#14. Find the derivative $g'(t)$:

$$g(t) = \frac{1+te^t}{1+t}$$

$$g'(t) = \left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} = \frac{(e^t+te^t)(1+t)-(1+te^t)\cdot 1}{(1+t)^2}$$

$$= \frac{e^t+te^t+\cancel{te^t}+t^2e^t-1-\cancel{te^t}}{(1+t)^2}$$

$$= \frac{(1+t+t^2)e^t-1}{(1+t)^2}$$

#20. Find the equation of the tangent line at x_0:

$$G(x) = (x-5)(x^3-x), \quad x_0 = -1$$

$$G'(x) = (x^3-x) + (x-5)(3x^2-1)$$

$$= x^3-x+3x^3-x-15x^2+5$$

$$= 4x^3-15x^2-2x+5$$

$$G'(-1) = -4-15+2+5 = -12$$

Tangent line:

$$y-y_0 = m(x-x_0)$$

$$y_0 = G(-1) = (-6)(-1+1) = 0$$

$$x_0 = -1, \quad m = -12$$

$$y = -12(x+1)$$

$$y = -12x-12.$$

#24. $g(x) = x \ln x \quad , \quad x_0 = 1$

$$g'(x) = (uv)' = u'v + uv' = \ln x + x \cdot \frac{1}{x}$$
$$= \ln x + 1$$

$$g'(1) = 0 + 1 = 1$$

Tangent line:

$$y - y_0 = m(x - x_0)$$

$$y_0 = g(1) = 1 \cdot \ln 1 = 0 \ ; \quad x_0 = 1, \quad m = 1$$

$$y = x - 1$$

#38. $$E(v) = \frac{0.074\,(v-35)^2 + 22}{v} \quad \left[\frac{cal}{km}\right]$$

$v \left[\frac{km}{h}\right]$.

(a) Find $E'(v)$ (inst. rate of change of the energy expenditure with respect to v)

$$E(v) = \frac{0.074\,(v^2 - 70v + 35^2) + 22}{v}$$
$$= 0.074\,v - 70 + \frac{0.074 \cdot 35^2 + 22}{v}$$
$$= 0.074\,v - 70 + \frac{112.65}{v}$$

$$E'(v) = 0.074 - \frac{112.65}{v^2}$$

(b) Find v such that $E'(v) = 0$

$$0.074 - \frac{112.65}{v^2} = 0$$

$$v^2 = \frac{112.65}{0.074} \Rightarrow v = \sqrt{\frac{112.65}{0.074}} \approx 39.02 \left[\frac{km}{h}\right]$$

HOMEWORK – SECTION 3.3

Find the derivatives:

#16.

$$y = xe^{-x^2}$$

$$\frac{dy}{dx} = (uv)' = u'v + uv' = 1\cdot e^{-x^2} + x\left(e^{-x^2}\right)'$$

$$\left(e^{-x^2}\right)' = \underbrace{\left(e^{-x^2}\right)}_{\text{outer}}\underbrace{(-2x)}_{\text{inner}}$$

$$\frac{dy}{dx} = e^{-x^2} - 2x^2e^{-x^2} = (1-2x^2)e^{-x^2}.$$

#18.

$$y = \sqrt{\frac{x^3-x}{4-x^2}}$$

$$\frac{dy}{dx} = \underbrace{\frac{1}{2\sqrt{\frac{x^3-x}{4-x^2}}}}_{\text{outer}} \cdot \underbrace{\left(\frac{x^3-x}{4-x^2}\right)'}_{\text{inner}} = \frac{1}{2}\sqrt{\frac{4-x^2}{x^3-x}} \cdot \left(\frac{x^3-x}{4-x^2}\right)'$$

$$\left(\frac{x^3-x}{4-x^2}\right)' = \left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} = \frac{(3x^2-1)(4-x^2)-(x^3-x)(-2x)}{(4-x^2)^2}$$

$$= \frac{12x^2-4-3x^4+x^2+2x^4-2x^2}{(4-x^2)^2}$$

$$= \frac{-x^4+11x^2-4}{(4-x^2)^2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{4-x^2}{x^3-x}}\,\frac{(-x^4+11x^2-4)}{(4-x^2)^2}$$

$$= \frac{1}{2}\,\frac{-x^4+11x^2-4}{\sqrt{x^3-x}\,(4-x^2)^{3/2}}$$

#22. Differentiate implicitly:

$$\frac{1}{y} + \frac{1}{x} = 1$$

$$\frac{d}{dx}\left(y^{-1} + x^{-1}\right) = \frac{d}{dx}\, 1$$

$$-y^{-2}y' - x^{-2} = 0$$

$$y' = -\frac{x^{-2}}{y^{-2}} = -\frac{y^2}{x^2}.$$

#24. $\ln(xy) = e^{2x}$

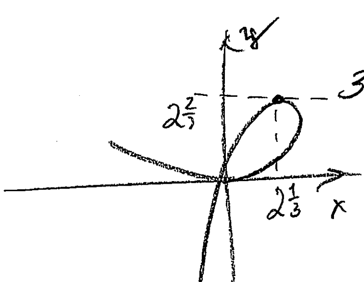
$$\frac{d}{dx}\ln(xy) = \frac{d}{dx}e^{2x}$$

$$\frac{1}{xy}\cdot\left(y + xy'\right) = 2e^{2x}$$

$$y + xy' = 2xye^{2x}$$

$$y' = \frac{2xye^{2x} - y}{x} = 2ye^{2x} - \frac{y}{x}.$$

#32. $x^3 + y^3 = 3xy$ Find points (x, y) such that $\frac{dy}{dx} = 0$.



$$3x^2 + 3y^2y' = 3y + 3xy'$$

$$\left(3y^2 - 3x\right)y' = 3y - 3x^2$$

$$y' = \frac{y - x^2}{y^2 - x} = 0 \quad \text{if} \quad y - x^2 = 0 \text{ but } y^2 - x \neq 0$$

Answer:
$(x, y) = (2^{\frac{1}{3}}, 2^{\frac{2}{3}})$.

$$y - x^2 = 0 \Rightarrow y = x^2 \Rightarrow \quad x^3 + x^6 = 3x \cdot x^2$$

$$x^6 = 2x^3 \Rightarrow \underbrace{x = 0}_{y' \text{ undefined}} \text{ or } \underbrace{x = \sqrt[3]{2}}_{\Rightarrow y = x^2 = 2^{2/3}}$$

#2:

$$g(x) = 2\sin x + \tan x$$

$$g'(x) = 2\cos x + (\tan x)'$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

$$g'(x) = 2\cos x + \frac{1}{\cos^2 x} \quad \text{or} \quad 2\cos x + 1 + \tan^2 x.$$

#10.

$$g(\theta) = \cos^2 \theta$$

$$g'(\theta) = \underbrace{2\cos\theta}_{\text{outer}} \cdot \underbrace{(\cos\theta)'}_{\text{inner}} = -2\cos\theta\sin\theta$$

$$= -\sin(2\theta)$$

#18.

$$y = \sin(2t^3+1)$$

$$\frac{dy}{dt} = \underbrace{\cos(2t^3+1)}_{\text{outer}} \cdot \underbrace{(2t^3+1)'}_{\text{inner}} = 6t\cos(2t^3+1)$$

#26.

$$y = \frac{\sqrt{\sin x}}{\cot x};$$

$$(\sin x)' = \cos x$$

$$(\cot x)' = \left(\frac{\cos x}{\sin x}\right)'$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x}$$

$$\frac{dy}{dx} = \frac{(\sqrt{\sin x})'\cot x - \sqrt{\sin x}\,(\cot x)'}{\cot^2 x}$$

$$= \frac{\dfrac{\cos x \cdot \cot x}{2\sqrt{\sin x}} + \dfrac{\sqrt{\sin x}}{\sin^2 x}}{\cot^2 x}$$

$$= \frac{\cos x}{2\sqrt{\sin x}\,\cot x} + \frac{1}{\sqrt{\sin x}\,\sin x \cdot \cot^2 x}$$

$$= \frac{1}{\sqrt{\sin x}\,\sin x}\left(\frac{1}{2} + \frac{1}{\cot^2 x}\right)$$

$$= \frac{1}{\sqrt{\sin x}\,\sin x}\left(\frac{1}{2} + \tan^2 x\right).$$