## Estimating integrals with Riemann sums using TI-84

**Problem** (#15 in Section 5.2)

Estimate the area under the curve $y = f(x)$ using a Riemann sum with 10 terms evaluated at right (left) endpoints:

$$f(x) = \cos x \quad \text{on} \quad [-\tfrac{\pi}{2}, 0].$$

**Solution:**

The right endpoint Riemann sum is

$$R_{10} = \sum_{i=1}^{10} f(x_i)\Delta x,$$

while the left endpoint sum is

$$L_{10} = \sum_{i=1}^{10} f(x_{i-1})\Delta x.$$

This corresponds to subdivision into 10 sub-intervals, so

$$\Delta x = \frac{0 - (-\pi/2)}{10} = \frac{\pi}{20},$$

and

$$x_i = -\frac{\pi}{2} + \frac{\pi}{20} i.$$

(Draw a diagram of the interval $[-\frac{\pi}{2}, 0]$ to visualize!)

To compute $R_{10}$ on TI-84 we follow these steps.

Enter the function $y = \cos x$: press `[Y=]` then enter `cos(X)` in the first line to obtain

$$\mathtt{Y_1} = \mathtt{cos(X)}$$

(return back by pressing `[2nd] [Quit]`). We then use functions `sum()` and `seq()` as follows:

$$\mathtt{sum(seq(Y_1(-\pi/2 + \pi/20 * X) * \pi/20, X, 1, 10, 1))}$$

The function `sum()` is available by pressing `[2nd] [List]`, choosing `[MATH]`, then `[5]` for `sum(`. To access function `seq()`, press `[2nd] [List]`, choose `[OPS]` then `[5]` for `seq(`. The order of arguments of the function `seq()` is as follows:

$$\texttt{Expression}: \ \texttt{Y}_1(-\pi/2+\pi/20*\texttt{X})*\pi/20; \ \texttt{Variable}: \ \texttt{X}; \ \texttt{Start}: \ 1; \ \texttt{End}: \ 10; \ \texttt{Step}\ 1.$$

The symbol for $\texttt{Y}_1$ is obtained by pressing `[Vars] -> [Y-Vars]` and choosing `[1]`. In the expression $\texttt{Y}_1(-\pi/2+\pi/20*\texttt{X})*\pi/20$ symbol `X` is used to denote the index $i$, which runs from 1 to 10 to produce the right endpoints. These values are then substituted in the function $\texttt{Y}_1$ and the result is multiplied by $\Delta x = \pi/20$. Notice that the negative sign in front of $\pi/2$ is produced by pressing `[(-)]`, not the subtraction button. After you're done with the input make sure the number of closing parentheses matches the number of opening ones. The command

$$\texttt{sum}(\texttt{seq}(\texttt{Y}_1(-\pi/2+\pi/20*\texttt{X})*\pi/20,\texttt{X},1,10,1)) \ \ [\texttt{ENTER}]$$

produces the answer 1.07648203.

To change the command for using left endpoints we use `[2nd] [EntrySolve]` (or `[Arrow Up]` works on some models) to get the previous input, then edit it, using `[<-]`, `[->]`, `[Del]` and `[2nd] [Ins]` as necessary. To change the calculation for using left endpoints we modify the input as follows:

$$\texttt{sum}(\texttt{seq}(\texttt{Y}_1(-\pi/2+\pi/20*\texttt{X})*\pi/20,\texttt{X},0,9,1)) \ \ [\texttt{ENTER}]$$

This produces the answer 0.91940317. Another possible modification to get the left endpoints is

$$\texttt{sum}(\texttt{seq}(\texttt{Y}_1(-\pi/2+\pi/20*(\texttt{X}-1))*\pi/20,\texttt{X},1,10,1)) \ \ [\texttt{ENTER}]$$

(same answer 0.91940317).

Optional: to test the method with midpoints of sub-intervals we could change the input to

$$\texttt{sum}(\texttt{seq}(\texttt{Y}_1(-\pi/2+\pi/20*(\texttt{X}-0.5))*\pi/20,\texttt{X},1,10,1)) \ \ [\texttt{ENTER}]$$

This produces a more accurate value for the approximate area 1.001028824.

The right endpoint sum is an overestimate since the function $y = \cos x$ is increasing on $[-\frac{\pi}{2}, 0]$; the left endpoint sum is an underestimate of the true area. The exact value of the area in this problem is 1, which can be checked by computing the integral

$$\int_{-\pi/2}^{0} \cos x \, dx$$

using the method of antiderivatives (the Fundamental Theorem of Calculus).