#8 $f(x) = x^3 + 1$, $a = 2$

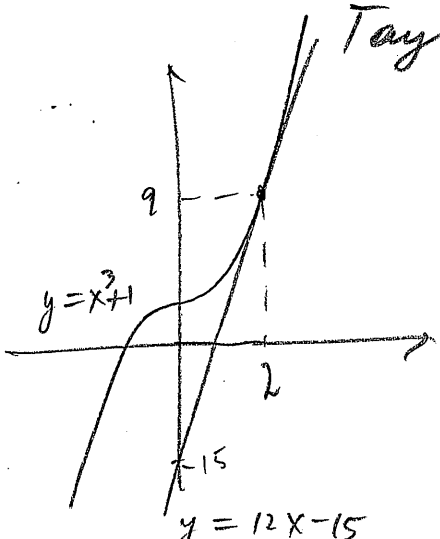
$$f'(a) = f'(2) = \lim_{h \to 0} \frac{\left[(2+h)^3 \cancel{+1}\right] - \left(2^3 \cancel{+1}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2^3} + 3 \cdot 4 \cdot h + 3 \cdot 2 \cdot h^2 + h^3 - \cancel{2^3}}{h}$$

$$= \lim_{h \to 0} \left(12 + 6h + h^2\right)$$

$$= 12$$

#18. $f(x) = x^3 + 1$, $a = 2$



Tangent line: $y - y_0 = m(x - x_0)$

$y_0 = f(2) = 2^3 + 1 = 9$

$x_0 = 2$; $m = f'(2) = 12$

$y - 9 = 12(x-2) = 12x - 24$

$y = 12x - 24 + 9$

$y = 12x - 15$

#27.

$$f(x) = \begin{cases} -2x, & x<1 \\ \sqrt{x}-3, & x \geq 1 \end{cases}$$

$$\lim_{x\to 1-} f(x) = (-2x)\Big|_{x=1} = -2$$

$$\lim_{x\to 1+} f(x) = (\sqrt{x}-3)\Big|_{x=1} = 1-3 = -2$$

The limits match and are equal the value $f(1)$.

$$f(1) = (\sqrt{x}-3)\Big|_{x=1} = -2$$

$\Rightarrow$ f is continuous at $x=1$.

However
$$\lim_{h\to 0-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0-} \frac{-2(1+h)+2}{h}$$

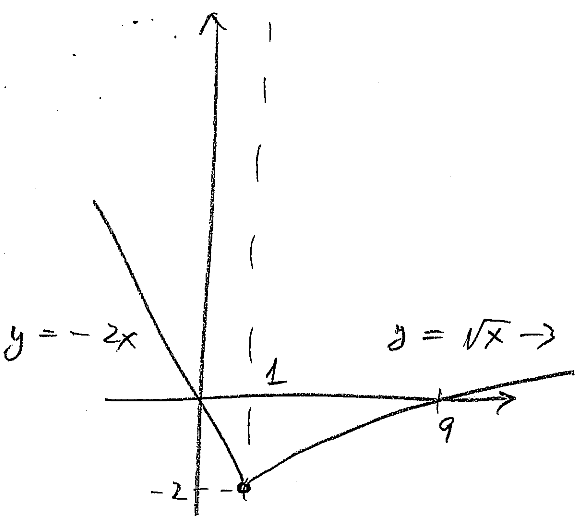
$$= \lim_{h\to 0-} \frac{-2h}{h} = -2$$

$$\lim_{h\to 0+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0+} \frac{\sqrt{1+h}-3+2}{h}$$

$$= \lim_{h\to 0+} \frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0+} \frac{(\sqrt{1+h}-1)(\sqrt{1+h}+1)}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0+} \frac{h}{h(\sqrt{1+h}+1)} = \lim_{h\to 0+} \frac{1}{\sqrt{1+h}+1}$$

$$= \frac{1}{1+1} = \frac{1}{2}$$



Therefore $\lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$ does not exist!

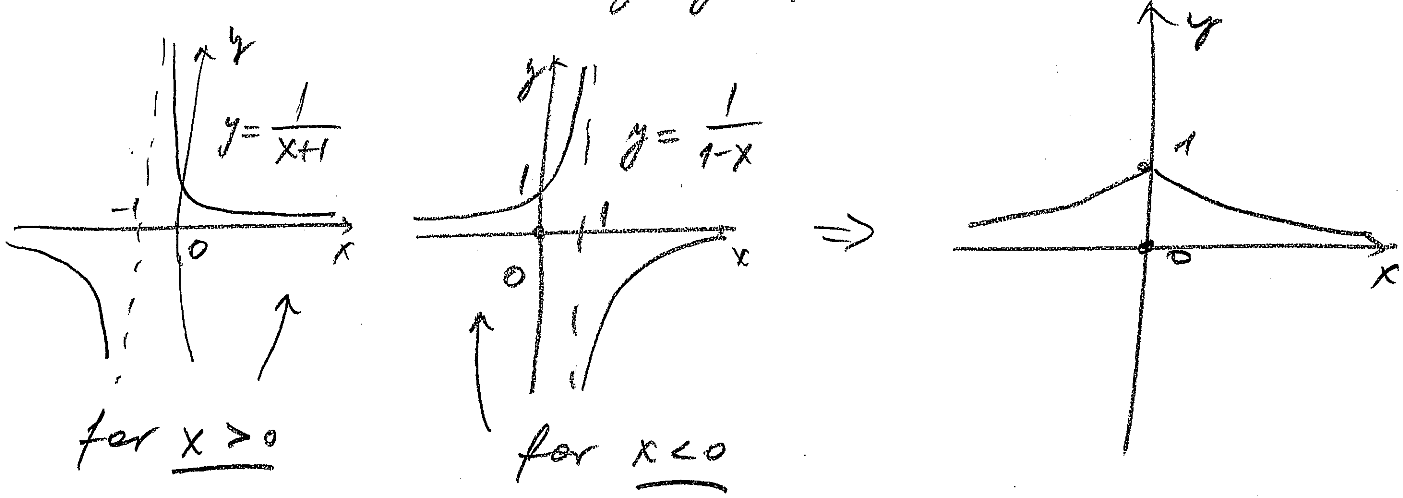
The slope on the left is -2

The slope on the right is $\frac{1}{2}$

#29.

$$f(x) = \begin{cases} \dfrac{1}{-x+1}, & x \le 0 \\ \dfrac{1}{x+1}, & x > 0 \end{cases}$$

Combine the two graphs:



The function is continuous at $x = 0$.
(limits from the left and from the right are both $= 1$)

The function is $\underline{\text{not}}$ differentiable at $x = 1$

The slope on the left is positive

The slope on the right is negative

$$\Rightarrow \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \text{ does not exist!}$$

The function is both continuous and differentiable at any other x-value.

#34. $S(t) = 7.292 + 0.023t - 0.004t^2$

$$S'(20) = \lim_{h \to 0} \frac{S(20+h) - S(20)}{h}$$

$$= \lim_{h \to 0} \frac{\left[7.292 + 0.023(20+h) - 0.004(20+h)^2\right] - \left(7.292 + 0.023 \cdot 20 - 0.004 \cdot 20^2\right)}{h}$$

$$= \underbrace{\lim_{h \to 0} \frac{7.\cancel{292} - 7.\cancel{292}}{h}}_{=0} + 0.023 \underbrace{\lim_{h \to 0} \frac{(\cancel{20}+h) - \cancel{20}}{h}}_{=1} - 0.004 \lim_{h \to 0} \frac{(20+h)^2 - 20^2}{h}$$

$$= 0.023 - 0.004 \lim_{h \to 0} \frac{\cancel{20^2} + 2 \cdot 20 \cdot h + h^2 - \cancel{20^2}}{h}$$

$$= 0.023 - 0.004 \lim_{h \to 0} (2 \cdot 20 + h)$$

$$= 0.023 - 0.004 \cdot 40 = 0.023 - 0.160$$

$$= -0.137$$

Comparing with $S'(30) = -0.237$

we see that after 20 years the rate at which ice cap is melting is less than the corresponding rate after 30 years.

So, the melting process is speeding up.

#8. $f(x) = \frac{1}{2x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{2(x+h)} - \frac{1}{2x}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\,\frac{1}{2}\left(\frac{1}{x+h} - \frac{1}{x}\right) = \lim_{h\to 0} \frac{1}{2h}\,\frac{x-(x+h)}{(x+h)x}$$

$$= \lim_{h\to 0} \frac{-h}{2\cdot h\,(x+h)\cdot x} = \lim_{h\to 0} \frac{-1}{2\,(x+h)\cdot x}$$

$$= -\frac{1}{2x^2}$$

#16. $\left.\frac{dy}{dx}\right|_{x=10}$ where $y = \frac{1}{2x}$

$$\left.\frac{dy}{dx}\right|_{x=10} = f'(10) = -\frac{1}{2\cdot 10^2} = -\frac{1}{200}$$

$$= -0.005$$

#20. $f(x) = x + x^2;$ $\quad f'(x) = \lim_{h\to 0} \frac{x+h+(x+h)^2 - x - x^2}{h}$

$[A,B] = [0,1].$

$$= \lim_{h\to 0} \frac{h + \cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} 1 + 2x + h$$

$$= 1 + 2x.$$

$$f'(x) = ARC = \frac{f(1)-f(0)}{1-0} = \frac{2-0}{1-0} = 2$$

$$1 + 2x = 2$$

$$2x = 1$$

$$x = 0.5$$

MATH 25A HOMEWORK PROBLEMS — SECTION 3.1

#8. $f(x) = 2x^2 - 5x^8 + 1$

$f'(x) = 2(x^2)' - 5(x^8)' + (1)'$

$= 2 \cdot 2x - 5 \cdot 8x^7 + 0 = 4x - 40x^7.$

#12. $f(t) = 82.1\,(1.85)^t$

$f'(t) = 82.1 \cdot \ln(1.85) \cdot (1.85)^t$

since $(b^t)' = (\ln b) \cdot b^t.$ $(b = 1.85)$

#18. $g(x) = x^3 + 35x^2 - 125x - 9{,}375$

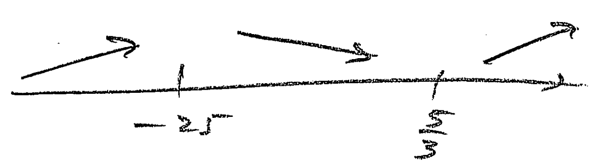
$g'(x) = 3x^2 + 35 \cdot 2x - 125 = 3x^2 + 70x - 125$

$= (3x-5)(x+25)$

$g'(x) = 0$ when $x = \frac{5}{3}$ or $x = -25$

g':

g:



g increases on $(-\infty, -25)$ or $(\frac{5}{3}, \infty)$

g decreases on $(-25, \frac{5}{3})$.

#22.

$$g(x) = x^2(x^3 - 3x)$$
$$= x^5 - 3x^3$$
$$g'(x) = (x^5 - 3x^3)' = (x^5)' - 3(x^3)'$$
$$= 5x^4 - 3 \cdot 3x^2$$
$$= 5x^4 - 9x^2.$$
$$(= x^2(5x^2 - 9))$$

#25.

$$h(t) = \frac{3^t + 3^{-t}}{2^t}$$
$$h(t) = \frac{3^t}{2^t} + \frac{3^{-t}}{2^t} = \left(\frac{3}{2}\right)^t + \frac{1}{3^t \cdot 2^t}$$
$$= \left(\frac{3}{2}\right)^t + \left(\frac{1}{6}\right)^t$$
$$h'(t) = \ln\left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right)^t + \ln\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)^t$$
$$= \frac{(\ln 3 - \ln 2)\, 3^t - (\ln 6)\, 3^{-t}}{2^t}$$