MATH 262 EXAM #1 Spring 2004

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (30 points) Consider the linear system

$$\begin{cases} x + y - z & = & 2 \\ x + 2y + z & = & 3 \\ x + y + (k^2 - 5)z & = & k \end{cases}$$

   where k is an arbitrary constant.

   a. For which value(s) of k this system is *inconsistent*?
   b. For which value(s) of k does this system have one solution? Find the solution.
   c. For which value(s) of k does this system have infinitely many solutions? Find all the solutions.

2. (10 points) Consider two (nonzero) perpendicular vectors $\vec{u}$, and $\vec{w}$ in $\mathbf{R}^2$. Show that the transformation

$$T(\vec{x}) = \vec{x} + (\vec{u} \cdot \vec{x})\vec{w}$$

   is a shear parallel to the line L spanned by $\vec{w}$.

3. (15 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

   a. The matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is in reduced row-echelon form (rref).
   b. A system of four equations in three unknowns is *always* inconsistent.
   c. If A is a 3×4 matrix and $\vec{v}$ is a vector in $\mathbf{R}^4$, then the product $A\vec{v}$ is a vector in $\mathbf{R}^4$.
   d. There is a 3×4 matrix with *rank* 4.
   e. If $\vec{u}$, $\vec{v}$, and $\vec{w}$ are non zero vectors in $\mathbf{R}^2$, then $\vec{w}$ *must* be a linear combination of $\vec{u}$ and $\vec{v}$.

HEY, THERE'S MORE—TURN THE PAGE OVER!

4. (15 points) How many types of 2×3 matrices in reduced row-echelon form are there?

5. (30 points) Let L be the line in $\mathbf{R}^3$ that consists of all scalar multiples of $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, and T a transformation from $\mathbf{R}^3$ to $\mathbf{R}^3$ such that

$$T(\vec{x}) \;=\; 2(proj_L \vec{x}) - \vec{x}$$

where $proj_L \vec{x}$ is the orthogonal projection of $\vec{x}$ onto L.

a. Show that T is a linear transformation.

b. Find the matrix corresponding to T.

c. Use question (b) to find $T(\vec{v})$ for $\vec{v} \;=\; \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.