#19 $f(x) = x^{3/2} + x^{1/2} + x^{-1}, \; x > 0$

$$F(x) = \frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + \ln x + C$$

$$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + \ln x + C$$

#26. $f(x) = 4 - 5x \; ; \quad F(0) = 4$

$$F(x) = 4x - \frac{5}{2}x^2 + C$$

$$F(0) = C = 4 \implies F(x) = 4x - \frac{5}{2}x^2 + 4$$

#29. (a) $F'(x) = 1 - 4x \; ;$

$$F(x) = x - 2x^2 + C$$

$$F(1) = 1 - 2 + C = C - 1 = 0 \implies C = 1$$

$$F(x) = x - 2x^2 + 1$$

$$= -2\left(x^2 - 2 \cdot \frac{1}{4}x + \left(\frac{1}{4}\right)^2\right) + \frac{1}{8} + 1$$

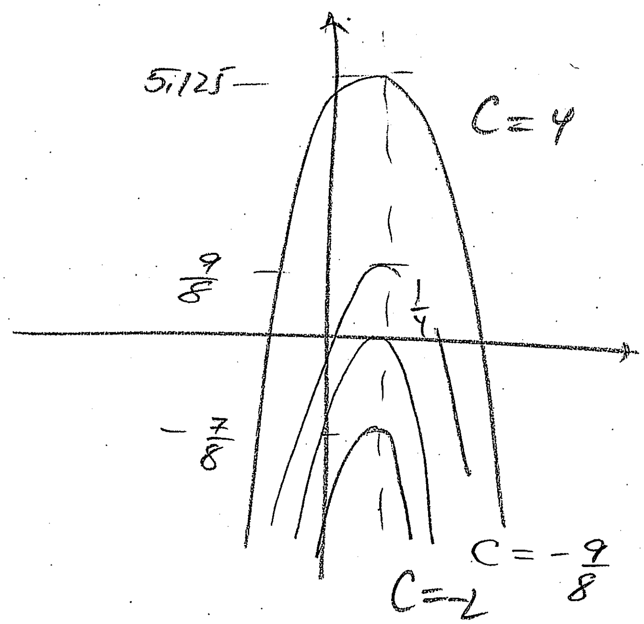
$$= -2\left(x - \frac{1}{4}\right)^2 + \frac{9}{8}$$

(c) If $C = -\frac{9}{8}$ then $F(x) + C$

$$= -2\left(x - \frac{1}{4}\right)^2$$

has the greatest value 0

(b)



#50. Level of CO pollution

$L(t)$ [ppm]

Rate of change (instantaneous)

$L'(t)$ [ppm/year]

$$L'(t) = 0.1t + 0.1$$

$$L(t) = \int 0.1t + 0.1 \, dt$$

$$= 0.1 \frac{t^2}{2} + 0.1t + C$$

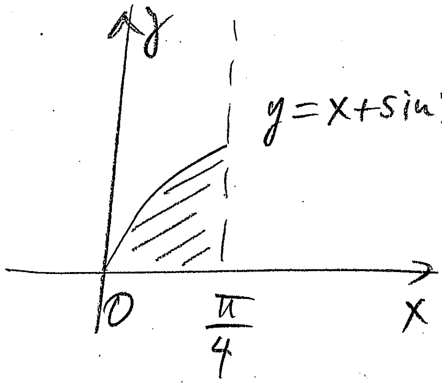
Current level : $t = 0$

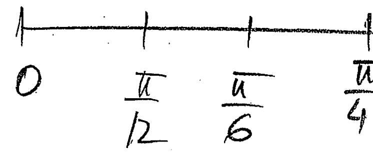
$$L(0) = 3.4 = C \implies C = 3.4$$

$$L(t) = 0.05t^2 + 0.1t + 3.4$$

\# 12. $f(x) = x + \sin x$ on $[0, \frac{\pi}{4}]$, $n=3$

Approximate using right endpoints.



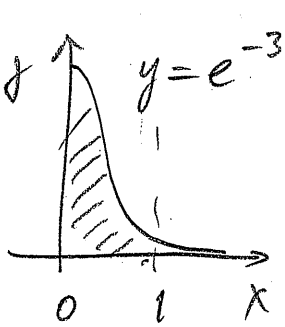


$$\Delta x = \frac{\pi/4}{3} = \frac{\pi}{12}$$

$$AUC \cong \frac{\pi}{12}\left(f\left(\frac{\pi}{12}\right) + f\left(\frac{\pi}{6}\right) + f\left(\frac{\pi}{4}\right)\right) = 0.795012$$

\#18 $f(x) = e^{-3x^2}$ on $[0,1]$, $n = 10$

right endpoints.



$$AUC \approx \sum_{i=1}^{10} f(x_i)\Delta x$$

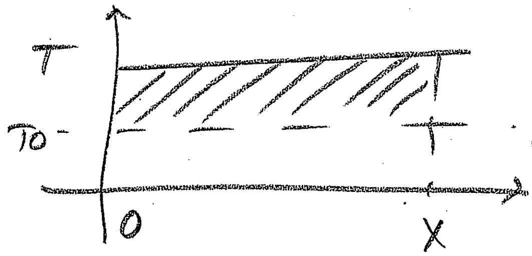
$$\Delta x = \frac{1}{10}; \quad x_i = \frac{i}{10}$$

$$AUC \approx \frac{1}{10}\Big(\underbrace{f\left(\tfrac{1}{10}\right) + f\left(\tfrac{2}{10}\right) + \dots + f\left(\tfrac{9}{10}\right) + f(1)}_{10 \text{ terms}}\Big)$$

$$= 0.456585$$

#26.

$T_0 = 50°F$



$GDD = 1,197$

$T = 72°F$

x – time in days.

$$GDD = (T - T_0) \cdot x = 1,197$$

$$22x = 1,197$$

$$x = \frac{1,197}{22} = 54.41 \quad [\text{days}]$$

#37.

$T_0 = 40°F$

$$T(t) = 50 + 20\cos(2\pi t/365) + 10\sin(2\pi t)$$

$$GDD = \int_0^{10} (T(t) - T_0)\,dt$$

$$= \int_0^{10} (10 + 20\cos(2\pi t/365) + 10\sin(2\pi t))\,dt$$

$\Delta t = 2$; $[0, 10]$; $n = 5$

Using right end points:

$$GDD \approx \sum_{i=1}^{5} T(t_i)\,\Delta t = 2\sum_{i=1}^{5} T(2i)$$

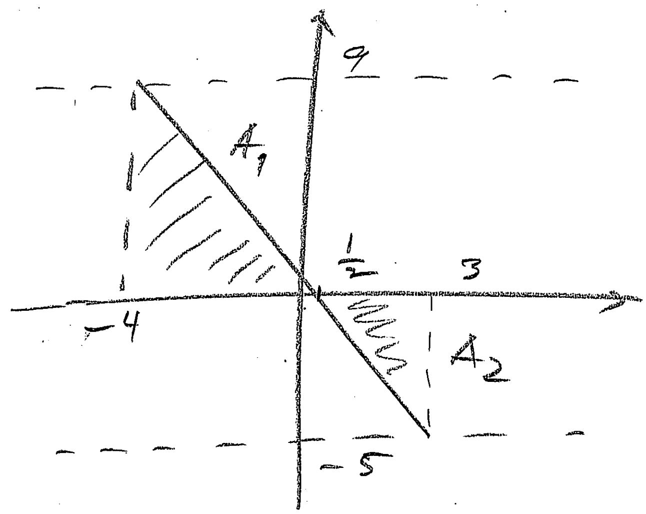
$$= 298.698$$

Using left end points:

$$GDD \approx \sum_{i=1}^{5} T(t_{i-1})\,\Delta t = 2 \cdot \sum_{i=1}^{5} T(2(i-1))$$

$$= 299.28$$

#13 $\displaystyle\int_{-4}^{3} (1-2x)\,dx$



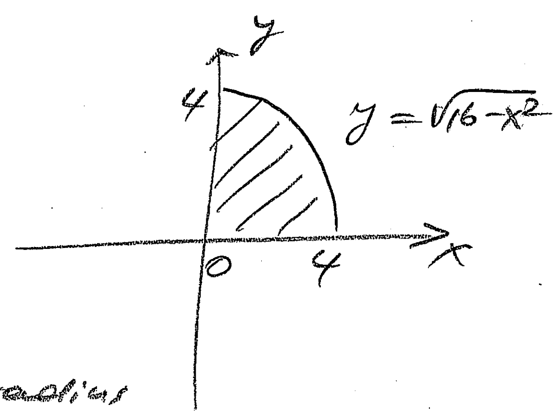
$f(x) = 1-2x$
$f(3) = -5$
$f(-4) = 9$

$$\int_{-4}^{3}(1-2x)\,dx = A_1 - A_2$$
$$= \frac{1}{2}\cdot 4.5 \cdot 9 - \frac{1}{2}\, 2.5 \cdot 5$$

(Area of right triangle $A = \frac{1}{2}\cdot l \cdot w$)

$$= 4.5^2 - 2.5^2 = 20.25 - 6.25$$
$$= 14.$$

#15. $\displaystyle\int_0^4 \sqrt{16-x^2}\,dx$



$y = \sqrt{16-x^2}$

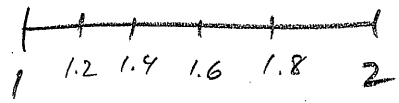
$\Longleftrightarrow y^2 = 16 - x^2, \quad y > 0$

$\Longleftrightarrow x^2 + y^2 = 16$ – circle of radius 4 about $(0,0)$
$y > 0$

$\frac{1}{4}\cdot$ Area of a circle $= \frac{1}{4}\cdot \pi \cdot 4^2 = 4\pi.$

#34

$\int_1^2 x^2 dx$, $n=5$, right endpoints

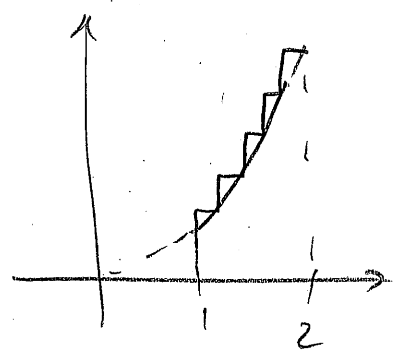


$\Delta x = \frac{1}{5} = 0.2$

$x_i = 1 + 0.2i$

$$\int_1^2 x^2 dx \approx 0.2\left(f(1.2) + f(1.4) + f(1.6) + f(1.8) + f(2.0)\right)$$

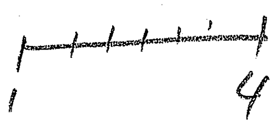
$$= 0.2 \sum_{i=1}^{5} f(1+0.2i) = 2.64$$



f increasing
right endpoints
pick up the greatest
value on each
sub-interval

$\Rightarrow$ Riemann sum is
an overestimate for AUC.

#35. $\int_1^4 \frac{dx}{x}$



$\Delta x = \frac{3}{5} = 0.6$

$x_i = 1 + 0.6i$

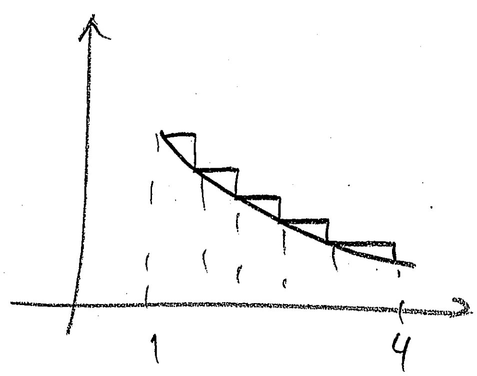
$$\int_1^4 \frac{1}{x}\,dx \approx \sum_{i=1}^{5} f(x_i)\Delta x$$

$$= 0.6 \sum_{i=1}^{5} f(1+0.6i)$$

$$= 1.1885$$

Use $Y_1 = 1/X$

sum(seq(Y₁(1+0.6I)*0.6, I, 1, 5, 1))



f. decreasing on $[1,4]$

right endpoints pick up least value on each sub interval

$\Rightarrow$ Riemann sum is an under-estimate for AUC.

#43. $T_0 = 50°F$

$GDD = 3{,}000$

$T(x) = 60 + 10\sin(2\pi x)$

from $x=0$ to $x=15$:

$$GDD = \int_0^{15} (60 + 10\sin(2\pi x) - 50)\,dx$$
$$= \int_0^{15} 10 + 10\sin(2\pi x)\,dx$$
$$= \underbrace{\int_0^{15} 10\,dx}_{\substack{15\cdot 10 \\ \text{by area} \\ \text{of a rectangle}}} + 10 \underbrace{\int_0^{15} \sin(2\pi x)\,dx}_{\substack{=0 \\ \text{by symmetry} \\ (\text{period} = 1 \text{ day})}}$$

$= 150. \quad °F \cdot \text{day}.$

$$\frac{150}{3{,}000} = 0.05 = 5\%$$

5% of the ripening took place during the first 15 days.

#4 (a) $\int_0^1 (x^3+6x^2)\,dx = \left[\frac{x^4}{4} + 6\frac{x^3}{3} + C\right]_0^1$

$= \left[\frac{x^4}{4} + 6\frac{x^3}{3}\right]_0^1 = \frac{1}{4} + \frac{6}{3}.$

(b) $\int_{-2}^{-1} \frac{6}{x^2}\,dx = \int_{-2}^{-1} 6x^{-2}\,dx$

$= \left[-6x^{-1}\right]_{x=-2}^{x=-1}$

$= -6\left((-1)^{-1} - (-2)^{-1}\right)$

$= -6\left(-1 + \frac{1}{2}\right) = 6 \cdot \frac{1}{2} = \frac{6}{2}.$

#13. (a) $\int \sqrt{x}\,(x+1)\,dx = \int \left(x^{3/2} + x^{1/2}\right)dx$

$= \frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + C$

$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$

(b) $\int \sqrt{t}\,(t - \sqrt{t})\,dt$

$= \int (\sqrt{t} - t)\,dt = \int \left(t^{1/2} - t\right)dt$

$= \frac{t^{3/2}}{3/2} - \frac{t^2}{2} + C$

$= \frac{2}{3}t^{3/2} - \frac{1}{2}t^2 + C.$

#22.

$$F(x) = \int_x^3 e^{t^2}\,dt$$

$$F(x) = -\int_3^x e^{t^2}\,dt \qquad \left(\begin{array}{c}\text{whether} \\ x > 3 \text{ or } x < 3\end{array}\right)$$

$$F'(x) = -\frac{d}{dx}\int_3^x e^{t^2}\,dt = -e^{x^2}$$