

HOMEWORK PROBLEMS - SECTION 1.7

①

#20.

Find  $a_5$ :

$$a_{n+1} = 2a_n(1-a_n); \quad a_1 = 1$$

$n$	1	2	3	4	5
$a_n$	1	0	0	0	0

#24:

Find equilibria, sketch cobwebbing diagrams

$$a_{n+1} = f(a_n); \quad a_1 = 3$$

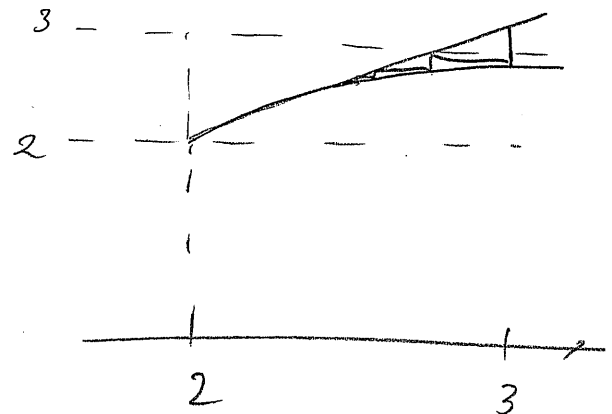
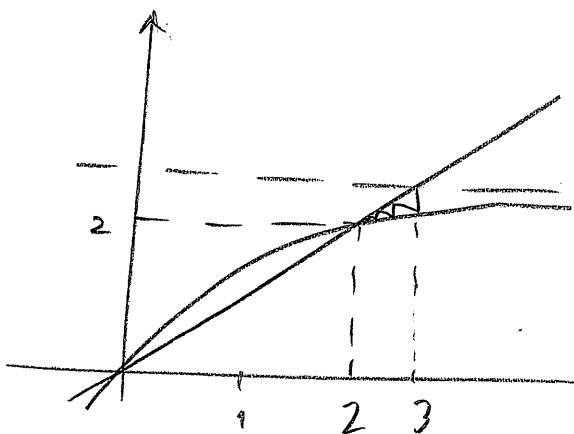
$$f(x) = \frac{3x}{1+x}$$

Equilibria:  $x = \frac{3x}{1+x}$  :  $x=0$  or

$$1 = \frac{3}{1+x}$$

$$1+x = 3$$

$$x = 2$$



$a_n$  is decreasing;  $a_n \rightarrow 2$  as  $n \rightarrow \infty$

#33: (a)

$$a_1 = 0$$

$$a_2 = (1-c)A$$

$$a_3 = (1-c)^2 A + (1-c)A$$

⋮

$$a_{n+1} = (1-c)a_n + (1-c)A$$

(b) Equilibrium:

$$a = (1-c)a + (1-c)A$$

$$ca = (1-c)A$$

$$a = \frac{1-c}{c} A$$

(c)

$$\frac{1-c}{c} A > A$$

$$\frac{1-c}{c} > 1$$

$$1-c > c$$

$$1 > 2c$$

$$c < \frac{1}{2}$$

If  $c < 0.5$   
(50%)

then the  
equilibrium  
is

$\frac{1-c}{c} A$  is  
more than  $A$ .

#26.

Equilibria:

2

$$x = \frac{x}{1+x}$$

$$(1+x)x = 1$$

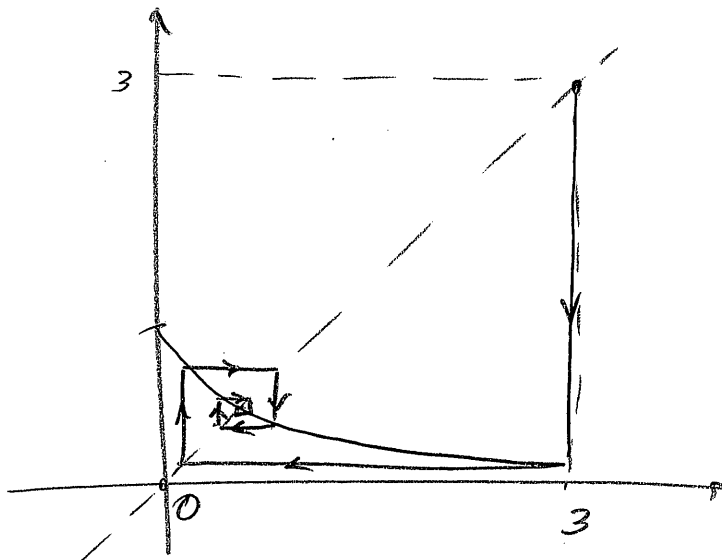
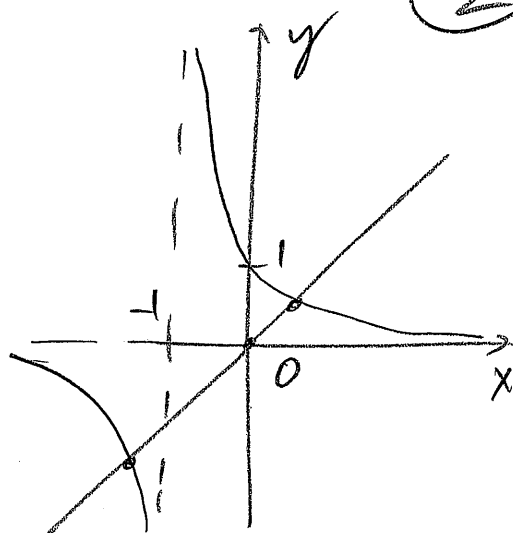
$$x^2 + x = 1$$

$$x^2 + x + \frac{1}{4} = \frac{5}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{5}{4}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{5}}{2} = -1.618, 0.618$$

$$a_{n+1} = f(a_n); \quad a_1 = 3$$



The sequence is zigzagging about equilibrium  $a = -\frac{1}{2} + \frac{\sqrt{5}}{2}$

$$a_n \rightarrow -\frac{1}{2} + \frac{\sqrt{5}}{2}, \quad n \rightarrow \infty.$$