ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, and (3) a grading table. You must work all the problems on the exam. Show ALL your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Find the polynomial $f(t)$ of degree 3 such that $f(1) = 1$, $f(2) = 5$, $f'(1) = 2$, and $f'(2) = 9$, where $f'(t)$ is the derivative of $f(t)$. Graph this polynomial.

2. (20 points) Consider the transformation T from $\mathbf{R}^2$ to $\mathbf{R}^3$ given by

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Is this transformation linear? If so, find its matrix.

3. (15 points) Consider the linear system

$$\begin{cases} x + y - z & = & -2 \\ 3x - 5y + 13z & = & 18 \\ x - 2y + 5z & = & k \end{cases}$$

where k is an arbitrary constant.

   i. For which value(s) of k, does this system have one or infinitely many solutions?
   ii. For each value of k you found in part (i), how many solutions does this system have?
   iii. Find all the solutions for each value of k.

4. (20 points) For which values of the constant k is the following matrix invertible?

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$$

Find the rank of the matrix A.

HEY, THERE'S MORE—TURN THE PAGE OVER!

5. (10 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

   a. The matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is in reduced row-echelon form (rref).

   b. A system of five equations in three unknowns is *always* inconsistent.

   c. If A is a 3×4 matrix and $\vec{v}$ is a vector in $\mathbf{R}^3$, then the product $A\vec{v}$ is a vector in $\mathbf{R}^3$.

   d. There is a 3×4 matrix with *rank* 4.

   f. The inverse of an $n \times m$ matrix is an $m \times n$ matrix.

   g. Let A and B be two invertible matrices. Then,

      i. $(A+B)^2 = A^2 + 2AB + B^2$
      ii. $(ABA^{-1})^3 = AB^3A^{-1}$
      iii. $ABA^{-1} = B$
      iv. $(A^{-1}B)^{-1} = B^{-1}A$
      v. $(A+B)^{-1} = A^{-1} + B^{-1}$

6. (15 points) If possible, compute the following matrix products.

   a. $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

   b. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

   c. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

   d. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

   e. $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$