HOMEWORK – SECTION 4.1

#6. $y = \dfrac{1}{x-1} + x$

Asymptotes: Vertical : $x = 1$

Oblique : $y = x$ $\left(\text{since } \dfrac{1}{x-1} \to 0 \text{ as } x \to \pm\infty \right)$

Intercepts: $x = 0 \Rightarrow y = -1$

$y = 0 \Rightarrow \dfrac{1}{x-1} + x = 0 \Rightarrow 1 + x(x-1) =$

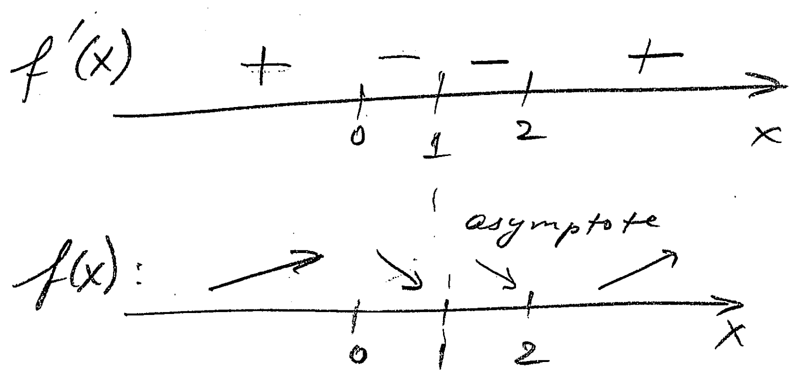
$x^2 - x + 1 = 0$ – has no real roots.

1st Derivative: $f'(x) = -\dfrac{1}{(x-1)^2} + 1$

$f'(x) = 0$ when $\dfrac{1}{(x-1)^2} = 1 \Rightarrow (x-1)^2 = 1$

$\Rightarrow x = 1 \pm 1$

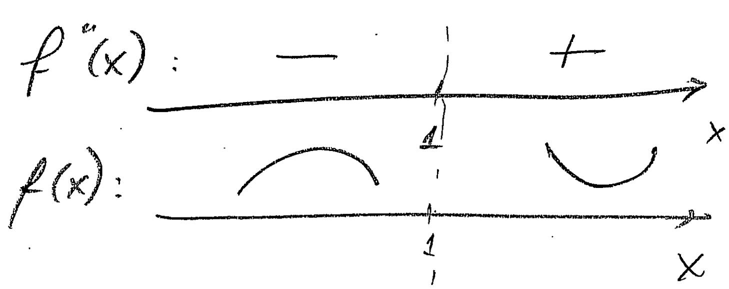
$\Rightarrow \underbrace{x = 0 \text{ or } x = 2}_{\text{critical points}}.$



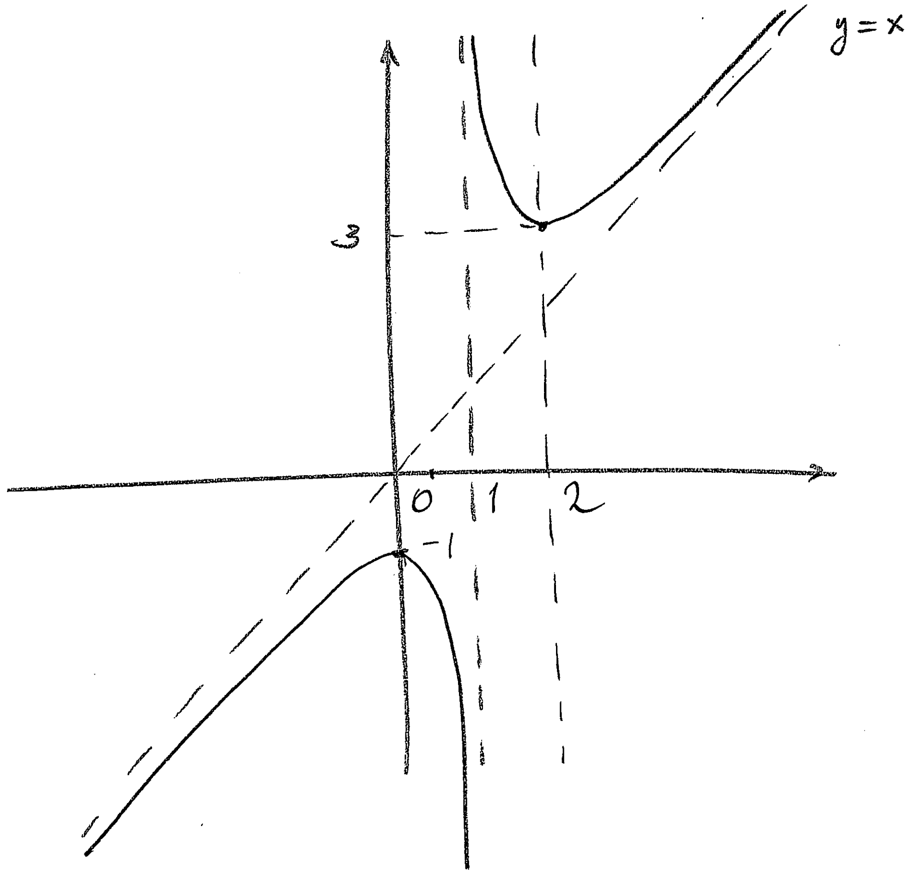
$f(0) = -1$

$f(2) = 3$

2nd Derivative: $f''(x) = \dfrac{2}{(x-1)^3} \neq 0$



Sketch a graph:



Key values: $f(0) = -1$, $f(2) = 3$

#8. $y = \frac{1}{3}x^3 - 9x + 2$

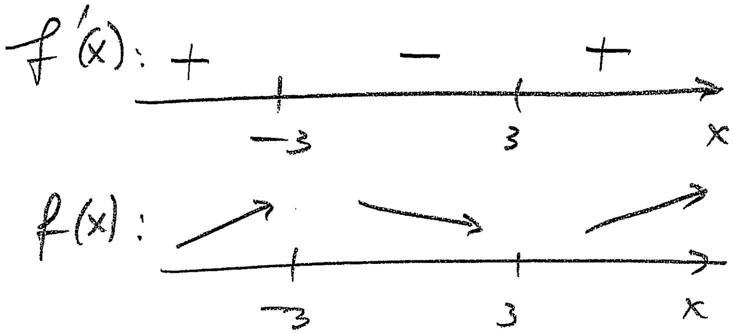
Asymptotes: none

Intercepts: $x = 0 \Rightarrow y = 2$

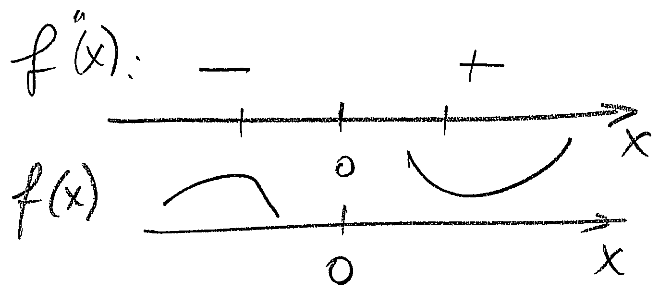
$y = 0$ not easily solved.

1^{st} Derivative: $f'(x) = x^2 - 9 = 0$

$\underbrace{x = \pm 3}_{\text{critical pts.}}$



2^{nd} Derivative: $f''(x) = 2x = 0$ when $x = 0$



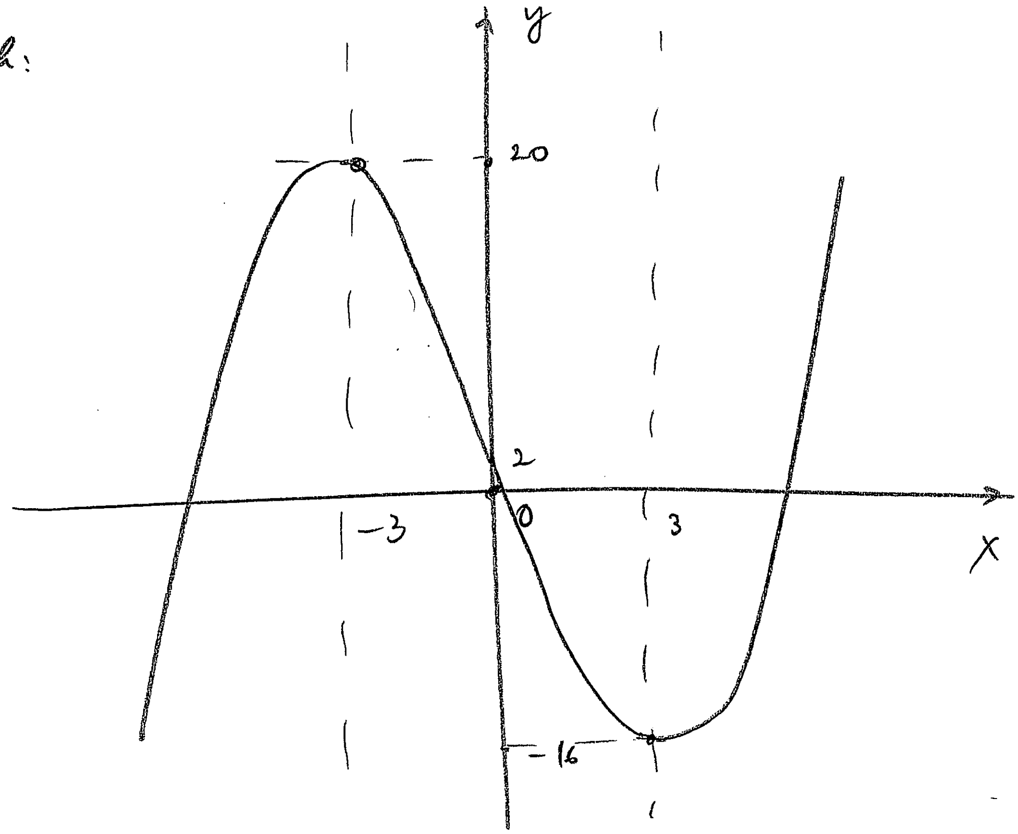
Key values: $f(0) = 2$ (intercept)

$f(-3) = -9 + 27 + 2 = 20$ (local max)

$f(3) = 9 - 27 + 2 = -16$

(local min)

Graph:



#12. $y = \dfrac{2+x}{1+x}$

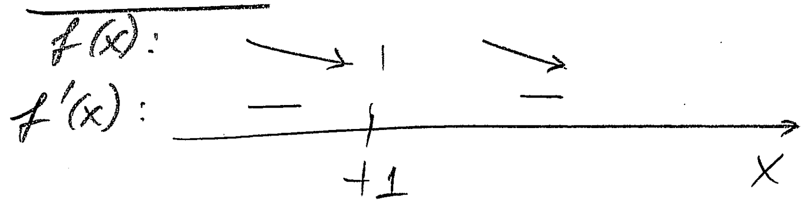
Asymptotes: Vertical: $x = -1$

Horizontal: $y = 1$ $\left(\lim\limits_{x \to \pm\infty} \dfrac{2+x}{1+x} = 1\right)$

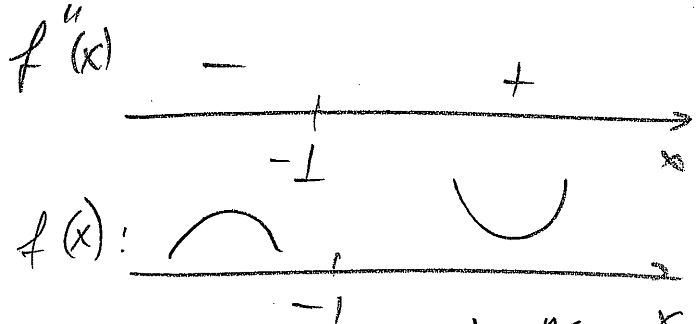
Intercepts: $x = 0 \Rightarrow y = 2$

$y = 0 \Rightarrow x = -2$

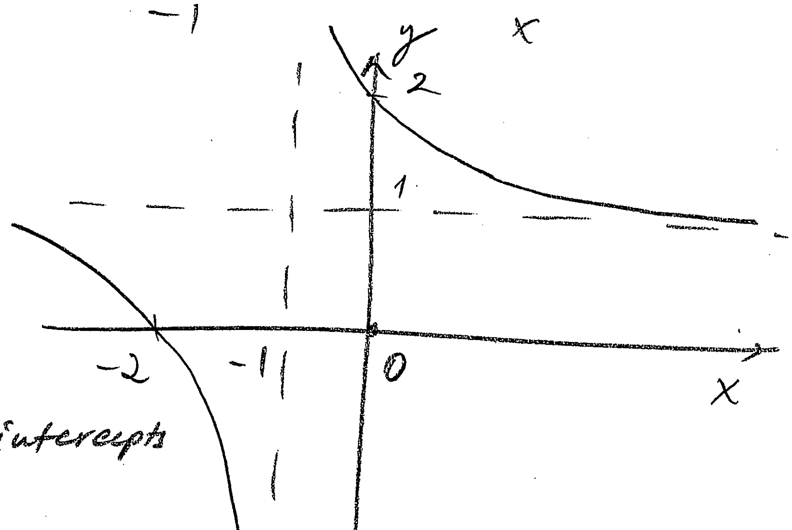
1st Derivative: $f'(x) = \dfrac{(1+x)-(2+x)}{(1+x)^2} = -\dfrac{1}{(1+x)^2}$



2nd Derivative: $f''(x) = \dfrac{2}{(1+x)^3}$



Graph.



Key values:

$\left.\begin{array}{l} f(0) = 2 \\ f(-2) = 0 \end{array}\right\}$ intercepts

Concentration of drug on bloodstream:

#35. $$C(t) = 23.725\left(e^{-0.5t} - e^{-0.7t}\right) \quad \left[\frac{mg}{ml}\right]$$

t – hours after taking the dose

(a) Asymptotes: horizontal:

$$y = 0 \quad \left(\lim_{t\to\infty} e^{-0.5t} = 0,\ \lim_{t\to\infty} e^{-0.7t} = 0\right)$$

vertical: none.

Intercepts: $C(0) = 0$

$$C'(t) = 23.725\left(-0.5e^{-0.5t} + 0.7e^{-0.7t}\right)$$

$C'(t) = 0$: $$0.5e^{-0.5t} = 0.7e^{-0.7t}$$

$$\frac{e^{-0.5t}}{e^{-0.7t}} = \frac{0.7}{0.5}$$

$$e^{0.2t} = 1.4$$

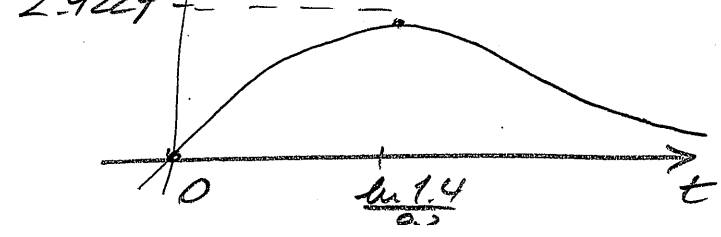
$$t = \frac{\ln 1.4}{0.2} = 1.6824$$

$C'(t)$: $+$ on $\left(0, \frac{\ln 1.4}{0.2}\right)$, $-$ after $\frac{\ln 1.4}{0.2}$

$C'(0) = 23.725 \cdot 0.2 > 0$

$C'(T) < 0$ for T large; > 0 since $\lim_{t\to\infty} C(t) = 0$.

Graph:



Key value: $C\left(\frac{\ln 1.4}{0.2}\right) \simeq 2.9229$ – global max.

(b) The concentration of drug in the bloodstream increases gradually after the drug is taken, it reaches a maximum of

$\approx 2.93 \ \frac{mg}{ml}$ at $t \simeq 1.68$ hr

after which the concentration decays toward zero.

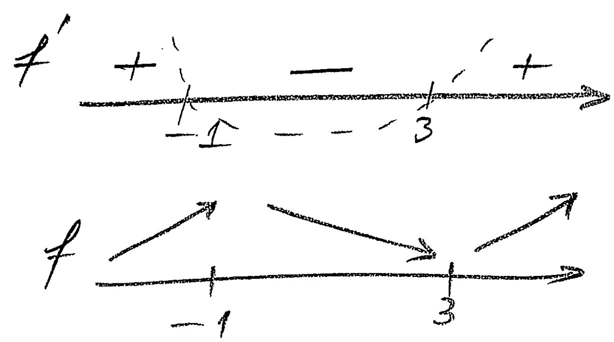
4.2.10

$$y = -3x - x^2 + \frac{x^3}{3} = f(x)$$

$$f'(x) = -3 - 2x + x^2 = (x-3)(x+1)$$

$$f'(x) = 0 \quad : \quad x = 3 \text{ or } x = -1$$

First derivative test:



(f' is an upward looking parabola)

$f(-1)$ is a local max

$f(3)$ is a local min

4.2.16

$$y = \frac{2x^2 - x^4}{4} = \frac{1}{2}x^2 - \frac{1}{4}x^4 = f(x)$$

$$f'(x) = x - x^3 = x(1-x^2) = x(1-x)(1+x)$$

$$f'(x) = 0 \quad : \quad x = 0,\ x = 1 \text{ or } x = -1$$

$$f''(x) = 1 - 3x^2$$

$f''(-1) = -2 < 0 \Rightarrow f(-1)$ is a local max

$f''(0) = 1 > 0 \Rightarrow f(0)$ is a local min

$f''(1) = -2 < 0 \Rightarrow f(1)$ is a local max.

4.2.20 Find global max/min on closed interval: 2

$$f(x) = xe^{-x} \quad \text{on} \quad [0, 100].$$

End points:

$$f(0) = 0; \quad f(100) = 100e^{-100} \approx 3.72 \cdot 10^{-42}$$

Critical points:

$$f'(x) = e^{-x} + x \cdot (-1) \cdot e^{-x}$$
$$= (1-x)\, e^{-x}$$
$$f'(x) = 0 \iff x = 1 \quad (e^{-x} \text{ is never } 0)$$

$$f(1) = e^{-1} \approx 0.36788$$

Compare the values of f:

$$0, \quad 0.368, \quad 3.72 \cdot 10^{-42}$$

Global minimum: $f(0) = 0$

Global maximum: $f(1) \approx 0.368$

4.2.22

Find global max / min of they exist:

$f(x) = x^3 - 12x + 2$ on $(0, \infty)$

End points:

$\lim_{x \to 0} f(x) = f(0) = 2$; $\lim_{x \to \infty} f(x) = \infty$

(polynomial of degree 3, leading term x^3)

Critical points:

$f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x-2)(x+2)$

$x = 2$ or $x = -2$ (not in the interval)

$f(2) = 8 - 24 + 2 = -14$

Compare the values of f and the limits:

$2, \ -14, \ \infty$

-14 is the least $\Rightarrow$ global minimum

$f(2) = -14$

there is no greatest value since $\lim_{x \to \infty} f(x) = \infty$

$\Rightarrow$ there is no global maximum.