February 13, 2019 MATH 655 Prof. V. Panferov

# Homework 4

Due on Wed. Feb 20, 2019.

1. (Ahlfors 3.3.1:1) Prove that the reflection $z \mapsto \bar{z}$ is not a fractional linear transformation.

2. (Ahlfors 3.3.1:4) Show that any fractional linear transformation which transforms the real axis into itself can be written with real coefficients.

3. (Ahlfors 3.3.2:4) Show that any four distinct points in the complex plane can be carried by a fractional linear transformation to positions $1$, $-1$, $k$, $-k$, where the value $k$ depends on the points. How many solutions are there and how are they related?

4. Show that a composition of two *reflections* (according to definition on page 81 in Ahlfors) is a fractional linear transformation.

5. Verify the computations on p. 81 in Ahlfors to show that the points symmetric relative to a finite circle $C$ satisfy equation 11 and are indeed located as shown in Figure 3-2.

6. (Ahlfors 3.3.3:4) Find the fractional linear transformation which carries the circle $|z| = 2$ into $|z + 1| = 1$, the point $-2$ into the origin, and the origin into $i$.

7. (Ahlfors 3.3.3:8) Find a fractional linear transformation which carries $|z| = 1$ and $x = 2$ into concentric circles. What is the ratio of the radii?

8. (Problem 1-5:5) Can a fractional linear transformation be found that will map two non-intersecting circles into concentric circles? Describe how to construct such a transformation.

9. (Problem 1-5:11) Describe branch-points, branch lines and Riemann surfaces for $w = z^{1/3}$, $w = \ln z$ and $w = (z(z-1))^{1/2}$.

10. (Problem 1-5:12) Verify that a suitably defined branch of

$$f(z) = \ln\left(5 + \sqrt{\frac{z+1}{z-1}}\right)$$

is single-valued in the $z$ plane outside a line joining the points $z = 1$ and $z = -1$. Show, however, that is one enters another sheet of the Riemann surface by crossing this line, there will be a branch point at $z = 13/12$.