MATH 255A — HOMEWORK PROBLEMS SECTION 3.2

#14.

$$g(t) = \frac{1+te^t}{1+t}$$

$$g'(t) = \frac{(1+te^t)'(1+t) - (1+te^t)\cdot 1}{(1+t)^2} =$$

$$= \frac{(1\cdot e^t + te^t)(1+t) - 1 - te^t}{(1+t)^2}$$

$$= \frac{e^t + te^t + \cancel{te^t} + t^2e^t - 1 - \cancel{te^t}}{(1+t)^2} = \frac{e^t(1+t+t^2)-1}{(1+t)^2}$$

#22.

$f(x) = e^x + e^{-x}$; $x_0 = 0$

$f'(x) = e^x - e^{-x}$; $f'(0) = 0$

$f(0) = 1+1 = 2$

Tangent line: $y - 2 = 0\cdot(x-0)$

$y = 2$.

#23.

$$F(x) = \frac{3x^2+5}{2x^2+x-3}, \quad x_0 = -1$$

$$F'(x) = \frac{6x(2x^2+x-3) - (3x^2+5)(4x+1)}{(2x^2+x-3)^2}$$

$$= \frac{\cancel{12x^3} + 6x^2 - 18x - \cancel{12x^3} - 3x^2 - 20x - 5}{(2x^2+x-3)^2}$$

$$= \frac{3x^2 - 38x - 5}{(2x^2+x-3)^2}$$

$$F'(-1) = \frac{36}{(-2)^2} = \frac{36}{4} = 9. \qquad F(-1) = \frac{8}{-2} = -4$$

Tangent line:

$$y+4 = 9(x+1)$$
$$y+4 = 9x+9$$
$$y = 9x+5$$

#33. $f(x) = \dfrac{3.36\,X}{0.46+X}$ $\left[\dfrac{\text{moose}}{\text{wolf}\cdot 100\text{days}}\right]$

X $\left[\dfrac{\text{moose}}{\text{km}^2}\right]$

$$f'(x) = \frac{3.36(0.46+X) - 3.36X}{(0.46+X)^2} = \frac{3.36\cdot 0.46}{(0.46+x)^2}$$

$$f'(0.5) = \frac{3.36\cdot 0.46}{(0.96)^2} = 1.677$$

$$f'(2) = \frac{3.36\cdot 0.46}{(2.46)^2} = 0.255$$

The units are $\left[\dfrac{\text{km}^2}{\text{wolf}\cdot 100\text{days}}\right]$

The number of killed moose increases with increased density of moose; more noticeably for $x = 0.5\ \frac{\text{moose}}{\text{km}^2}$ than for $x = 2.0\ \frac{\text{moose}}{\text{km}^2}$.

#18.

$$y = xe^{-x^2}$$

$$\frac{dy}{dx} = 1 \cdot e^{-x^2} + x\left(e^{-x^2}\right)'$$

$$= e^{-x^2} + x \cdot (-2x) \cdot e^{-x^2}$$

$$= e^{-x^2} - 2x^2 e^{-x^2} = (1-2x^2)e^{-x^2}.$$

#22.

$$\frac{1}{y} + \frac{1}{x} = 1$$

$$\frac{d}{dx}\left(\frac{1}{y} + \frac{1}{x}\right) = \frac{d}{dx} 1$$

$$-\frac{1}{y^2} \cdot y' - \frac{1}{x^2} = 0$$

$$\frac{y'}{y^2} = -\frac{1}{x^2} \Rightarrow y' = -\frac{y^2}{x^2}.$$

#24

$$\ln(xy) = e^{2x}$$

$$\frac{1}{xy} \cdot (y + xy') = 2e^{2x}$$

$$y + xy' = 2xy\, e^{2x}$$

$$xy' = 2xy\, e^{2x} - y$$

$$y' = \frac{2xye^{2x} - y}{x} = 2y e^{2x} - \frac{y}{x}.$$

#43.

$$h(t) = 32 + 0.19t \quad [in]$$

$$t \; [months]$$

$$W(h) = 0.0024\, h^{2.6} \quad [lb]$$

$$\frac{dh}{dt} = 0.19 \quad \left[\frac{in}{month}\right]$$

$$t = 10 \text{ years} = 120 \text{ months}$$

$$h(120) = 54.8 \quad [in]$$

$$\frac{dW}{dt} = \frac{dW}{dh}\frac{dh}{dt}$$

$$= 0.0024 \cdot 2.6 \cdot h^{1.6} \cdot 0.19$$

$$\left.\frac{dW}{dt}\right|_{\substack{t=120 \\ (h=54.8)}} = 0.0011856 \cdot 54.8^{1.6}$$

$$= 0.71777 \quad \left[\frac{lb}{month}\right]$$

\# 7. $y = e^{-x}\sin x$

$$\frac{dy}{dx} = -e^{-x}\sin x + e^{-x}\cos x$$

$$= e^{-x}(\cos x - \sin x).$$

\# 8. $y = (\tan x)^2$

$$\frac{dy}{dx} = 2\tan x\,(\tan x)' = 2\,\frac{\sin x}{\cos x}\cdot\frac{1}{\cos^2 x} = \frac{2\sin x}{\cos^3 x}$$

$$\left[\,(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}\,\right]$$

$$\left\{\,\frac{dy}{dx} = 2\tan x\,\sec^2 x \text{ is also OK.}\right\}$$

\# 13. $P(t) = (t^2+2)\sin t$

$$P'(t) = 2t\sin t + (t^2+2)\cos t$$

\# 16. $f(x) = \dfrac{\sin x}{1-\cos x}$

$$f'(x) = \frac{\cos x(1-\cos x) - \sin x(\sin x)}{(1-\cos x)^2}$$

$$= \frac{\cos x - \cos^2 x - \sin^2 x}{(1-\cos x)^2}$$

$$= \frac{\cos x - 1}{(1-\cos x)^2} = \frac{-1}{1-\cos x} = \frac{1}{\cos x - 1}.$$

#34.

$$H(t) = 1.8\cos\left(\frac{\pi}{6}(t-11)\right) + 2.2$$

$$H'(t) = 1.8(-\sin)\left(\frac{\pi}{6}(t-11)\right)\cdot\frac{\pi}{6} = -0.3\pi\sin\left(\frac{\pi}{6}(t-11)\right)$$

$$H'(6) = -0.3\pi\sin\left(-\frac{5\pi}{6}\right)$$

$$= -0.3\cdot\pi\cdot(-0.5) = 0.15\cdot\pi$$

$$\cong 0.47 \quad \left[\frac{ft}{hour}\right]$$

After 6 hours (at 6 am)
the tide height is increasing
at the rate of about 0.47 ft
per hour.