#6. $f(x) = \dfrac{-2}{x+1}$, $[1,5]$

$$ARC = \big(f(5)-f(1)\big)/(5-1)$$

$$= \left(-\frac{2}{6}+\frac{2}{2}\right)/4 = \left(1-\frac{1}{3}\right)/4$$

$$= \frac{2}{3}\cdot\frac{1}{4} = \boxed{\frac{1}{6}}$$

#12. $$IRC = f'(1) = \lim_{h\to 0}\frac{1}{h}\left(-\frac{2}{2+h}+\frac{2}{2}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left(1-\frac{2}{2+h}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{\cancel{2}+h-\cancel{2}}{2+h}\right)$$

$$= \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(2+h)} = \lim_{h\to 0}\frac{1}{2+h} = \boxed{\frac{1}{2}}$$

#26 $f(x) = \cos\dfrac{\pi x}{2}$; $a = 0.5$

Tangent line: $y - y_0 = m(x - x_0)$

$x_0 = a = 0.5$; $y_0 = f(a) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \approx 0.707$

$$m = f'(a) = \lim_{b\to a}\frac{f(b)-f(a)}{b-a}$$

Estimate numerically:

a	b	$\frac{f(b)-f(a)}{b-a}$
0.5	0.49	-1.102
0.5	0.499	-1.110
0.5	0.501	-1.112
0.5	0.51	-1.119

$f'(a) \approx -1.11$

$$y - 0.707 = -1.11(x-0.5) \Rightarrow \underline{y = -1.11x + 1.262}$$

#24.

$$\lim_{x\to 4} \frac{(x-4)^2}{|x-4|} = \lim_{x\to 4} \begin{cases} \frac{(x-4)^2}{x-4}, & x>4 \\ \frac{(x-4)^2}{-(x-4)}, & x<4 \end{cases}$$

$$= \lim_{x\to 4} \begin{cases} x-4, & x>4 \\ -(x-4), & x<4 \end{cases}$$

$$= 0$$

Since $\lim_{x\to 4+} (x-4) = 0$, $\lim_{x\to 4-} -(x-4) = 0$ (one-sided limits match.)

#42

$$f(x) = \begin{cases} 6.25x, & x<150 \\ ax+b, & 150<x<300 \\ 1300, & x>300 \end{cases}$$

Choose a, b (if possible) in such a way that the limits as $x\to 150$ and as $x\to 300$ exist.

$x=150$: one-sided limits must match

$$6.25\cdot 150 = a\cdot 150+b$$

$x=300$: one-sided limits must match

$$a\cdot 300+b = 1300$$

$$-\begin{cases} a \cdot 300 + b = 1300 \\ a \cdot 150 + b = 937.5 \end{cases}$$

$$a \cdot 150 = 362.5$$

$$a = 2.4167 \quad ; \quad b = 1300 - a \cdot 300$$

$$= 1300 - 725$$

$$= 575.$$

HOMEWORK PROBLEMS 2.2

\# 20

$$\lim_{x\to 1} \frac{\ln x}{x-1} = \lim_{x\to 1} \ln x^{\frac{1}{x-1}} \; ; \quad \left[\begin{array}{l} \text{set } x-1=t \\ \Rightarrow x = 1+t \\ t\to 0 \\ \text{since } x\to 1 \end{array}\right]$$

$$= \lim_{t\to 0} \ln (1+t)^{\frac{1}{t}}$$

$$= \lim_{m\to\infty} \ln \left(1+\frac{1}{m}\right)^m \quad \left[\begin{array}{l} \text{set } m=\frac{1}{t} \\ \text{then } t=\frac{1}{m} \\ m\to\infty \\ \text{since } t\to 0 \end{array}\right]$$

$$= \ln\left(\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^m\right) = \ln(e) = 1.$$

Also OK: estimate numerically:

x	$y = \frac{\ln x}{x-1}$
1.01	0.995
1.001	0.9995
0.999	1.0005
0.99	1.005

$$\Rightarrow \lim_{x\to 1} \frac{\ln x}{x-1} = 1$$

\#22.

$$\lim_{x\to 0} \frac{x}{|x|} = \lim_{x\to 0} \begin{cases} \frac{x}{x}, & x>0 \\ \frac{x}{-x}, & x<0 \end{cases}$$

$$= \lim_{x\to 0} \begin{cases} 1, & x>0 \\ -1, & x<0 \end{cases}$$

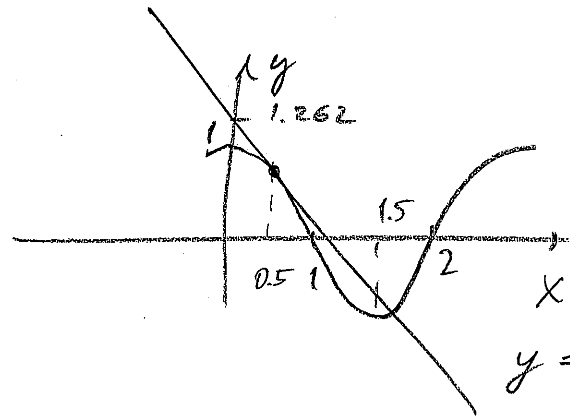
does not exist,

since

$$\left.\begin{array}{l} \lim_{x\to 0+} \frac{x}{|x|} = 1 \\ \lim_{x\to 0-} \frac{x}{|x|} = -1 \end{array}\right\}$$

one sided limits do not match $\Rightarrow$ two-sided limit D.N.E.

Graph:



#42: $f(x) = \sqrt{5x}$; $a = 5$

Tangent line: $y - y_0 = m(x - x_0)$

$y_0 = f(5) = \sqrt{25} = 5$

$x_0 = 5$

$$m = f'(5) = \lim_{h \to 0} \frac{\sqrt{5(5+h)} - \sqrt{25}}{h}$$

$$= \lim_{h \to 0} \frac{5\cancel{h}}{\cancel{h}\left(\sqrt{25+5h} + \sqrt{25}\right)}$$

$$= \frac{5}{5+5} = \frac{1}{2}$$

$$y - 5 = \frac{1}{2}(x - 5) \Rightarrow y = \frac{1}{2}x - \frac{5}{2} + 5$$

$$\underline{y = \frac{1}{2}x + \frac{5}{2}}.$$

#2. Find $\lim_{x\to 1-} f(x)$, $\lim_{x\to 1+} f(x)$, $\lim_{x\to 1} f(x)$ (if exist)

$$f(x) = \begin{cases} 3x+2 & x \le 1 \\ 5 & 1 < x \le 3 \\ 3x^2-1, & x > 3 \end{cases}$$

$$\lim_{x\to 1-} f(x) = \lim_{x\to 1-} 3x+2 = 3+2=5$$

$$\lim_{x\to 1+} f(x) = \lim_{x\to 1+} 5 = 5$$

$$\Rightarrow \lim_{x\to 1} f(x) = 5.$$

#8. $$\lim_{x\to 0} \frac{(x+1)^2-1}{x} = \lim_{x\to 0} \frac{x^2+2x+\cancel{1}-\cancel{1}}{x}$$

$$= \lim_{x\to 0} \frac{x^2+2x}{x} = \lim_{x\to 0} x+2 = 0+2 = 2.$$

#21 Find the value that should be assigned to $f(2)$, if any, to ensure that $f(x)$ is continuous at 2:

$$f(x) = \begin{cases} 2x+5, & x > 2 \\ 15-x^2, & x < 2 \end{cases}$$

$$\lim_{x\to 2-} f(x) = \lim_{x\to 2-} 2x+5 = 4+5 = 9$$

$$\lim_{x \to 2+} f(x) = \lim_{x \to 2+} 15 - x^2 = 15 - 4 = 11$$

The limits do not match, so no matter what value is assigned to $f(2)$, f is not continuous at 2.

#28. $1 + \sin x + x^3 = 0$

$$f(x) = 1 + \sin x + x^3$$

To use intermediate value thm, notice that

* f is continuous everywhere (combination of elementary functions; domain $= \mathbb{R}$)
* $f(0) = 1 > 0$
* $f(-1) = 1 + \sin(-1) - 1 = \sin(-1) = -\sin(1) < 0$

Therefore there is a value c in $(-1, 0)$ such that $f(c) = 0$.

Using calculator [CALC] → zero found $x \approx -0.705694$

OR : [TRACE]

x	y
-0.70592	$-5E-4$
-0.70305	0.00594

$\Rightarrow -0.7059 < c < -0.70305 \Rightarrow c \simeq 0.705$