Homework 9

Due on Wed. Apr 10, 2019.

1. (Problem 2-7:1) Expand $1/\sin z$ in powers of z for $0 < |z| < \pi$, and also for $\pi < |z| < 2\pi$.

   Hint: Let a_k be the Laurent coefficients for the region $0 < |z| < \pi$ and b_k the Laurent coefficients for the region $\pi < |z| < 2\pi$. Set up the difference $b_k - a_k$ as a contour integral and use the Residue Theorem to compute it.

2. (Problem 2-7:5) Expand the function $\exp(t(z + z^{-1}))$ in a Laurent series around the origin of the z plane. Express coefficients as simple trigonometric integrals.

   Hint: In the definition of the Laurent coefficients use the unit circle $|z| = 1$ as the contour of integration. Simplify the expressions for the coefficients. (All of them are real since the function is real-valued for t and z in $\mathbb{R}$!)

3. (Problem 2-7:6) Obtain the principal part of the Laurent series for $z^{1/2}(1 + \sin z)^{-1}$ in an annual region centered on the point $-\pi/2$. Describe the character of each singularity of this function including that at the point at ∞. In what regions is it possible to represent the function as a sum of a Laurent series?

4. (Problem 2-7:8) Show that no limit point (including the point at ∞) of poles can be a pole. Discuss the behavior of $\csc(1/z)$ as $z \to 0$.