MATH 350 EXAM #3 Fall 2004

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are <u>not permitted</u>.

1. (20 points) Let I be an open interval that contains 0 and $f : I \longrightarrow \mathbf{R}$. If there exists an $\alpha > 1$ such that $|f(x)| \leq |x|^\alpha$ for all $x \in I$, prove that f is differentiable at 0. What happens when $\alpha = 1$?

2. (20 points) Evaluate the following limits

   a. $\lim\limits_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x$

   b. $\lim\limits_{x \to 0} \dfrac{\sin \frac{1}{x}}{e^{\frac{1}{x}} + 1}$

   c. $\lim\limits_{x \to \infty} x^2 \left(e^{\frac{1}{x}} - e^{\frac{1}{x+1}}\right)$

   d. $\lim\limits_{x \to a} \dfrac{x^x - a^x}{x - a}$; $a > 0$

3. (30 points)

   a. Show that
   $$1 + x \leq e^x \ ; \quad \forall x \in \mathbf{R}$$

   b. For $k \in [0\ ,\ 1)$, consider the following sequence
   $$u_n \ = \ (1+k)(1+k^2)\cdots(1+k^n) \ ; \quad \forall n \in \mathbf{N}$$
   Prove that (u_n) is a convergent sequence.

4. (30 points) Find all $\mathcal{C}^2$ functions $f : \mathbf{R} \longrightarrow \mathbf{R}$ such that
$$\exists\, \theta \in [0\ ,\ 1]\ ,\ \forall x \in \mathbf{R} \text{ and } \forall h \in \mathbf{R} \ : \quad f(x+h) \ = \ f(x) + hf^{'}(x + \theta h)$$

5. **(Bonus Question.)**(20 points) For $n \in \mathbf{N}$, find the n-th derivative of
$$f(x) \ = \ \frac{x^2+1}{(x+1)^3}$$