MATH 255A

HOMEWORK – SECTION 2.1

#4. Average Rate of Change

$$f(x) = -2x^2 + x + 4 \quad \text{on } [1, 4]$$

$$\frac{f(b)-f(a)}{b-a} = \frac{(-2\cdot 4^2 + 4 + 4) - (-2\cdot 1^2 + 1 + 4)}{4-1}$$

$$= \frac{-30+3}{3} = -9.$$

#10. Approximate the inst. rate of change.

$$f(x) = -2x^2 + x + 4 \quad \text{at } x = 4$$

$$f'(4) = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h}$$

h	$4+h$	$\frac{f(4+h)-f(4)}{h}$
0.01	4.01	−15.02
−0.01	3.99	−14.98
0.001	4.001	−15.002
−0.001	3.999	−14.998
0.0001	4.0001	−15.0002
−0.001	3.9999	−14.9998

Estimate

$$f'(4) \simeq -15$$

#28. Estimate the tangent by evaluating secant slopes

$$y = \sin\frac{\pi x}{2}, \quad a = 1$$

h	$1+h$	$\frac{\sin\frac{\pi}{2}(1+h) - \sin\frac{\pi}{2}}{h}$
0.01	1.01	−0.0123
−0.01	0.99	0.0123
0.001	1.001	−0.0012
−0.001	0.999	0.0012
0.0001	1.0001	−0.0001
−0.0001	0.9999	0.0001

$$y - y_0 = m(x - x_0)$$

$$x_0 = a = 1$$

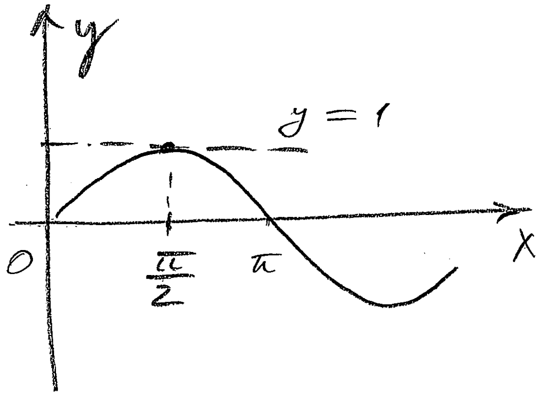
$$y_0 = f(a) = \sin\frac{\pi}{2} = 1$$

$$m = f'(a)$$

Estimate $f'(a) \simeq 0$

Tangent line:

$y = 1$ (horiz. line)



Tangent line:

$y = 1$.

#38. Determine the tangent line by evaluating $f'(a)$ algebraically.

$y = x^2$ at $a = -1$

$$y - y_0 = m(x - x_0);$$

$x_0 = a = -1$
$y_0 = f(a) = 1$
$m = f'(a) = f'(-1)$

$$f'(-1) = \lim_{h \to 0} \frac{(-1+h)^2 - (-1)^2}{h}$$

$$= \lim_{h \to 0} \frac{1 - 2h + h^2 - 1}{h} = \lim_{h \to 0} -2 + h$$

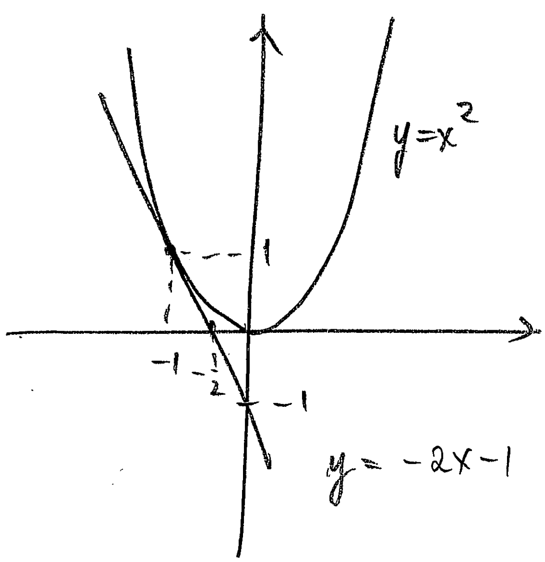
$$= -2.$$

Tangent line:

$$y - 1 = -2(x + 1)$$

$$y = 1 - 2x - 2$$

$$y = -2x - 1$$



MATH 255A

HOMEWORK - SECTION 2.2

#14. Approximate the limit using a table:

$$\lim_{x \to 2^-} g(x); \quad g(x) = \frac{x^3 - 8}{x^2 + 2x + 4}$$

x	1.0	1.5	1.9	1.99	1.999	1.9999
y	−1	−0.5	−0.1	−0.01	−0.001	−0.0001

y-Values appear to approach 0

Estimate: $\lim_{x \to 2^-} g(x) = 0$.

#21. Determine the limit:

$$\lim_{x \to 2} \frac{\sqrt{x+2} - 2}{x - 2} = \lim_{x \to 2} \frac{(\sqrt{x+2} - 2)(\sqrt{x+2} + 2)}{(x-2)(\sqrt{x+2} + 2)}$$

$$= \lim_{x \to 2} \frac{(x+2) - 4}{(x-2)(\sqrt{x+2} + 2)} = \lim_{x \to 2} \frac{x-2}{(x-2)(\sqrt{x+2} + 2)}$$

$$= \lim_{x \to 2} \frac{1}{\sqrt{x+2} + 2} = \left. \frac{1}{\sqrt{x+2} + 2} \right|_{x=2} = \frac{1}{2+2} = \frac{1}{4}.$$

#24.
$$\lim_{x \to 4} \frac{(x-4)^2}{|x-4|} = \lim_{x \to 4} \frac{|x-4|^2}{|x-4|} = \lim_{x \to 4} |x-4|$$

($(x-4)^2$ same as $|x-4|^2$) $\quad = \left. |x-4| \right|_{x=4} = 0.$

HOMEWORK - SECTION 2.3

#4: Determine the limits $\lim_{x\to a-} f(x)$, $\lim_{x\to a+} f(x)$, $\lim_{x\to a} f(x)$ if they exist.

$$f(x) = \frac{x^2}{|x|} \quad ; \quad a = 0$$

$$f(x) = \frac{|x|^2}{|x|} = |x| \quad (x \neq 0)$$

$$\lim_{x\to 0-} f(x) = \lim_{x\to 0-} |x| = 0$$

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} |x| = 0$$

$\lim_{x\to 0} f(x) = 0$ since both of the limits above exist and are $= 0$.

#10. Find the limit:

$$\lim_{t\to 0} \frac{\sqrt{4-t^2}-2}{t^2} = \lim_{t\to 0} \frac{(\sqrt{4-t^2}-2)(\sqrt{4-t^2}+2)}{t^2(\sqrt{4-t^2}+2)}$$

$$= \lim_{t\to 0} \frac{4-t^2-4}{t^2(\sqrt{4-t^2}+2)} = \lim_{t\to 0} \frac{-t^2}{t^2(\sqrt{4-t^2}+2)}$$

$$= \lim_{t\to 0} \frac{-1}{\sqrt{4-t^2}+2} = \left.\frac{-1}{\sqrt{4-t^2}+2}\right|_{t=0} = \frac{-1}{2+2} = -\frac{1}{4}$$

#20. Find the value that needs to be assigned to $f(2)$, to guarantee that f will be continuous at $x=2$.

$$f(x) = \sqrt{\frac{x^2-4}{x-2}} = \sqrt{\frac{(x-2)(x+2)}{x-2}}$$

$$= \sqrt{x+2}\ , \quad x \neq 2$$

$$\lim_{x\to 2} \sqrt{x+2} = \sqrt{x+2}\Big|_{x=2} = \sqrt{4} = 2$$

So if we define

$$\tilde{f}(x) = \begin{cases} \sqrt{\dfrac{x^2-4}{x-2}}\ , & x \neq 2 \\ 2\ , & x = 2 \end{cases}$$

then $\tilde{f}(x)$ is continuous at $x=2$.

#26. Use IVT to <u>prove</u> that the equation

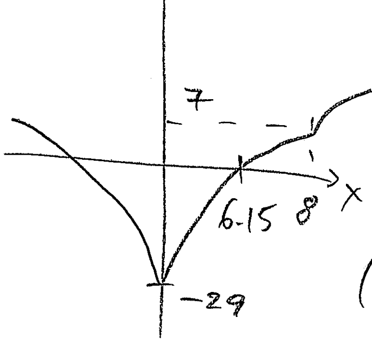
$$\sqrt[3]{x-8} + 9x^{2/3} = 29$$

Set $f(x) = (x-8)^{1/3} + 9x^{2/3} - 29$

$f(x)$ is large and positive as $x \to \pm\infty$ (because of $9x^{2/3}$-term.

$f(0) = (-8)^{1/3} - 29 = -2 - 29 = -31 < 0$

$f(8) = 9\cdot 2^2 - 29 = 7 > 0$



Therefore there must be at least one c in $(0,8)$ for which $f(c)=0$

(i.e. $\sqrt[3]{x-8} + 9x^{2/3} = 29$ for $x=c$.)

Calculator: $x \simeq 6.1548652$

#28. Use IVT to prove that the equation

$$1+\sin x + x^3 = 0$$ has a solution

x^3 large positive when x large positive

x^3 large negative when x large negative

$0 \le 1+\sin x \le 2$.

when $x=0$: $f(x) = 1+\sin x + x^3 = 1 > 0$

try $x<0$:

$x = -\frac{\pi}{2} \Rightarrow f\left(-\frac{\pi}{2}\right) = 1-1+\left(-\frac{\pi}{2}\right)^3 = -\left(\frac{\pi}{2}\right)^3 < 0$

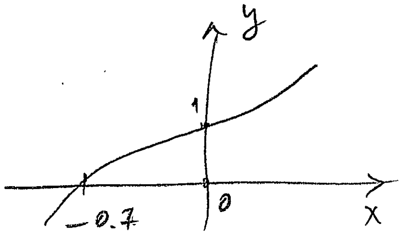
$f(x) = 1+\sin x + x^3$ is continuous

(elementary function: polynomial + trig.)

Therefore there must be at least one value c in $\left(-\frac{\pi}{2}, 0\right)$

such that $f(c)=0$.

$\Rightarrow 1+\sin c + c^3 = 0$.



verify by graphing: $x \approx -0.705694$.

#30. $\lim\limits_{x\to 1} (x-1)\sin\frac{1}{x-1}$

$$-1 \le \sin\frac{1}{x-1} \le 1$$

$$-|x-1| \le (x-1)\sin\frac{1}{x-1} \le |x-1|$$

$\lim\limits_{x\to 1}(-|x-1|) = 0$; $\lim\limits_{x\to 1}(|x-1|) = 0$

Therefore $\lim\limits_{x\to 1} (x-1)\sin\frac{1}{x-1}$ exists and is $= 0$.