Tue, Sep. 17, 2019 **MATH 255A Midterm 1, Version 2** Prof. V. Panferov

Name: (print) Solutions.

CSUN ID No. : ______________________

This test includes 7 questions (46 points in total), on 7 pages. Page 8 is a formula sheet. The duration of the test is 1 hour 15 minutes.

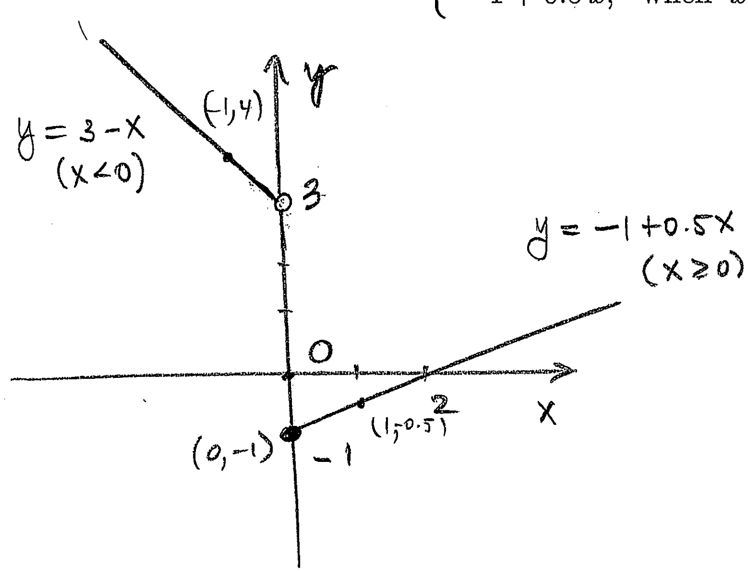
Your scores: (do not enter answers here)

1	2	3	4	5	6	7	total

Important: One single-sided page of notes is allowed. No electronic devices except an approved model of graphing calculator. All cellphones must be off and put away completely for the duration of the exam. Show all your work.

1. (6 points) (a) Find the domain and the range of the function. Sketch a graph:

$$f(x) = \begin{cases} 3 - x, & \text{when } x < 0 \\ -1 + 0.5\,x, & \text{when } x \geq 0. \end{cases}$$



Domain: $(-\infty, 0) \cup [0, \infty) = (-\infty, \infty)$ (all of $\mathbb{R}$)

Range: $[-1, \infty)$

(b) For the above function, find the values $f(-1)$, $f(0)$, $f(1)$ if defined.

$f(-1) = 3 - (-1) = 4$

$f(0) = -1 + 0.5 \cdot 0 = -1$

$f(1) = -1 + 0.5 = -0.5$

2. (8 points) (a) Suppose a patient had x [mg] of drug in his/her bloodstream 24 hours ago and just took an oral dose of 30 [mg] of the same drug. It is estimated that 72% of drug is removed from the body every 24 hours. Write a formula for the amount of drug in patient's body 24 hours from now.

$[100\% - 72\% = 28\% = 0.28]$

24 hours ago: x

now: $0.28x + 30$

24 hours after: $0.28(0.28x + 30)$

$= 0.0784x + 8.4$

(b) The initial amount of \$1,000 is invested at the annual rate 8%. Calculate the future value after 4 years assuming interest is compounded (i) monthly; (ii) continuously.

(i) $\underset{(\text{monthly})}{A(4)} = 1{,}000 \cdot \left(1 + \frac{0.08}{12}\right)^{12 \cdot 4} = 1{,}375.67$

(ii) $\underset{(\text{cont})}{A(4)} = 1{,}000 \cdot e^{0.08 \cdot 4} = 1{,}000 \cdot e^{0.32}$

$= 1{,}377.13$

3. (6 points) (a) Are the following power functions? If yes, write them in the form $y = ax^p$. If no, justify your answer.

(i) $y = \dfrac{\sqrt{49x^3}}{x^2}$ (ii) $y = \dfrac{x}{5} + \dfrac{5}{x}$.

YES: $y = \dfrac{\sqrt{49}\sqrt{x^3}}{x^2} = \dfrac{7x^{\frac{3}{2}}}{x^2} = 7x^{\frac{3}{2}}x^{-2} = 7x^{-\frac{1}{2}}$

$a = 7,\ p = -\frac{1}{2}$.

No: $y = 0.2 \cdot x^1 + 5x^{-1}$

$= \dfrac{0.2x^2 + 5}{x} \neq ax^p$

[For otherwise $0.2x^2 + 5 = ax^{p+1}$

plug in $x = 0$:

$5 = ax^{p+1}\Big|_{x=0}$

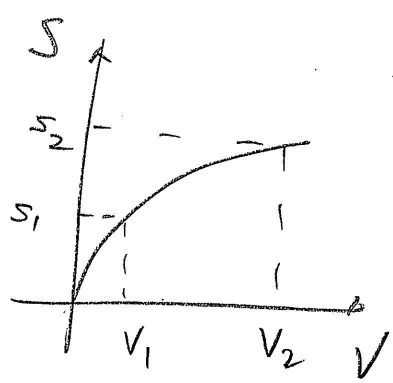
impossible.]

sum of different powers cannot be combined into a single power.

(b) If $V \propto L^3$ and $S \propto L^2$, how is S related to V? [Find a relation of the form $S \propto V^p$.] If S increases 9-fold, what happens to V?

$V \propto L^3 \Rightarrow L \propto V^{\frac{1}{3}}$

$S \propto L^2 \Rightarrow S \propto \left(V^{\frac{1}{3}}\right)^2 = V^{\frac{2}{3}}$



$\dfrac{S_2}{S_1} = 9$ (increases 9-fold)

$V \propto S^{\frac{3}{2}}$

$\dfrac{V_2}{V_1} = \dfrac{cS_2^{\frac{3}{2}}}{cS_1^{\frac{3}{2}}} = \left(\dfrac{S_2}{S_1}\right)^{\frac{3}{2}} = 9^{\frac{3}{2}} = 3^3 = 27$

V increases 27-fold.

4. (8 points)

(a) Solve the equation using any method. Show work:

$$140(0.82)^t = 70.$$

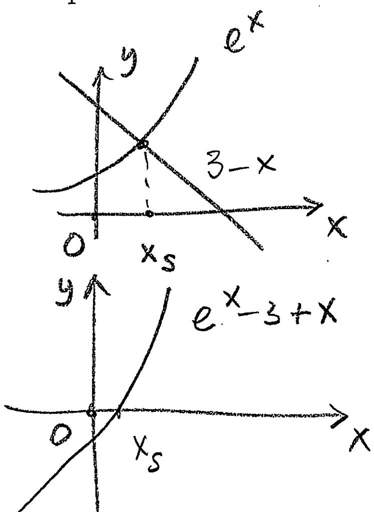
$$(0.82)^t = \frac{70}{140} = \frac{1}{2}$$

$$\ln (0.82)^t = \ln \frac{1}{2}$$

$$t \ln 0.82 = \ln \frac{1}{2}$$

$$t = \frac{\ln \frac{1}{2}}{\ln 0.82} \simeq \underline{3.4928}$$

(b) Solve the equation using a graphing calculator. Give answer accurate to two decimal places. Show work:



$$e^x = 3 - x.$$

Find intersection point

$\boxed{\text{CALC}}$ → 5: INTERSECT

Or solve $e^X - 3 + X = 0$!

$Y_1 = e^X - 3 + X$ $\boxed{\text{GRAPH}}$

$\boxed{\text{CALC}}$ → 2: zero

$X = \underline{0.79206}$

(c) Simplify using properties of logarithms. Show work without referring to graphing calculator.

$$\ln e^4 + 5\ln 1 - \ln e^{256}.$$

$$4 \ln e + 5 \ln 1 - 256 \ln e$$

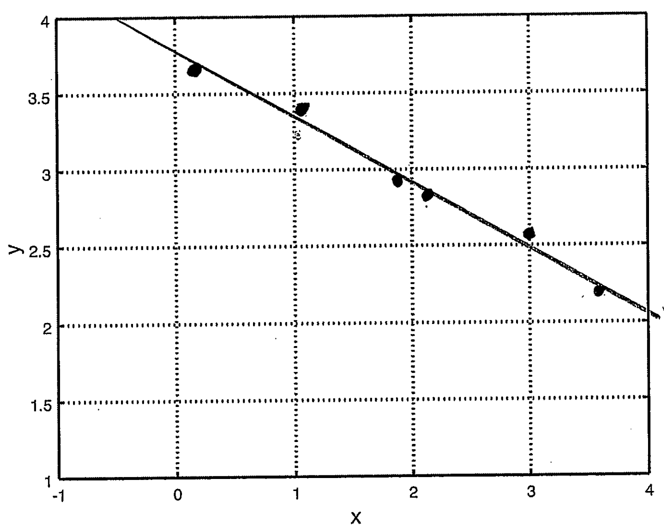
$$\ln e = 1; \ \ln 1 = 0$$

$$4 - 256 = \underline{252}$$

5. (6 points) Consider the following dataset:

x	0.12	1.02	1.87	2.17	2.97	3.53
$f(x)$	3.67	3.45	2.99	2.78	2.54	2.19

(a) Sketch a scatter plot of the data



(b) Find an equation of a straight line through the first and the last data points.

$$m = \frac{\Delta y}{\Delta x} = \frac{2.19 - 3.67}{3.53 - 0.12} = \frac{-1.48}{3.41} = -0.434$$

$$y - 3.67 = -0.434(x - 0.12)$$

$$y = -0.434x + 3.722$$

(c) Using calculator find an equation of the line of best fit. Show the line on the graph above.

$$y = ax + b,$$

$$a = -0.441$$

$$b = 3.795$$

(d) Based on part (c) obtain a prediction for the value $f(5)$.

$$y = -0.441x + 3.795 \Big|_{x=5} = 1.59$$

6. (6 points) The tide height, in ft, at a Southern California beach are given in the following table:

Time	Height (ft)	Tide
1:00am	3.4	High
7:00am	1.1	Low
1:00pm	3.4	High

Let

$$T(t) = A\cos(B(t+C)) + D$$

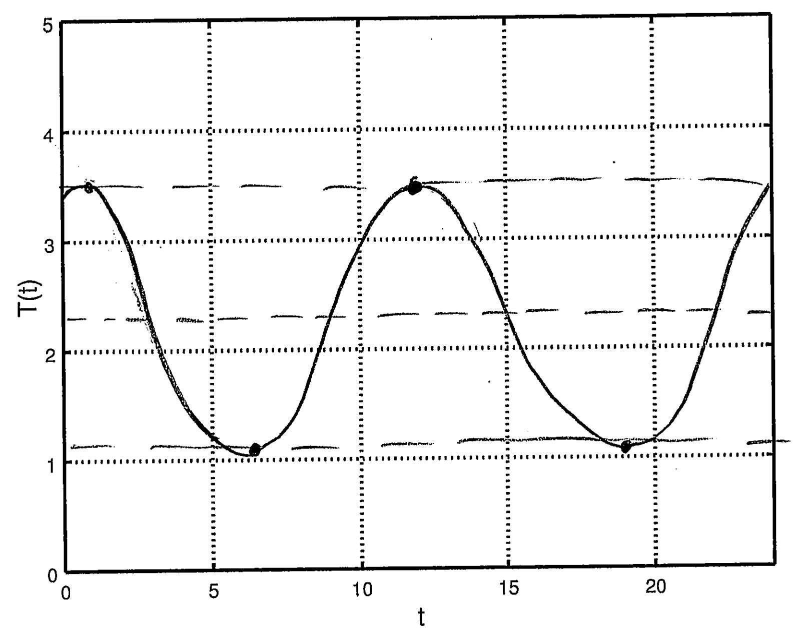
denote the height of the tide t hours after midnight.

(a) Find values of A, B, C, and D such that the function fits the data.

$D = \frac{3.4+1.1}{2} = 2.25$ $\quad B = \frac{2\pi}{12} = \frac{\pi}{6}$

$A = \frac{3.4-1.1}{2} = 1.15$ $\quad C = -1$

(b) Sketch a graph of the function over the interval [0, 24].



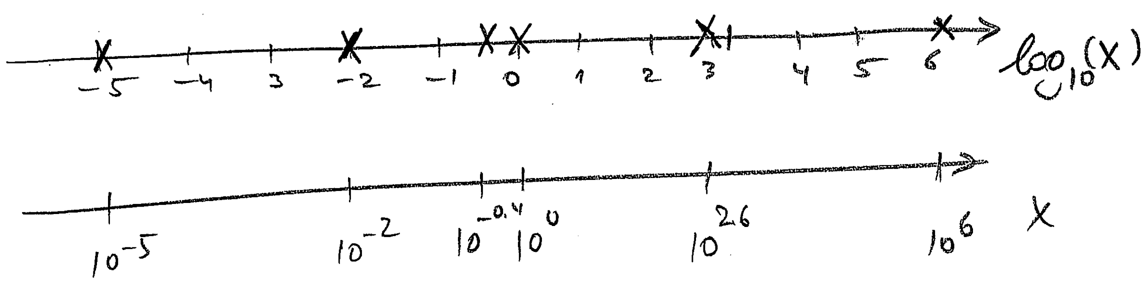
(c) According to this model, what was the tide height at 4:00am?

$T(4) = 2.25 + 1.15\cos\left(\frac{\pi}{6}(4-1)\right) = 2.25\,(ft)$

7. (6 points) (a) Sketch the indicated points on a logarithmic scale (use base 10):

$$X: \quad 10^{-5},\ 0.01,\ 0.4,\ 1,\ 400,\ \underbrace{1,000,000}_{10^6}$$

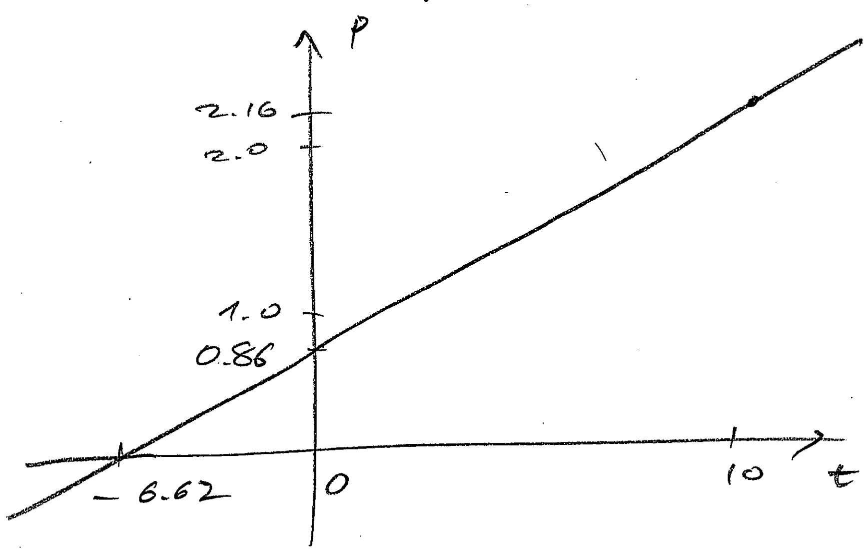
$$\log_{10} X: \ -5,\ -2,\ -0.4,\ 0,\ 2.6,\ 6$$



(b) If $P = 7.3(1.35)^t$ and $p = \log_{10}(P)$, find the equation satisfied by p and t. Sketch a graph in the (p, t)-plane.

$$p = \log_{10}\left(7.3\,(1.35)^t\right) = \log_{10} 7.3 + t \log_{10}(1.35)$$

$$= 0.8633 + t \cdot 0.1303$$



Graph is a straight line.

Linear Regression (line of best fit): $y = ax + b$; Linreg$(ax+b)$ L_1, L_2

Trigonometric Functions $y = A\cos(B(t+C)) + D$

$A = \frac{y_{\max} - y_{\min}}{2}$ – amplitude; $D = \frac{y_{\max} + y_{\min}}{2}$ – mean value

$B = \frac{2\pi}{\text{period}}$; $t = -C$ – location of maximum.

Proportionality $y \propto x^p$ means $y = cx^p$

$$y \propto x^p \quad \Leftrightarrow \quad x \propto y^{1/p}$$

$$y \propto x^p,\ z \propto y^q \quad \Rightarrow \quad z \propto x^{pq}.$$

Compound Interest

$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ – compounded n times per year

$A(t) = Pe^{rt}$ – compounded continuously.

Half-life and doubling time

$$P(t) = P_0\, b^t \quad \Rightarrow \quad t_D = \frac{\ln 2}{\ln b} \quad (b > 1)$$

$$t_{1/2} = \frac{\ln \frac{1}{2}}{\ln b} \quad (b < 1).$$

Laws of exponents

$$x^p x^q = x^{pq}$$

$$\frac{x^p}{x^q} = x^{p-q}; \qquad (xy)^p = x^p y^p$$

$$\left(x^p\right)^q = x^{pq}; \qquad \left(\frac{x}{y}\right)^p = \frac{x^p}{y^p}.$$

Laws of logarithms

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^p = p \log_a x.$$

Change of base

$$\log_a x = \frac{\log_b x}{\log_b x}.$$

The end.