MATH 262 EXAM #3 Spring 2004

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (15 points) Find a basis for the space of all lower triangular 2×2 matrices.

2. (25 points) Consider V the subset of P_2 defined by

$$V = \left\{ p(t) : \int_0^1 p(t)\,dt = 0 \right\}$$

   a. Show that V is a subspace of P_2.

   b. Find a basis for V.

3. (30 points) Consider the linear transformation

$$T(M) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}^{-1} M \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

   from $\mathbf{U}^{2\times 2}$ to $\mathbf{U}^{2\times 2}$, where $\mathbf{U}^{2\times 2}$ is the space of upper triangular 2×2 matrices.

   a. Show that T is linear.

   b. Find the matrix of T with respect to the basis $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

4. (30 points) Consider the transformation $T(f(t)) = t(f'(t))$ from P_2 to P_2.

   a. Show that the transformation T is linear.

   b. Find the kernel and the nullity of the transformation T.

   c. Use part (b) to find the rank of the transformation T.

   d. Is the transformation T an isomorphism?