MATH 255A

HOMEWORK — SECTION 2.6

#6. Use the definition of the derivative to find

$$f'(2), \qquad f(x) = \frac{1}{x+1}.$$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\left(\frac{1}{(2+h)+1} - \frac{1}{2+1}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{1}{3+h} - \frac{1}{3}\right)}{h} = \lim_{h\to 0} \frac{\left(\frac{3-(3+h)}{(3+h)3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{-h}{h(3+h)\cdot 3} = \lim_{h\to 0} \frac{-1}{(3+h)3} = -\frac{1}{9}.$$

#16. Find the tangent line at $x=2$:

$$f(x) = \frac{1}{x+1} \;;\quad x_0 = 2 ;\; y_0 = f(x_0) = \frac{1}{3}$$

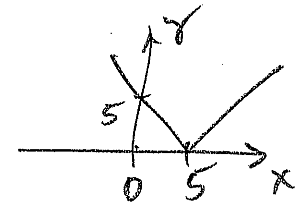
$$(y - y_0) = m(x - x_0) \;;\quad m = f'(x_0) = f'(2) = -\frac{1}{9}$$

$$y - \frac{1}{3} = -\frac{1}{9}(x-2)$$

$$y = -\frac{1}{9}x + \frac{2}{9} + \frac{1}{3} = -\frac{1}{9}x + \frac{5}{9}$$

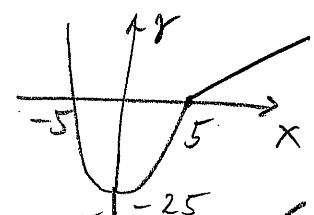
#28. An example of a function continuous on $(-\infty, \infty)$, but not differentiable at $x=5$.

$$f(x) = |x-5|.$$



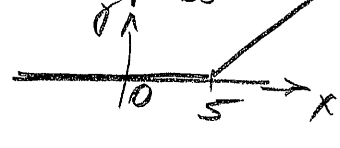
OR

$$f(x) = \begin{cases} x^2 - 25, & x \le 5 \\ 2(x-5), & x > 5 \end{cases}$$



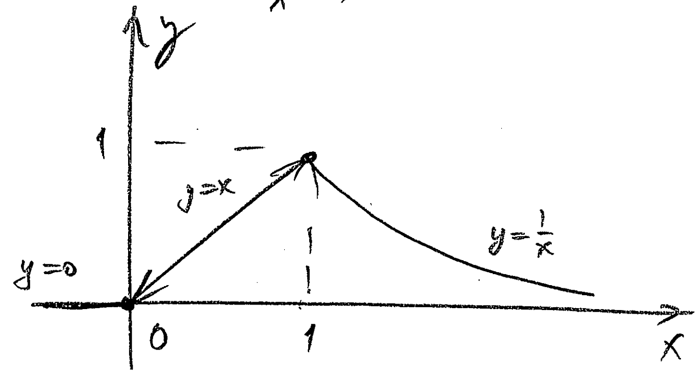
OR

$$f(x) = \begin{cases} 0, & x < 5 \\ x-5, & x \ge 5 \end{cases}$$



#30.

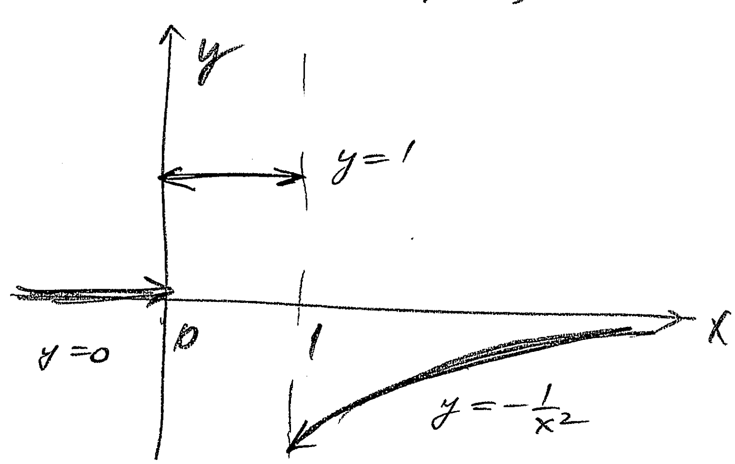
$$f(x) = \begin{cases} 0, & x \le 0 \\ x, & 0 < x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$$



Continuous everywhere (right and left limits exist and match at $x=0, 1$)

Not differentiable at $x=0$ or $x=1$
(right and left slopes are different)

$$f'(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 < x < 1 \\ -\frac{1}{x^2}, & x > 1 \end{cases}$$



$f'(x)$ undefined at $x=0$, $x=1$.

#6. $f(x) = x^3 - x$; Use the definition of the derivative to find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{\left((x+h)^3 - (x+h)\right) - (x^3 - x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x} - h - \cancel{x^3} + \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{(3x^2-1)h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \to 0} (3x^2 - 1) + 3xh + h^2$$

$$= 3x^2 - 1.$$

#14. $f'(2) = \left.\frac{dy}{dx}\right|_{x=2} = \left. 3x^2 - 1 \right|_{x=2} = 3 \cdot 4 - 1 = 11.$

#22. At what point is the instantaneous rate of change equal the average rate of change?

$f(x) = \frac{1}{2x}$; $I = [1, 4]$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{2(x+h)} - \frac{1}{2x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} - \cancel{2x} - 2h}{2(x+h)\,2 \cdot x \cdot h}$$

$$= \frac{1}{2} \lim_{h \to 0} \frac{-h}{(x+h) \cdot x \cdot h}$$

$$= -\frac{1}{2x^2}$$

Solve: $-\frac{1}{2x^2} = \frac{1/8 - 1/2}{3}$

$-\frac{1}{2x^2} = -\frac{1}{8}$

$x^2 = 4$

$x = \pm 2$

$x = 2$ is in the interval $(1, 4)$.

MATH 255A HOMEWORK - SECTION 3.1

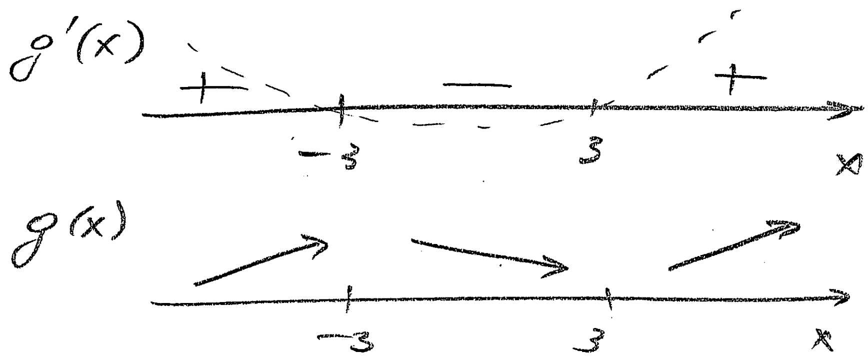
#16. $g(x) = \frac{1}{3}x^3 - 9x + 2$

On what intervals is $g(x)$ increasing (decreasing)?

[Find $g'(x)$. Solve $g'(x) = 0$. Solve $g'(x) > 0$ or < 0]

$$g'(x) = \frac{1}{3} \cdot 3 \cdot x^2 - 9 = x^2 - 9$$

$$g'(x) = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3.$$



Increasing : $(-\infty, -3)$ or $(3, \infty)$

Decreasing : $(-3, 3)$

#22. $g(x) = x^2(x^3 - 3x)$

$$g(x) = x^5 - 3x^3$$

$$g'(x) = (x^5)' - 3(x^3)' = 5x^4 - 3 \cdot 3 \cdot x^2$$

$$= 5x^4 - 9x^2.$$

#24. $f(x) = (e^x - 1)(e^x + 1)$

$$f(x) = (e^x)^2 - 1^2 = e^{2x} - 1$$

$$f'(x) = (e^{2x})' - (1)' = 2e^{2x} - 0 = 2e^{2x}.$$

#35

L(cm) – length of a pumpkin
W(cm) – width of a pumpkin.

$$L = 1.12\, W^{0.95}$$

How rapidly is length changing with regard to width when $W = 5$ cm ?
vs. when $W = 50$ cm ?

$$\frac{dL}{dW} = 1.12 \cdot 0.95\, W^{-0.05}$$
$$= 1.064\, W^{-0.05}$$

$$\left.\frac{dL}{dW}\right|_{W=5} = 0.9817$$

$$\left.\frac{dL}{dW}\right|_{W=50} = 0.8750$$

For a small pumpkin (5cm) the length is changing at essentially same rate as width ($\frac{dL}{dW} = 0.98$)

For a larger pumkin (50cm)
the growth is more assymetric, and the length is growing at a ~ 12.5% (1−0.875) slower rate.