

**Midterm 1: Study Guide****Textbook coverage:**

- 1.1:** Real Numbers and Functions. Interval notation, domains, ranges, intervals of increase and decrease. Examples 2, 3, 7.
- 1.2:** Data Fitting with Linear and Periodic Functions; Examples 2-6.
- 1.3:** Power functions and scaling laws. Equations of lines, linear regression, trigonometric functions, transformations. Proportionality notation. Geometric similarity. Examples 1-5.
- 1.4:** Exponential functions. Exponential growth and decay. Doubling time and half-life. Compound interest. Solving exponential equations with logarithms. Examples 1, 2, 5, 6.
- 1.5:** Function building. Transformations of graphs, compositions. Examples 1, 2, 4, 5, 7, 8.
- 1.6:** Inverse functions and logarithms; Examples 3, 4, 6-12.

**Review Questions**

1. (see also 1.1: 13-18) Find the domains of the functions and compute the indicated values (or state that the corresponding values are not in the domain)

(a)  $f(x) = (1 - 3x)^{-1/3}$ ,  $f(0)$ ,  $f(\frac{1}{3})$ ,  $f(3)$ .

(b)  $f(x) = \begin{cases} 3x + 5, & x < 2 \\ 3x^2 - 1, & x > 2 \end{cases}$ ,  $f(1)$ ,  $f(2)$ ,  $f(3)$ .

Sketch the graphs and determine the ranges of the functions.

2. (see also 1.1: 44-47) *Friend's rule* is a method for calculating pediatric drug dosages in terms of child's age (up to  $12\frac{1}{2}$  years). If  $A$  is the adult dose (in mg) and  $n$  is the age of the child (in years) then the child's dose is given by

$$D(n) = \frac{2}{25}nA.$$

- (a) What is the domain of the function defined by  $(n, D)$ ?
- (b) Graph this function for  $A = 600$  mg.
- (c) If a 3-year-old child receives 150 mg of a certain drug, what is the corresponding dose for a 7 year old? What is the adult dose?
3. (see also 1.2: 1-6, 11-18)

- (a) Find a linear function ( $y = mx + b$ ) that passes through the points  $(-1, 4)$  and  $(1, 3)$ .
- (b) Find an exponential function ( $y = ab^x$ ) that passes through the same points.
- (c) Find an equation for a straight line through the points  $(-1, 4)$  and  $(-1, 3)$ . Does this equation represent a function  $y = f(x)$ ? Does it represent a function  $x = g(y)$ ?
4. (see also 1.2: 47) The following Table presents the data on the average carbon dioxide ( $\text{CO}_2$ ) level in the atmosphere, measured in parts per million (ppm), at Mauna Loa Observatory from 2004 to 2012.

Year	$\text{CO}_2$ level (in ppm)
2004	377.5
2006	381.9
2008	385.6
2010	389.9
2012	393.8

- (a) Use the linear regression method to find the best-fitting data line. Plot the line and the points.
- (b) Use the linear equation obtained in part (a) to obtain a prediction for the  $\text{CO}_2$  level at Mauna Loa Observatory in 2016.
5. (see also 1.3: 11-16, 34, 35) If  $x \propto y^6$  and  $y \propto z^{1/3}$  then how does  $z$  change as  $x$  decreases from 16 to 4?
6. (see also 1.3: 25-27) Consider a cylinder of radius  $r$  and height  $5r$ . Express the volume  $V$  and the surface area  $S$  of this cylinder as a function of  $r$ . If  $r$  is doubled, what happens to  $V$ ? If  $V$  is decreased by a factor of 64, what happens to  $S$ ?
7. (see also 1.3: 36-38) A sample based on nineteen mountain ash trees of different sizes yielded a relationship between the leaf area,  $A$  [ $\text{m}^2$ ], of the tree, and the stem diameter at breast height (DBH),  $d$ , [cm]. The relationship obtained was  $A \propto d^{2.89}$ . If one of the points that this relationship passed through was  $(d, A) = (30, 78)$ , find the equation and sketch the graph.
8. (see also 1.4: 31, 32) Assume \$1,000 is invested for  $t = 10$  years at a rate  $r = 0.08$  compounded (a) annually (b) daily (c) continuously. Calculate the future value. (Round your answers to two significant digits.)

9. (see also 1.4: 35, 36) The height of beer froth is modeled by the function  $H(t) = 17(0.94)^{t/15}$  where  $t$  is measured in seconds. Estimate (accurate to 1 second) at what time the froth is (a) one half of its original height (b) one tenth of its original height.
10. (see also 1.4: 33, 39-42, 1.5: 37, 38) If a bacterial population initially has 20 individuals and doubles every 9.3 hours then how many individuals will it have after three days?
11. (see also 1.5: 34, 35; 1.2: 31-36, 47, 48) The tides at La Selva Beach, California, on Saturday September 17, 2016 are given by the following table (actual data have been approximated for simplicity of calculation):

Time	Height (ft)	Tide
11:30am	5.41	High
5:30pm	0.69	Low
11:30pm	5.41	High

Let

$$T(t) = A \cos(B(t - C)) + D$$

denote the height of the tide  $t$  hours after 12:00 noon.

- (a) Find values of  $A$ ,  $B$ ,  $C$ , and  $D$  such that the function fits the La Selva Beach data.
- (b) Sketch the function over the interval  $[-12, 12]$ .
12. (see also 1.6: 15-30; 1.3: 1-10) Write the expressions in terms of base  $e$  and simplify where possible:

(a)  $5^{2x}$

(c)  $\log_2(x^2 - 2)$

(b)  $3^{x^2}$

(d)  $\log_{10}(ex + e)$

Note:  $3^{x^2}$  is usually interpreted as  $3^{(x^2)}$ , while  $(3^x)^2$  is the same as  $3^{2x}$ .

13. Which one is greater:  $\log_5 127$  or  $\log_{10} 999$ ? Answer without using calculator!
14. Solve the equations:
- (a)  $\log_5 x + \log_5(x - 2) = 1$ .
- (b)  $e^{x^2 - 4x + 5} = e$ .
- (c)  $e^{-x} = \frac{x}{2} + 1$ . [Solve graphically, do not use calculator!]
- (d)  $e^{-x} = \frac{x}{2}$ . [Use equation solving function on a graphing calculator, show work!]

15. (see also 1.6: 9-14) Find the inverses of the functions. State the domain and the range of the inverse.

- (a)  $y = x/(x + 1)$ ; (c)  $y = \exp(-x^2)$  on  $[0, \infty)$ .  
 (b)  $y = 0.1 e^{3x-5}$ ;

Note:  $\exp(x)$  is another notation for  $e^x$ .

16. (see also 1.6: 31-34)

- (a) Sketch the indicated points on the logarithmic scale (use base 10): 0.01, 0.2, 10, 25,000.  
 (b) Sketch a log-log graph of the function  $y = \frac{10}{x^{1/2}}$  over the interval  $[0.01, 100]$ . On the vertical axis, mark the values that correspond to  $x = 10^{-2}, 10^0, 10^2$ .

17. (1.6: 44, 46) From one individual to another within the same species of mammal it is found that the brain volume  $V$  varies with the body weight  $W$  according to the power law  $V = aW^b$ . The following data are obtained from measurements of a number of adult chimpanzees. Show, using a log-log plot that the power law fits these data quite well, and obtain the values of  $a$  and  $b$ :

$W$ , kg	31	36	38	42	47	48	53
$V$ , cm <sup>3</sup>	365	380	382	397	410	415	427

### Answers:

- (a) Domain:  $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$ . Values: 1, undefined,  $-\frac{1}{2}$ ;  
 (b) Domain  $(-\infty, 2) \cup (2, \infty)$ . Values: 8, undefined, 26.
- (a) Domain:  $[0, 12.5]$ . Note: algebraically, the domain is  $\mathbb{R}$  (i.e.  $(-\infty, \infty)$ ). However, it's meaningless to apply the formula for  $n$  outside the interval  $[0, 12.5]$ , therefore the answer above is correct.  
 (b) The graph is a straight line that raises from 0 to 600 [mg] in the interval from 0 to 12.5 years.  
 (c) 350 mg; 625 mg.
- (a)  $y = -0.5x + 3.5$ .  
 (b)  $y = \sqrt{12} \left(\frac{3}{4}\right)^{t/2}$  ( $a = \sqrt{12}$ ,  $b = \sqrt{\frac{3}{4}}$ ).

- (c)  $x = -1$ . Not a function  $y = f(x)$  (the vertical line test fails). Works OK as a function  $x = g(y)$  ( $g(y)$  is a constant  $-1$ ).
4. (a)  $y = 2.03x - 3690.5$ .  
(b) 401.98.
5.  $x \propto z^2$ ,  $x$  is decreased by a factor of 4, therefore  $z$  is decreased by a factor of 2.
6. If  $r$  is doubled,  $V$  increases by a factor of 8. If  $V$  is decreased by a factor of 64 then  $S$  is decreased by a factor of 16.
7.  $A = 0.0042d^{2.89}$ ; the graph has appearance similar to the cubic parabola  $y = x^3$ , passes through (30, 78).
8. \$2,158.92, \$2,225.35, \$2,225.54.
9. 168 seconds; 558 seconds.
10. 4,281.
11.  $A = 2.36$ ;  $B = \frac{\pi}{6}$ ,  $C = -0.5$ ,  $D = 3.05$ . (Hint for the graph: plot the points of the data and match with a graph of cosine with a horizontal shift, vertical stretching and a vertical shift.)
12. (a)  $e^{2x \ln 5}$ ; (b)  $e^{x^2 \ln 3}$ ; (c)  $\ln(x^2 - 2)/\ln(2)$ ; (d)  $(1 + \ln(x + 1))/\ln(10)$ .
13.  $\log_5 127 > \log_{10} 999$ .
14. (a)  $1 + \sqrt{6}$ ; (b) 2; (c) 0; (d)  $x \approx 0.852606$ .
15. (a)  $y = \frac{x}{1-x}$  (note that the inverse is expressed in the form  $y = f(x)$ ); (b)  $y = \frac{1}{3}(5 + \ln(10x))$ ; (c)  $y = \sqrt{-\ln x}$  on  $(0,1]$ .
16. (b) The graph is a straight line  $Y = -\frac{1}{2}X + 1$  that passes through the points  $(-2, 2)$ ,  $(0, 1)$ ,  $(2, 0)$ . The values marked on the  $y$  axis are 100, 10, 1.
17.  $a = 130.62$ ,  $b = 0.298$ .