MATH 581 EXAM #2 Fall 2005

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section, and (5) a grading table. You must work all the problems on the exam. Show ALL your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are NOT permitted.

1. (20 points) Solve the following linear systems twice. First, use Gaussian elimination and give the factorization $A = LU$. Second, use Gaussian elimination with scaled row pivoting and determine the factorization of the form $PA = LU$

$$\begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \frac{1}{2} \end{bmatrix}$$

2. (30 points)

   a. Assume that A is an invertible matrix. Prove that A has an LU decomposition if and only if all principal minors of A are nonsingular.

   b. Prove that if A is a symmetric matrix whose leading principal minors are nonsingular, then A has a factorization LDL^T in which L is a unit lower triangular matrix and D is diagonal.

3. (20 points) Count the number of multiplications and/or divisions needed to invert a unit $n \times n$ lower triangular matrix.

4. (10 points) Consider the linear system

$$\begin{bmatrix} 2 & 0 & -1 \\ -2 & -10 & 0 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$$

   Using the starting vector $x^{(0)} = (0,0,0)^T$, carry out two iterations of the Jacobi method.

5. (20 points) Using Q as in the Gauss-Seidel method, prove that if A ($n \times n$ matrix) is strictly diagonally dominant, then $||I - Q^{-1}A||_\infty < 1$. Conclude!