^2/C^2)(1.60\times10^{-19})^2}{2\times(6.63\times10^{-34}\, J.s)}$$

$$= 1.09\times10^{-6}\, m/s$$

2) Kinetic energy

$$K_2 = \frac{1}{2} m v_2^2$$

$$= \frac{1}{2}(9.11\times10^{-31}\, kg)(1.09\times10^{6}\, m/s)^2 = 5.4\times10^{-19}\, J$$

31-3 Bohr's Model of the Hydrogen Atom

The total mechanical energy of an electron in a Bohr orbit is the sum of its kinetic and potential energies.

After some algebraic manipulation, and substituting known values of constants, we find for hydrogen atom:

$$E_n = -(13.6 \cdot eV)\frac{1}{n^2}, \quad n = 1, 2, 3, \ldots$$

1 eV = 1.60×10^{-19} Joule

The lowest energy is called the ground state. Most atoms at room temperature are in the ground state.

The Origin of Spectral Series in Hydrogen: Lyman Series

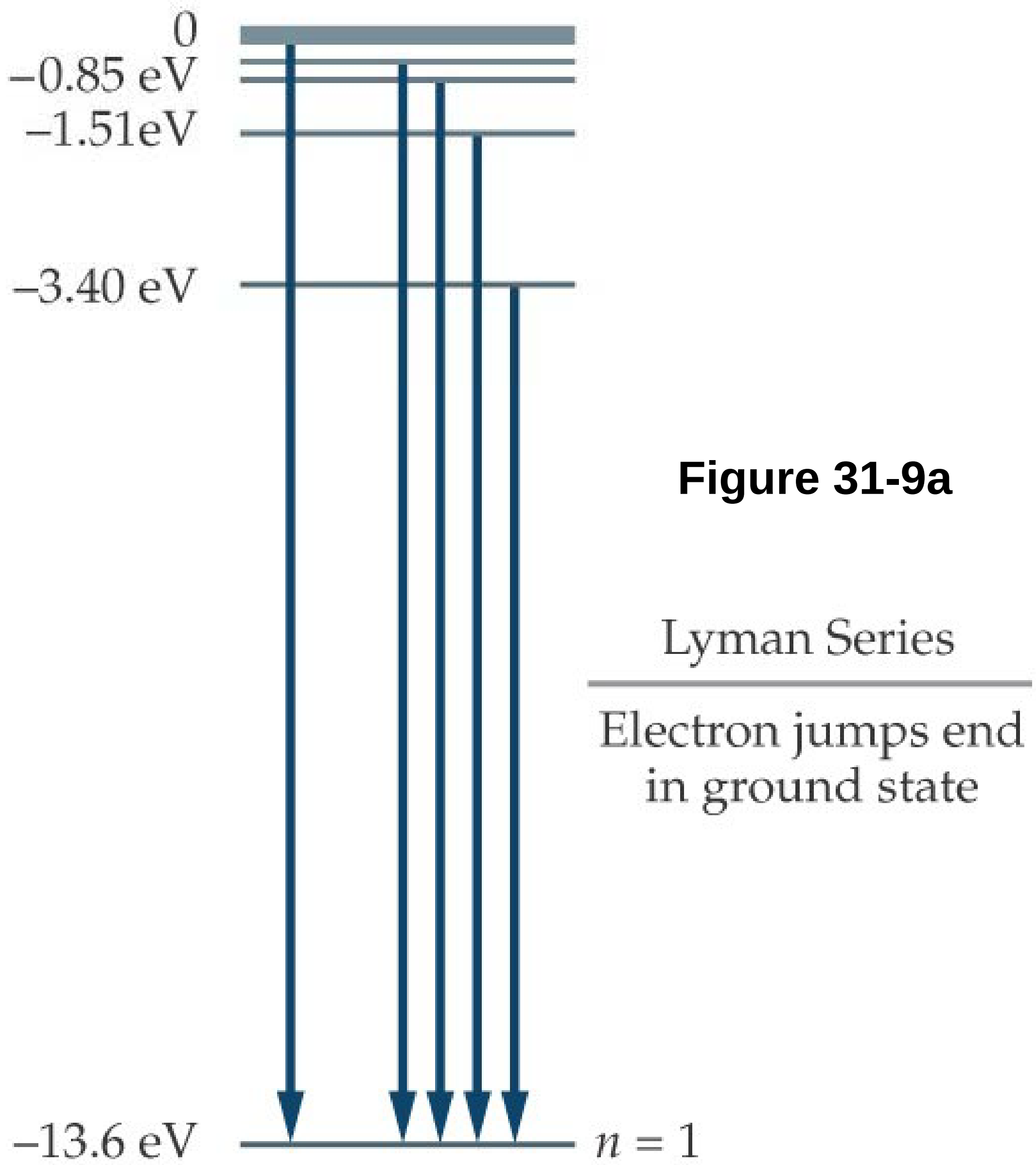

Figure 31-9a

The Origin of Spectral Series in Hydrogen: Balmer Series

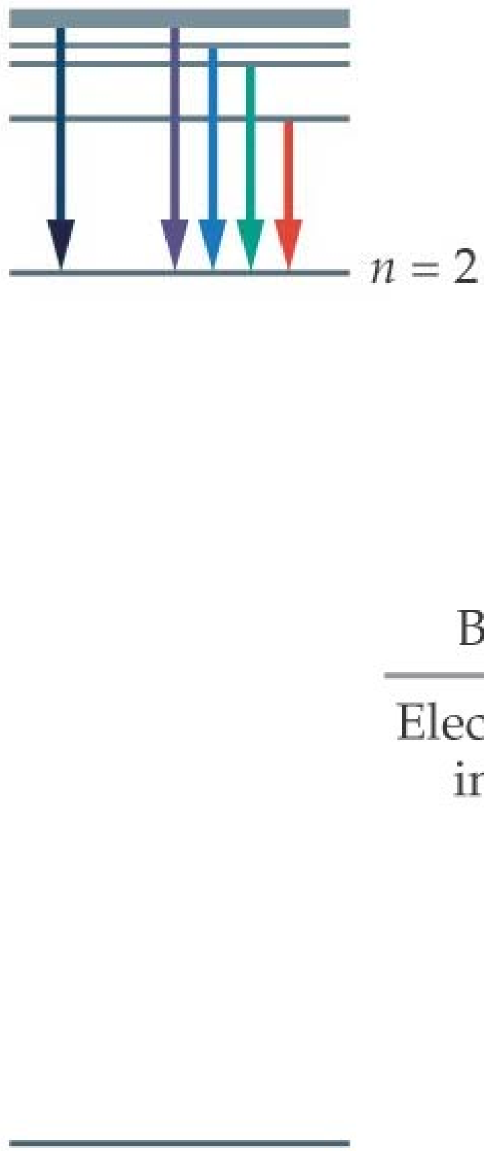

Figure 31-9b

Balmer Series
Electron jumps end in first excited state

The Origin of Spectral Series in Hydrogen: Paschen Series

Figure 31-9c

Paschen Series
Electron jumps end in second excited state

Active Exam 31-2 Absorbing a photon and what is the wavelength?

Find the wavelength of a photon that will be absorbed, if a electron is to raise in a hydrogen atom from the n=3 state to the n=5 state.

Solution:

1) The energy at n=5 state is

$$E_5 = -(13.6 \cdot eV)\frac{1}{n^2} = -(13.6 \cdot eV)\frac{1}{5^2} = -0.544 \quad eV$$

$$= -0.544 \times 1.6 \times 10^{-19} J = -0.87 \times 10^{-20} J$$

2) The energy at n=3 state is

$$E_3 == -(13.6 \cdot eV)\frac{1}{5^2} = -2.42 \times 10^{-19} J$$

3) The energy difference is

$$\Delta E_{5,3} = 1.55 \times 10^{-19} J$$

4) The wavelength is

$$hf = \Delta E_{5,3} = 1.55 \times 10^{-19} J \Rightarrow f = 2.34 \times 10^{14} Hz$$

$$f\lambda = c \Rightarrow \lambda = 1.28 \times 10^{-6} \quad m$$

Chapter Summary

The total mechanical energy of an electron in a Bohr orbit is the sum of its kinetic and potential energies. For Hydrogen atom, the total energy is

$$E_n = -(13.6 \cdot eV)\frac{1}{n^2}, \quad n = 1, 2, 3, \ldots$$