Review for Final Exam

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Mechanical Engineering 483
Alternative Energy Engineering II

May 5, 2010

Final Exam

- Monday, May 10, 3–5 pm
- Open book and notes
  - No books other than course text
  - No homework solutions or in-class exercise solutions
- Will be problems similar to those on homework and in-class exercise
- More credit for correct approach than for details of algebra or arithmetic
- No questions on material since second midterm

What is energy?

- Energy and power (energy/time) units
  - Energy units: joules (J), kilowatt·hours (kWh), British thermal units (Btu)
    - 1 Btu = 1055.056 J
  - Power units: watts (W), Btu/hr
    - 1 W = 1 J/s = 3.412 Btu/hr
  - Fuel equivalencies: 1 ft^3 natural gas ≈ 1000 Btu; 1 bbl crude = 5.8 MMBtu; 1 Mtoe oil = 41.868×10^{15} J = 0.0387 quads
  - World energy use (2006) was 466 quads

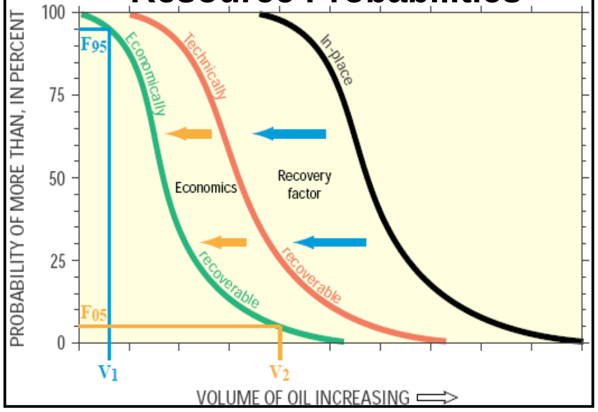
Energy Costs

- Home costs (San Fernando Valley 2008)
  - Electricity: \$0.115/kWh = \$32/GJ
    - Increase from \$0.11/kWh to \$0.12/kWh
  - Natural gas: \$1.07/therm = \$11/GJ
    - One therm = 10^5 Btu is approximately the energy in 100 standard cubic feet of natural gas
    - Range was \$0.69 to \$1.22 per therm
  - Gasoline at \$3.00 per gallon (including taxes) costs \$26/GJ
    - Assumes energy content of gasoline is 5.204 MMBtu per (42 gallon) barrel
    - \$100/bbl oil costs \$6.20/GJ (5.80 MMBtu/bbl)
  - Energy cost without California gasoline taxes (\$0.585/gallon) is \$21/GJ

Resources *vs*. Reserves

	Known	**Unknown**
Economical to Recover	Reserves	Resources
Not economical to recover	Resources	Resources

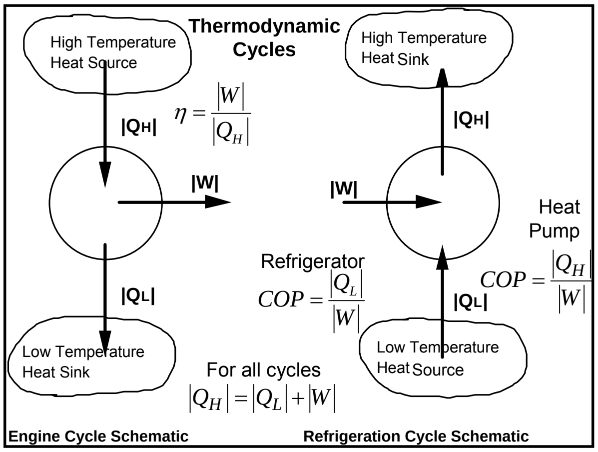
Resource Probabilities



Hubbert Peak

- Analysis due to M. King Hubbert
- Main publications in 1949 and 1956
- Correctly predicted peak in US oil production in early 1970s
- Not so accurate in other predictions
- Some recent applications show world oil production peak in next ten years
- Many other studies show later peak

Thermodynamic Cycles



$$\eta = \frac{|W|}{|Q_H|}$$

Refrigerator

$$COP = \frac{|Q_L|}{|W|}$$

$$COP = \frac{|Q_H|}{|W|}$$

For all cycles

$$|Q_H| = |Q_L| + |W|$$

Basic Combustion Analysis

- General fuel formula: $C_xH_yS_zO_wN_v$
- x, y, z, w, and v from ultimate analysis or analysis of gas mixtures
- Ultimate analyses:
  - x = wt%C/12.0107, y = wt%H/1.00794, z = t%S/32.065, w = wt%O/16.0004, v = wt%N/14.0067, m_{fuel} = 100
  - M_{fuel} = 12.0107x + 1.00794y + 32.065z + 15.9994w + 14.0067v = m_{fuel}(1 – %MM)
- For mixture of compounds (ω_k = mole fraction)

$$x = \sum_{species} \omega_k x_k \quad y = \sum_{species} \omega_k y_k \quad M_{fuel} = \sum_{species} \omega_k M_k$$

Combustion Air

- A = x + y/4 + z – w/2 = stoichiometric moles O_2/mole fuel
- Need input data on Actual O_2/Stoichiometric O_2 = Relative air/fuel ratio = λ
- Air/fuel ratio = m_{air}/m_{fuel} =138.28λA/m_{fuel}
- $C_xH_yS_zO_wN_v + \lambda A(O_2 + 3.77\ N_2) \rightarrow xCO_2 + (y/2)H_2O + zSO_2 + (\lambda - 1)AO_2 + 3.77\lambda A + v/2)N_2$

Exhaust Oxygen and λ

- Can relate these two quantities with fuel properties
- Can compute theoretical $\%O_2$ for given λ
- Dry exhaust has water removed to protect chemical analyzers

$$\frac{\%O_2|_{dry}}{100} = \frac{(\lambda - 1)A}{x + 4.77\lambda A - A + z + \frac{v}{2}}$$

$$\lambda = \frac{A + \frac{\%O_2|_{dry}}{100}\left[x - A + z + \frac{v}{2}\right]}{A\left(1 - 4.77\frac{\%O_2|_{dry}}{100}\right)}$$

Emission Rates

- Often stated as pollutant mass per unit heat input from fuel
- Equation used: $E_i = \rho_{i,d} F_d \frac{20.9}{20.9 - \%O_{2,d}}$
- Compute $\rho_{i,d} = y_{i,d}M_iP_{std}/R_uT_{std}$
- F_d is dry exhaust volume/heat input
  - Use default values or compute by equation
    - Feb 3 notes have values of K's and default F_d's

$$F_d = \frac{K(K_C\%C + K_H\%H + K_O\%O + K_S\%S + K_N\%N)}{Q_c}$$

Other Equations

- Pollutant mass per unit heat input

$$\frac{m_{CO_2}}{Q_{fuel}} = \frac{3.6642}{Q_c}\frac{wt\%\,C}{100} \qquad \frac{m_{SO_2}}{Q_{fuel}} = \frac{1.9979}{Q_c}\frac{wt\%\,S}{100}$$

- Combustion Efficiency (definitions on next slide)

$$\eta_{comb} = \frac{|q|}{|q|_{\max}} = 1 - \frac{\left[1+\frac{Air}{Fuel}\right]}{Q_c}\int_{T_{in}}^{T_{out}} c_{p_{Air}}\,dT' - \frac{xf\Delta h_{CO}}{M_{fuel}Q_c}$$

Combustion Efficiency

- – Air/fuel is the air to fuel (mass) ratio
- – $C_{p,air}$ = 0.24 Btu/lb$_m$▪R = 1.005 kJ/kg▪K
- – f = molar exhaust ratio CO/(CO + CO$_2$)
- – x = carbon atoms in fuel formula, C_xH_y…
- – Q_c = heat of combustion (Btu/lb$_m$ or kJ/kg)
  - Use lower heating value for water vapor (usual case)
- – Δh_{CO} 282,990 kJ/kgmol = 121,665 Btu/lbmol
- – M_{fuel} is combustible fuel molar mass lb$_m$/lbmol or kg/kmol

Energy Economics

- Look at balance between initial cost and ongoing costs
  - Uses interest rate to consider time value of money
- Key formula relates equivalence between initial cost, P (present value), and ongoing payment stream, A (annual cost)

$$A/P = \frac{i}{1-(1+i)^{-n}} \qquad P/A = \frac{1-(1+i)^{-n}}{i}$$

Using the A/P formula

- Formula applies to any time period so long as i is interest rate per time period
- *E. g.*, for monthly costs with i = 6%/yr = 0.5%/month for N months

$$A/P = \frac{0.005}{1-(1+0.005)^{-N}}$$

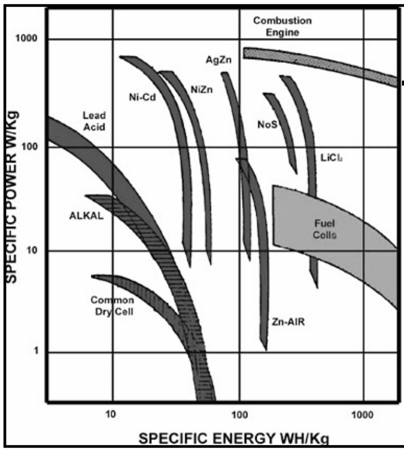
- Need trial-and-error solution (or financial calculator) to find i, given n and A/P
- Can find n for given i and A/P

$$n = -\frac{\ln\left(1-\frac{Pi}{A}\right)}{\ln(1+i)}$$

Energy Storage Measures

- Energy per unit mass (kJ/kg; Btu/lb$_m$)
- Energy per unit volume (kJ/m^3; Btu/ft^3)
- Rate of delivery of energy to and from storage (kW/kg; Btu/hr·lb$_m$)
- Efficiency (energy out/energy in)
- Life cycles – how many times can the storage device be used
  - Particularly important for batteries

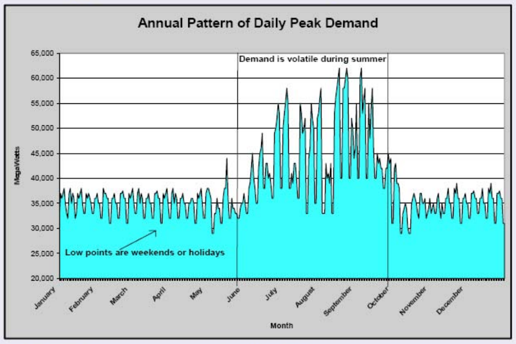
Compare



Batteries versus other motive power

http://www.nap.edu/books/0309092612/html/40.html

Typical Annual Electricity Demand

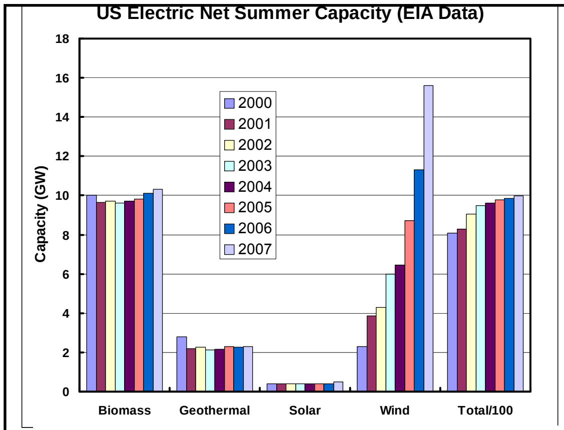


Energy Commission October 18, 2007

Renewable/Alternative

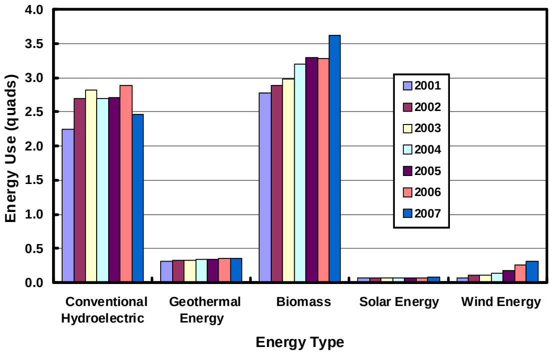
- Alternative or renewable resources
  - Solar energy
  - Wind energy
  - Ocean energy (tides, waves and temperature gradients)
  - Geothermal energy
  - Hydropower especially small hydro
  - Biomass fuels
  - Conservation as an alternative resource
    - Reduced usage and improved efficiencies including vehicle fuel economy

US Electric Net Summer Capacity (EIA Data)



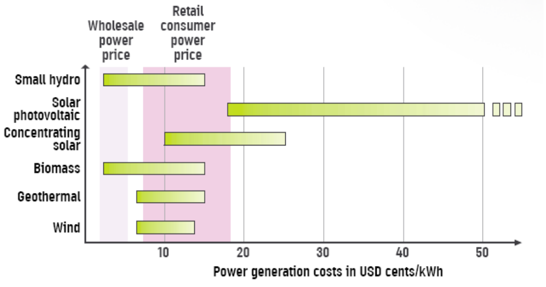
http://www.eia.doe.gov/cneaf/solar.renewables /page/prelim_trends /rea_prereport.html

US Renewable Energy Use 2001-2007



http://www.eia.doe.gov/cneaf/solar.renewables /page/prelim_trends /rea_prereport.html

Power Generation Costs

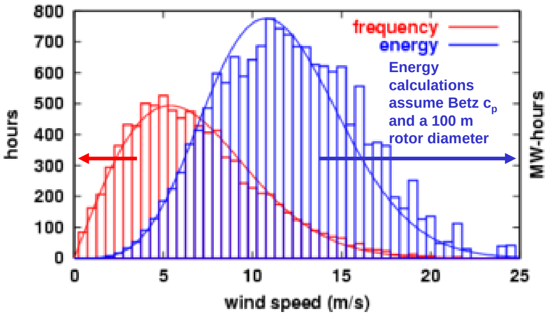


http://www.iea.org/textbase/papers/2006/renewable_factsheet.pdf

Wind Power and Betz Limit

- Power in incoming air = $\dot{m}e = \dot{m}V^2/2 = (\rho VA)V^2/2 = \rho AV^3/2 = P_0$
  - Air density, $\rho \approx 1.2$ kg/m^3
  - A = swept area of rotor = $\pi(D_{rotor})^2/4$
  - V = wind velocity
- c_p = power coefficient = turbine power divided by power in wind
  - Alternative: (generator power) / (wind power)
- Betz Limit: Maximum theoretical c_p = $16/27 \approx 0.593$

Effect of V^3 Dependence



http://www.sandia.gov/wind/other/LeeRanchData-2002.pdf and http://en.wikipedia.org/wiki/Wind_power for plot

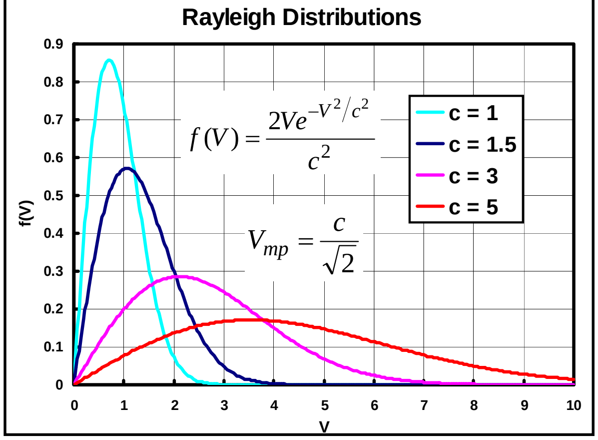
Rayleigh Distribution

- At least three variations are used

$$V_{mp} = \beta = \frac{c}{\sqrt{2}} \qquad f(V) = \frac{Ve^{-V^2/2\beta^2}}{\beta^2} \qquad 0 \le V < \infty$$

$$2\beta^2 = c^2 \qquad \boxed{f(V) = \frac{2Ve^{-V^2/c^2}}{c^2}} \qquad 0 \le V < \infty$$

$$\bar{V} = \beta\sqrt{\frac{\pi}{2}} = \frac{c}{2}\sqrt{\pi} \qquad f(V) = \frac{\pi V e^{-\pi V^2/4\bar{V}^2}}{2\bar{V}} \qquad 0 \le V < \infty$$

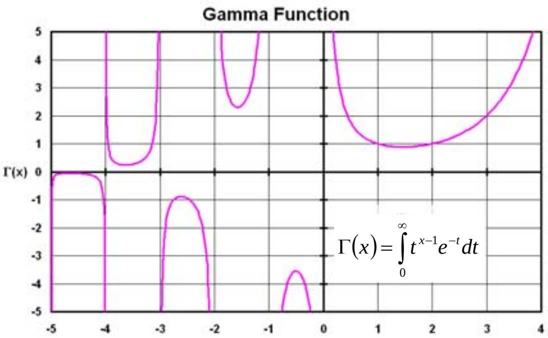


Weibull Distribution

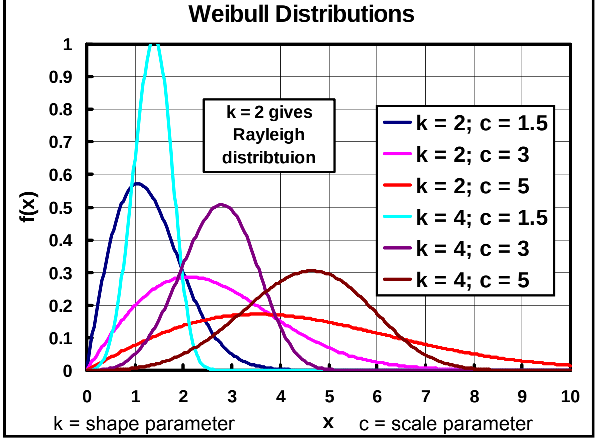
- A two-parameter distribution with shape parameter, k, and scale parameter, c
- Rayleigh distribution is Weibull distribution with k = 2
- Mean = $c\Gamma(1 + k^{-1})$
- Variance = $c^2[\Gamma(1 + 2k^{-1}) - \Gamma^2(1 + k^{-1})]$

Γ is the gamma function

$$f(V) = \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} e^{-(V/c)^k} \qquad 0 \le V < \infty$$



$$\Gamma(x+1) = x\Gamma(x) \qquad \Gamma(1) = \Gamma(2) = 1 \qquad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$



k = shape parameter c = scale parameter

Wind Power

- Instantaneous wind power: $P_0 = \rho V^3 A/2$
- Total or average wind power: $\overline{P}_0 = \frac{\rho A \overline{V^3}}{2} = \frac{\rho A}{2}\int_0^\infty V^3 f(V)dV$
- Total or average turbine power: $\overline{P}_{total} = c_p \overline{P}_0 = c_p \rho A \overline{V^3}\big/2$

$$\left(\overline{V^3}\right)_{Weibull} = c^3\Gamma\left(\frac{3}{k}+1\right)$$

$$\left(\overline{V^3}\right)_{Rayleigh} = c^3\Gamma\left(\frac{3}{2}+1\right) = c^3\frac{3}{2}\Gamma\left(\frac{3}{2}\right) = c^3\frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = c^3\frac{3\sqrt{\pi}}{4}$$

Wind Power Distribution

- Wind power between V_1 and V_2
  - Weibull (Set k = 2 for Rayleigh)

$$Wind\ \overline{P}\ between\ V_1\ and\ V_2 = \frac{\rho A c^3}{2}\int_{(V_1/c)^k}^{(V_2/c)^k} y^{\frac{3}{k}} e^{-y} dy$$

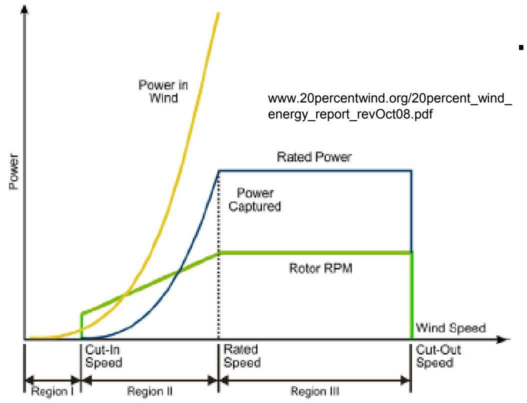
  - Found by numerical integration with results in tables

$$\begin{bmatrix} Wind\ \overline{P}\ between \\ V_1\ and\ V_2 \end{bmatrix} = \left[f_P(V_2) - f_P(V_1)\right]\frac{\rho A c^3}{2}\Gamma\left(\frac{3}{k}+1\right)$$

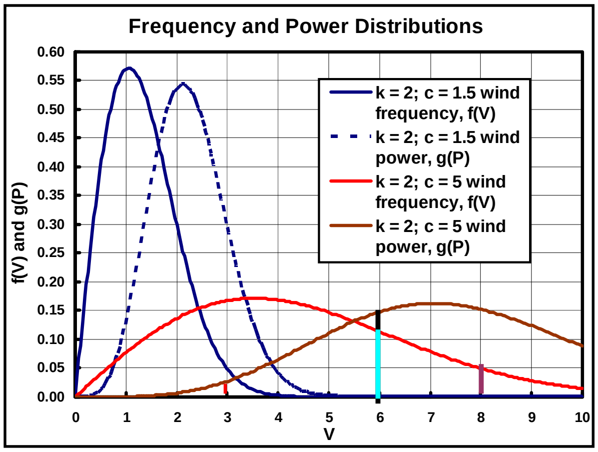
Wind Turbine Operation

- No operation until wind velocity reaches a minimum called the cut-in velocity
- Then operate at full turbine output power until turbine output is greater than generator can accept
- Limit turbine output power to full generator power at high wind speeds
- No operation above maximum velocity called cut-out velocity

Figure 2-8. Typical power output versus wind speed curve



www.20percentwind.org/20percent_wind_energy_report_revOct08.pdf



Average Operating Power

- Generator uses turbine power between $V_{cut\text{-}in}$ and rated (maximum power) velocity, $V_{Pmax} = [2P_{max}/(c_p\rho A)]^{1/3}$
  - Power coefficient c_p = generator power divided by wind power
- Between V_{Pmax} and $V_{cut\text{-}out}$ operate at maximum power

$$\overline{P}_{operation} = \int_{V_{cut-in}}^{V_{P\max}} \frac{c_p \rho A V^3}{2} f(V)dV + \int_{V_{P\max}}^{V_{cut-out}} P_{\max} f(V)dV$$

Average Operating Power II

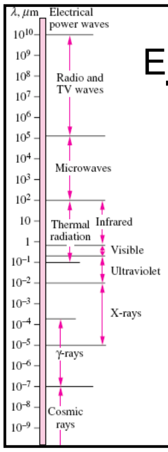
- Using power fraction table

$$\int_{V_{cut-in}}^{V_{P\max}} \frac{c_p \rho A V^3}{2} f(V)dV = \left[f_P(V_{cut-in}) - f_P(V_{P\max})\right]\frac{c_p \rho A c^3}{2}\Gamma\left(\frac{3}{k}+1\right)$$

- Using cumulative distribution

$$\int_{V_{P\max}}^{V_{cut-out}} P_{\max} f(V)dV = P_{\max}\left[\left(1-e^{-(V_{cut-out}/c)^k}\right)-\left(1-e^{-(V_{P\max}/c)^k}\right)\right]$$

$$\overline{P}_{operation} = \left[f_P(V_{cut-in}) - f_P(V_{P\max})\right]\frac{c_p \rho A c^3}{2}\Gamma\left(\frac{3}{k}+1\right) + P_{\max}\left(e^{-(V_{P\max}/c)^k} - e^{-(V_{cut-out}/c)^k}\right)$$



Electromagnetic Radiation

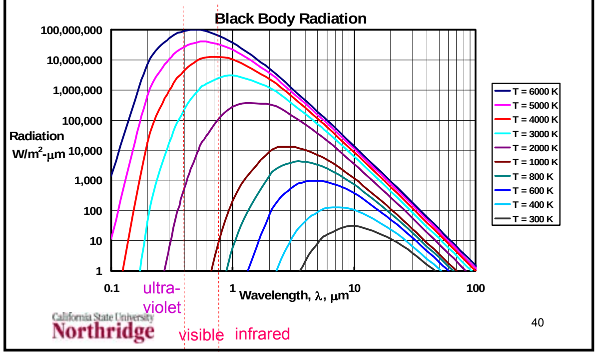
- Radiation heat transfer by electromagnetic radiation
  - Part of much larger spectrum
  - Thermal radiation transfers heat without contact
    - Use of fire or electric resistance heating are best examples
    - Thermal radiation lies in infrared and visible part of spectrum (with some in ultraviolet)

Figure 12-3 from Çengel, *Heat and Mass Transfer*

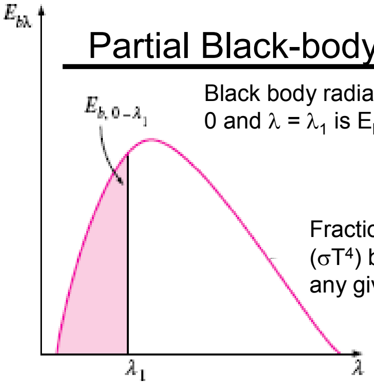
Black-body Radiation Spectrum

- Basic black body equation: $E_b = \sigma T^4$
  - E_b is total black-body radiation energy flux W/m² or Btu/hr·ft²; σ is the Stefan-Boltzmann constant
- $E_{b\lambda}$ is spectral radiation
  - Units are W/(m²·μm)
  - $E_{b\lambda}d\lambda$ is fraction of black body radiation in range dλ about wavelength λ
- Maximum occurs at λT = 2897.8 μm·K
  - T increase shifts peak shift to lower λ

Spectral $E_{b\lambda}$



Partial Black-body Power



Black body radiation between λ = 0 and λ = λ_1 is $E_{b,0-\lambda1}$

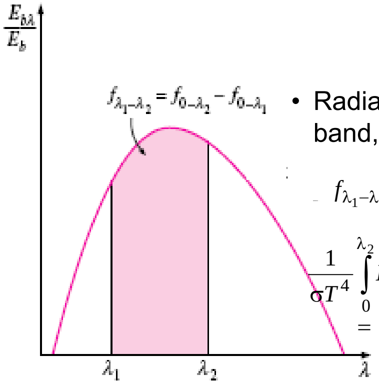
$$E_{b,0-\lambda_1} = \int_0^{\lambda_1} E_{b\lambda}d\lambda$$

Fraction of total radiation (σT^4) between λ = 0 and any given λ is f_λ

$$f_\lambda = \frac{1}{\sigma T^4}\int_0^{\lambda} E_{b\lambda}d\lambda'$$

Figure 12-13 from Çengel, *Heat and Mass Transfer*

Δλ



- Radiation in finite band, Δλ

$$f_{\lambda_1-\lambda_2} = \frac{1}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{b\lambda}d\lambda = \frac{1}{\sigma T^4}\int_0^{\lambda_2} E_{b\lambda}d\lambda - \frac{1}{\sigma T^4}\int_0^{\lambda_1} E_{b\lambda}d\lambda = f(\lambda_2 T) - f(\lambda_1 T)$$

Figure 12-14 from Çengel, *Heat and Mass Transfer*

Emissivity

- Emissivity = ratio of actual radiated power to that of black body
  - Diffuse surface – emissivity does not depend on direction
  - Gray surface – emissivity does not depend on wavelength
  - Gray, diffuse surface – emissivity is the does not depend on direction or wavelength
    - Simplest surface to handle and often used in radiation calculations

Properties

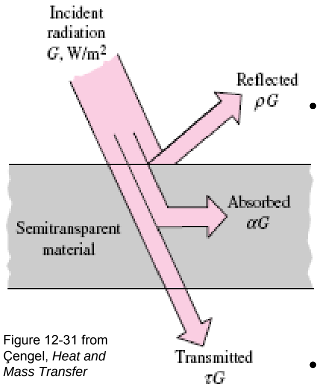


Figure 12-31 from Çengel, *Heat and Mass Transfer*

- When radiation, G, hits a surface a fraction ρG is reflected; another fraction, αG is absorbed, a third fraction τG is transmitted
- Energy balance: $\rho + \alpha + \tau = 1$

Kirchoff's Law

- Absorptivity equals emissivity (at the same temperature) $\alpha_\lambda = \varepsilon_\lambda$
- True only for values in a given direction and wavelength
- Assuming total hemispherical values of α and ε are the same simplifies radiation heat transfer calculations, but is not always a good assumption

Effect of Temperature

- Emissivity, ε, depends on surface temperature
- Absorptivity, α, depends on source temperature (e.g. $T_{sun} \approx 5800$ K)
- For surfaces exposed to solar radiation
  - high α and low ε will keep surface warm
  - low α and high ε will keep surface cool
  - Does not violate Kirchoff's law since source and surface temperatures differ

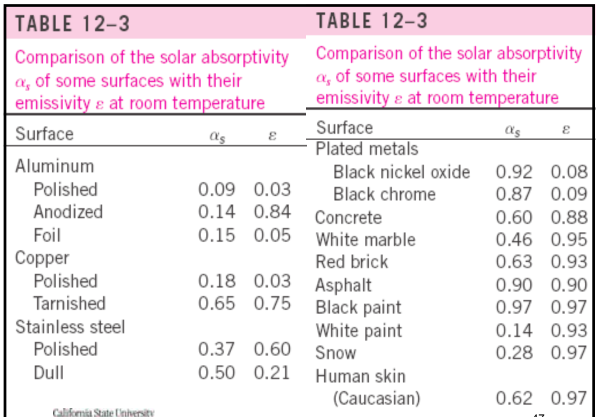


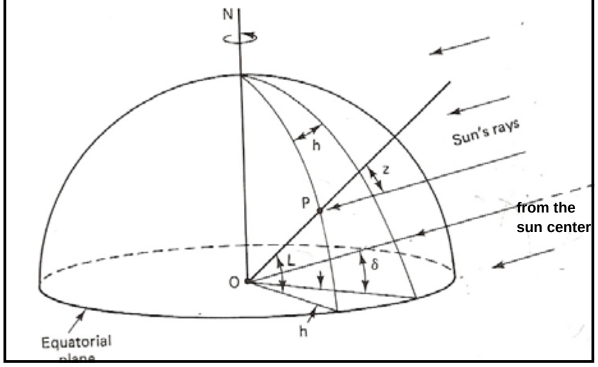
TABLE 12–3

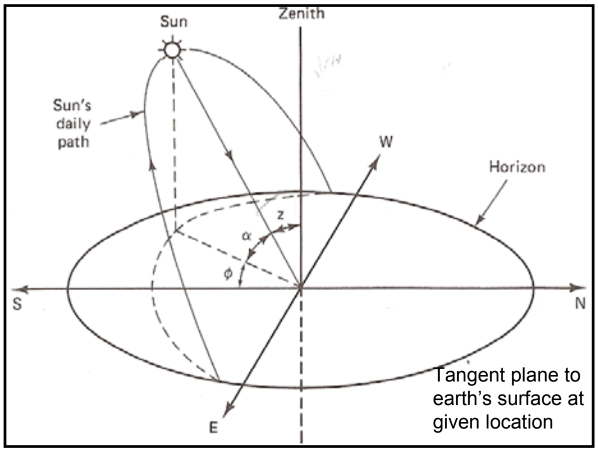
Comparison of the solar absorptivity α_s of some surfaces with their emissivity ε at room temperature

Surface	α_s	ε
Aluminum		
Polished	0.09	0.03
Anodized	0.14	0.84
Foil	0.15	0.05
Copper		
Polished	0.18	0.03
Tarnished	0.65	0.75
Stainless steel		
Polished	0.37	0.60
Dull	0.50	0.21
Plated metals		
Black nickel oxide	0.92	0.08
Black chrome	0.87	0.09
Concrete	0.60	0.88
White marble	0.46	0.95
Red brick	0.63	0.93
Asphalt	0.90	0.90
Black paint	0.97	0.97
White paint	0.14	0.93
Snow	0.28	0.97
Human skin (Caucasian)	0.62	0.97

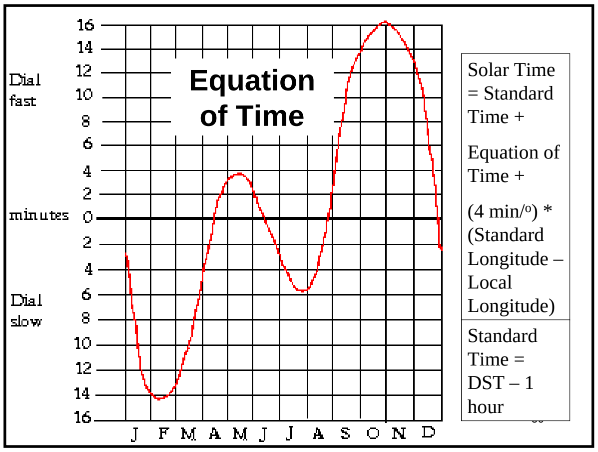
From Çengel, *Heat and Mass Transfer*

Solar Angles III



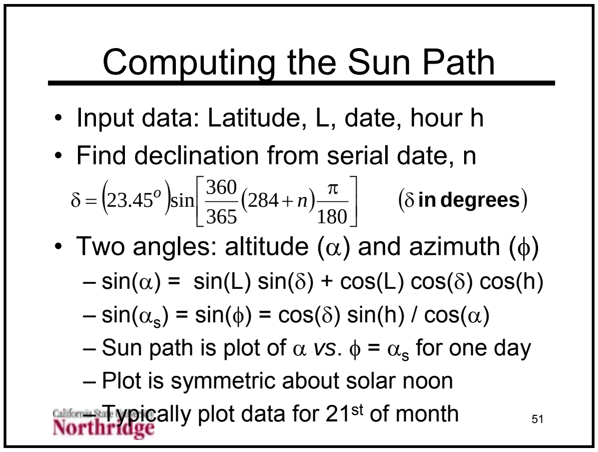


Tangent plane to earth's surface at given location



Solar Time = Standard Time + Equation of Time + (4 min/°) * (Standard Longitude – Local Longitude)

Standard Time = DST – 1 hour



Computing the Sun Path

- Input data: Latitude, L, date, hour h
- Find declination from serial date, n

$$\delta = \left(23.45^o\right)\sin\left[\frac{360}{365}(284+n)\frac{\pi}{180}\right] \qquad (\delta\ \textbf{in degrees})$$

- Two angles: altitude (α) and azimuth (ϕ)
  - $\sin(\alpha) = \sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(h)$
  - $\sin(\alpha_s) = \sin(\phi) = \cos(\delta)\sin(h) / \cos(\alpha)$
  - Sun path is plot of α *vs*. $\phi = \alpha_s$ for one day
  - Plot is symmetric about solar noon
  - Typically plot data for 21st of month

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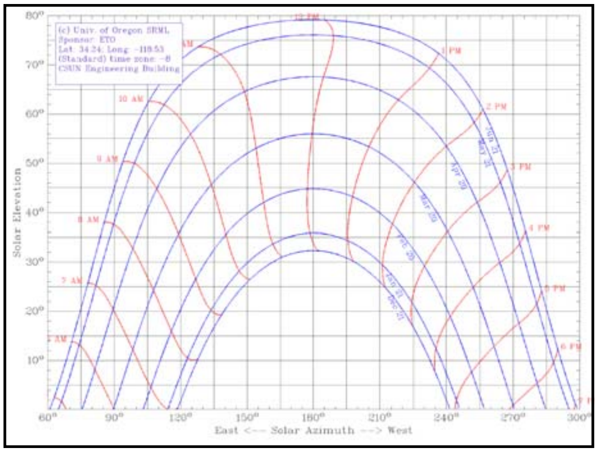

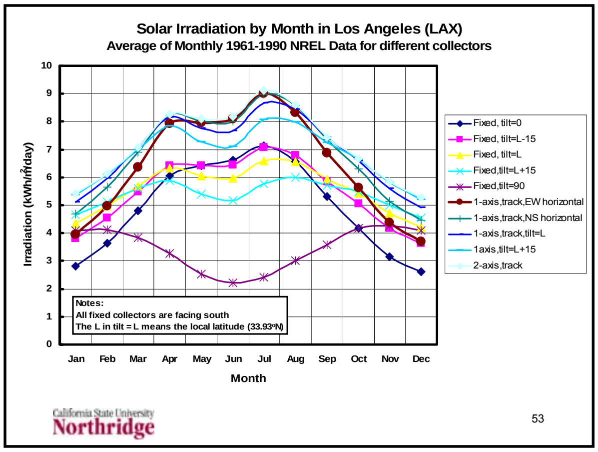

Notes:
All fixed collectors are facing south
The L in tilt = L means the local latitude (33.93°N)

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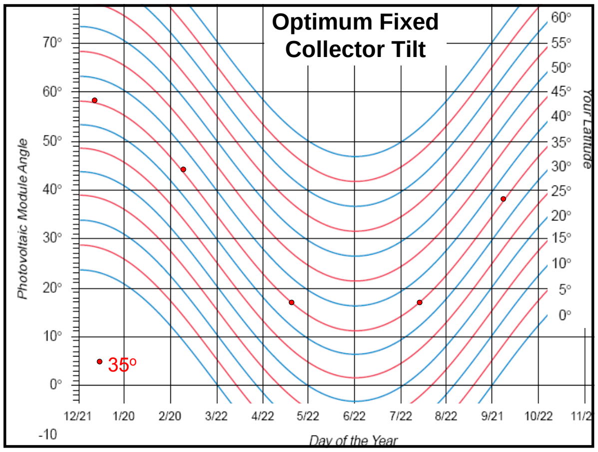
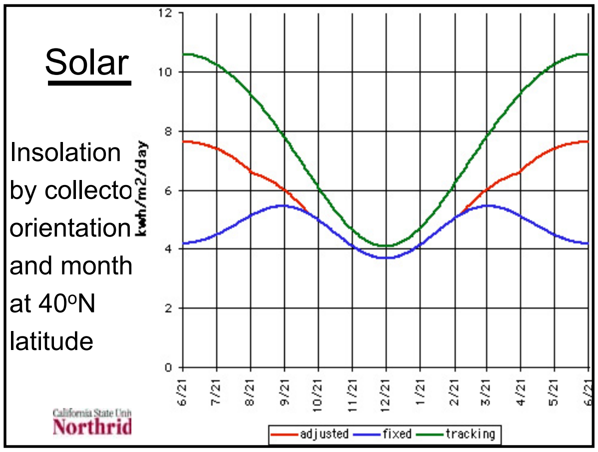
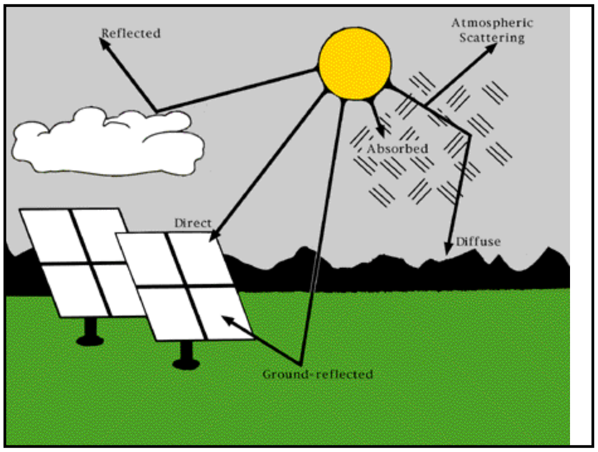
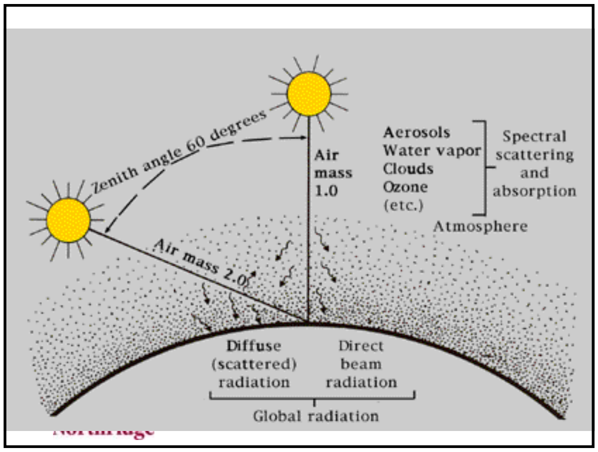


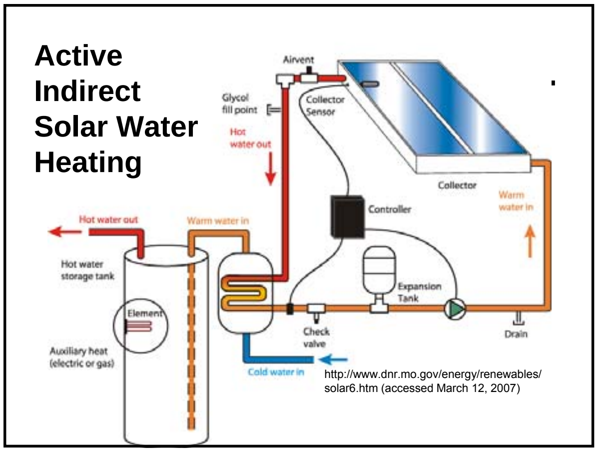


Solar

Insolation by collecto orientation and month at 40°N latitude

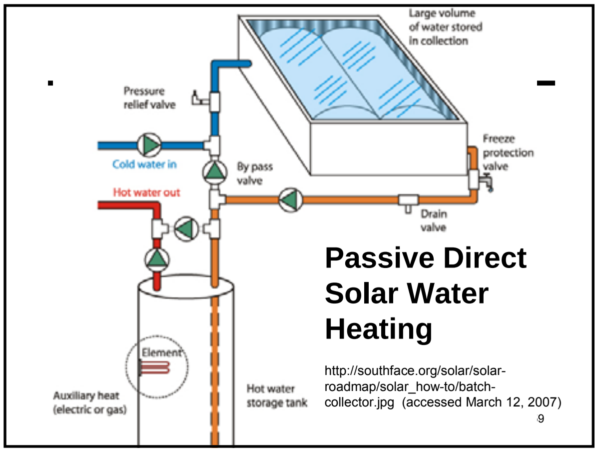






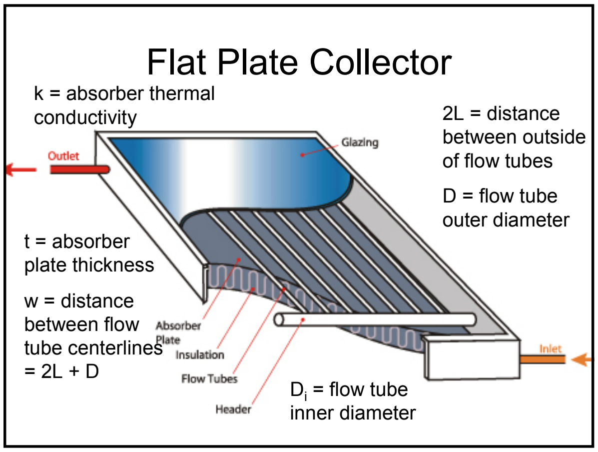
Active Indirect Solar Water Heating

http://www.dnr.mo.gov/energy/renewables/solar6.htm (accessed March 12, 2007)



Passive Direct Solar Water Heating

http://southface.org/solar/solar-roadmap/solar_how-to/batch-collector.jpg (accessed March 12, 2007)



Flat Plate Collector

k = absorber thermal conductivity

2L = distance between outside of flow tubes

D = flow tube outer diameter

t = absorber plate thickness

w = distance between flow tube centerlines = 2L + D

D_i = flow tube inner diameter

Solar Collector Analysis

- Three analysis steps for solar energy to heat fluid (Hottel-Whillier-Bliss equation)
  - – Solar energy into plate flows across plate to location of tubes at some line on plate
  - – At same line heat flow into collector fluid from plate is determined
  - – Integrate heat flow into fluid from inlet to exit to get total useful heat transfer to fluid

$$\dot{Q}_u = F_R A_c\left[H_a - U_c\left(T_{f.in} - T_a\right)\right]$$

$$\dot{Q}_u = \dot{m}_f\left(T_{f.out} - T_{f.in}\right)$$

Loss Through Top

- Analyze set of series thermal resistances with common Q_{top}
  - – Heat transfer between absorber plate and lower glass plate shown below

$$Q_{top} = \left(h_{p-g2} + h_{r,p-g2}\right)A_c\left(T_P - T_{g2}\right) = \frac{T_P - T_{g2}}{R_{p-g2}}$$

$$h_{r,p-g2} = \frac{A_c\sigma\left(T_P^2 + T_{g2}^2\right)\left(T_P + T_{g2}\right)}{\frac{1}{\varepsilon_P} + \frac{1}{\varepsilon_{g2}} - 1}$$

Remaining Top Loss Path

- Between glass plates

$$Q_{top} = \left(h_{g2-g1} + h_{r,g2-g1}\right)A_c\left(T_{g2} - T_{g1}\right) = \frac{T_{g2} - T_{g1}}{R_{g2-g1}}$$

$$h_{r,g2-g1} = \frac{A_c\sigma\left(T_{g1}^2 + T_{g2}^2\right)\left(T_{g1} + T_{g2}\right)}{\frac{1}{\varepsilon_{g1}} + \frac{1}{\varepsilon_{g2}} - 1}$$

- Top plate to ambient

$$Q_{top} = \left(h_{g1-a} + h_{r,g1-a}\right)A_c\left(T_{g1} - T_a\right) = \frac{T_{g1} - T_a}{R_{g2-g1}}$$

$$h_{r,g1-a} = \frac{A_c\sigma\left(T_{g1}^2 + T_{sky}^2\right)\left(T_{g1} + T_{sky}\right)}{\frac{1}{\varepsilon_{g1}} + \frac{1}{\varepsilon_{g2}} - 1}\frac{T_{g1} - T_{sky}}{T_{g1} - T_a}$$

Loss Through Top/Bottom

- Combine three resistances in series to get $R_{top} = R_{P\text{-}g2} + R_{g2\text{-}g1} + R_{g1\text{-}a}$
  - – $Q_{top} = (T_P – T_a)/R_{top} = U_{top}A_c(T_P – T_a)$
- Loss through bottom is conduction through insulation (k_{ins}, Δx_{ins}) in series with convection to ambient with $h_{b\text{-}a}$

$$Q_{bottom} = \frac{T_P - T_a}{R_{ins} + R_{conv}} = \frac{T_P - T_a}{\frac{k_{ins}}{\Delta x_{ins}A_c} + \frac{1}{h_{b-a}A_c}} = U_{bottom}A_c\left(T_P - T_a\right)$$

Total Loss

- $Q_{sides} = U'_{side}A_{side}(T_P – T_a)$
  - – Can estimate U'_{side} = 0.5 W/m²·K
  - – Use $U_{side}A_c = U'_{side}A_{side}$ for common area
  - – $Q_{sides} = U_{side}A_c(T_P – T_a) = (T_P – T_a)/A_{side}$
- Total is sum of individual losses
- $Q_{loss} = U_cA_c(T_P – T_a)= (T_P – T_a)/R_c$
- Overall conductance and resistance
- $U_c = U_{top} + U_{bottom} + U_{sides}$

$$\frac{1}{R_c} = \frac{1}{R_{top}} + \frac{1}{R_{bottom}} + \frac{1}{R_{side}}$$

Approximate U_{top} Equation

$$U_{top} = \frac{1}{\frac{A'}{T_p}\left(\frac{T_p - T_a}{N + B}\right)^{0.33} + \frac{1}{h_w}} + \frac{\sigma\left(T_p + T_a\right)\left(T_p^2 + T_a^2\right)}{\frac{1}{\varepsilon_p + 0.05N\left(1 - \varepsilon_p\right)} + \left(\frac{2N + B - 1}{\varepsilon_g}\right) - N}$$

N = number of glass covers

A’ = 250[1 – 0.0044(s – 90)]

s = tilt angle (degrees)

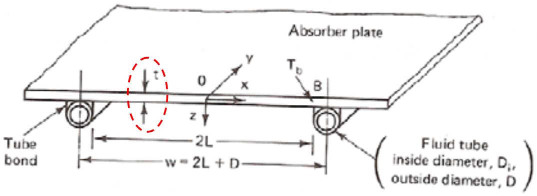
B = (1 – 0.04h_w + 0.0005h_w^2)(1 + 0.091N)

h_w = heat transfer coefficient from top to ambient

Other symbols have previous definitions

Equation uses SI units: U_c and h in W/m²·K, T in K, σ = 5.670x10⁻⁸ W/m²·K⁴, ε_g is same for all glass covers

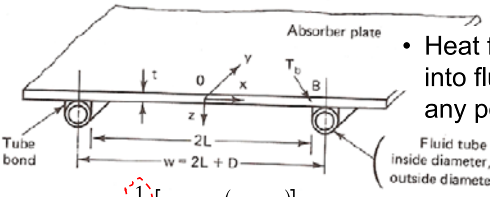
Absorber Plate Analysis



- Define **$m^2 = U_c/(tk_{plate})$**
- Effectiveness factor, **F = tanh(mL) / (mL)**
- Total (useful) heat transfer per unit length of tube

$$q_{total} = (2LF + D)[H_a - U_c(T_b - T_a)] = q_u^{'}$$

Absorber Plate Analysis II



- Heat flow into fluid at any point

$$q_u^{'} = \frac{\frac{1}{U_c}\left[H_a - U_c\left(T_f - T_a\right)\right]}{\left[\frac{1}{(2LF + D)U_c} + \left(\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}\right)\right]} = wF'\left[H_a - U_c\left(T_f - T_a\right)\right]$$

Factors, F’ and F_R

- Collector efficiency factor, F’

$$F' = \frac{\text{Thermal Resistance Between Plate and Ambient}}{\text{Thermal Resistance Between Fluid and Ambient}}$$

$$F' = \frac{1/U_c}{w\left[1/\left[(2LF + D)U_c\right] + \left(1/C_B + 1/\left(h_{c,i}\pi D_i\right)\right)\right]}$$

- Heat removal factor, F_R

$$\frac{F_R}{F'} = \frac{\dot{m}c_p}{U_c A_c F'}\left(1 - e^{-\frac{U_c A_c F'}{\dot{m}c_p}}\right) = \frac{1}{a}\left(1 - e^{-a}\right) \quad a = \frac{U_c A_c F'}{\dot{m}c_p}$$

$$F_R/F' \to [1, 1/a] \; as \; a \to [0, \infty]$$

Summary of Results

- Q_u = useful heat transfer to working fluid

$$m^2 = \frac{U_c}{k_P t_P} \qquad F' = \frac{1}{\frac{w}{U_c}\left[\frac{1}{U_c(2LF + D)} + \frac{1}{C_B} + \frac{1}{\pi D_i h_i}\right]}$$

$$F = \frac{\tanh mL}{mL}$$

$$H_a = H_i\left(\overline{\tau\alpha}\right) \qquad F_R = \frac{\dot{m}c_p}{U_c A}\left[1 - e^{-\frac{U_c AF'}{\dot{m}c_p}}\right]$$

$$Q_u = AF_R\left[H_a - U_c\left(T_{f,in} - T_a\right)\right] \qquad T_{f,out} = T_{f,in} + \frac{Q_u}{\dot{m}c_p}$$

Collector Efficiency, $\eta_c = Q_u/A_cH_i$

- Start with Hottel-Whillier-Bliss Equation

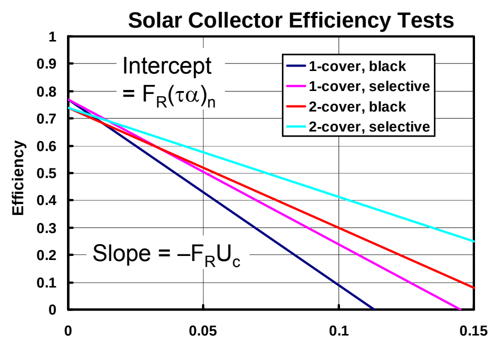
$$Q_u = F_R A_c\left[H_a - U_c\left(T_{f.in} - T_a\right)\right]$$

- Replace H_a by $H_i\tau\alpha$

$$Q_u = F_R A_c\left[H_i\tau\alpha - U_c\left(T_{f.in} - T_a\right)\right]$$

- Substitute into efficiency equation

$$\eta_c = \frac{Q_u}{A_c H_i} = \frac{F_R A_c\left[H_i\tau\alpha - U_c\left(T_{f.in} - T_a\right)\right]}{A_c H_i} = F_R\tau\alpha - \frac{F_R U_c\left(T_{f.in} - T_a\right)}{H_i}$$



Sample Rating Sheet

SOLAR COLLECTOR
CERTIFICATION AND RATING

SRCC OG-100

CERTIFIED SOLAR COLLECTOR

SUPPLIER: **Heliodyne, Inc.**
4910 Seaport Avenue
Richmond, CA 94804

MODEL: Heliodyne Gobi 408
COLLECTOR TYPE: Glazed Flat-Plate
CERTIFICATION #: 100-1981-085A

COLLECTOR THERMAL PERFORMANCE RATING

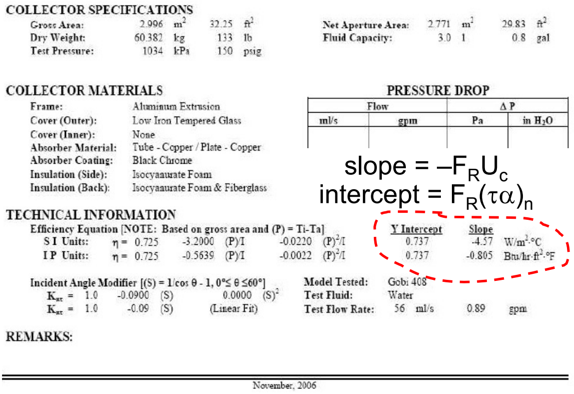
Megajoules Per Panel Per Day				Thousands of Btu Per Panel Per Day			
CATEGORY (Ti-Ta)	CLEAR DAY 23 MJ/m²·d	MILDLY CLOUDY 17 MJ/m²·d	CLOUDY DAY 11 MJ/m²·d	CATEGORY (Ti-Ta)	CLEAR DAY 2000 Btu/ft²·d	MILDLY CLOUDY 1500 Btu/ft²·d	CLOUDY DAY 1000 Btu/ft²·d
A (-5°C)	49	37	25	A (-9°F)	46	35	24
B (5°C)	45	33	21	B (9°F)	43	32	20
C (20°C)	39	27	15	C (36°F)	37	25	14
D (50°C)	24	14	4	D (90°F)	23	13	4
E (80°C)	10	2		E (144°F)	10	2	

A-Pool Heating (Warm Climate) B-Pool Heating (Cool Climate) C-Water Heating (Warm Climate) D-Water Heating (Cool Climate) E-Air Conditioning

Original Certification Date: August 1, 1983

http://www.builditsolar.com/References/Ratings/SRCCRating.htm

Sample Rating Sheet II



COLLECTOR SPECIFICATIONS

Gross Area:	2.996	m²	32.25	ft²
Dry Weight:	60.382	kg	133	lb
Test Pressure:	1034	kPa	150	psig
Net Aperture Area:	2.771	m²	29.83	ft²
Fluid Capacity:	3.0	l	0.8	gal

COLLECTOR MATERIALS

Frame:	Aluminum Extrusion
Cover (Outer):	Low Iron Tempered Glass
Cover (Inner):	None
Absorber Material:	Tube - Copper / Plate - Copper
Absorber Coating:	Black Chrome
Insulation (Side):	Isocyanurate Foam
Insulation (Back):	Isocyanurate Foam & Fiberglass

PRESSURE DROP

Flow		ΔP	
ml/s	gpm	Pa	in H_2O

slope = $-F_R U_c$
intercept = $F_R(\tau\alpha)_n$

TECHNICAL INFORMATION

Efficiency Equation [NOTE: Based on gross area and (P) = Ti-Ta]

						Y Intercept	Slope	
S I Units:	η = 0.725	-3.2000	(P)/I	-0.0220	(P)²/I	0.737	-4.57	W/m²·°C
I P Units:	η = 0.725	-0.5639	(P)/I	-0.0022	(P)²/I	0.737	-0.805	Btu/hr·ft²·°F

Incident Angle Modifier [(S) = 1/cos θ - 1, 0°≤ θ ≤60°]
$K_{\alpha\tau}$ = 1.0 -0.0900 (S) 0.0000 (S)²
$K_{\alpha\tau}$ = 1.0 -0.09 (S) (Linear Fit)

Model Tested: Gobi 408
Test Fluid: Water
Test Flow Rate: 56 ml/s 0.89 gpm

REMARKS:

November, 2006

f-chart Method

- Predicts fraction of demand over a time period (usually monthly) than can be supplied by solar
- Two empirical pamameters, X and Y
  - X is ratio of reference collector loss to total heating load
  - Y is ratio of absorbed solar energy to total heating load

$$X = \frac{A_c F_R' U_c \Delta t}{D}\left(T_{ref} - \overline{T}_a\right) \qquad Y = \frac{A_c F_R' \overline{\tau\alpha}}{D} H_{i,total}$$

Computing X (dimensionless)

$$X = A_c\left[F_R U_c \frac{F_R'}{F_R}\frac{\Delta t}{D}\left(T_{ref} - \overline{T}_a\right)\right]$$

- A_c = collector area (m²)
- $F_R U_c$ (W/m²·K) from slope of collector test data
- F'_R/F_R computed or assumed = 0.97
- Usual averaging period, Δt = 1 month, converted to seconds
- D = heating demand for averaging period (J)
- Tref = 100°C; $\overline{T}_a$ from NREL data

Computing Y (dimensionless)

$$Y = A_c\left[F_R(\tau\alpha)_n \frac{F_R'}{F_R}\frac{\overline{\tau\alpha}}{(\tau\alpha)_n}\frac{H_{i,total}}{D}\right]$$

- A_c = collector area (m²)
- $F_R(\tau\alpha)_n$ from intercept of collector test
- F'_R/F_R computed or assumed = 0.97
- Ratio $\overline{\tau\alpha}/(\tau\alpha)_n$ = 0.94 (October – March), = 0.90 (April – September) or computed
- $H_{i,total}$ is available from NREL data for Δt = 1 month (convert to J/m²)
- D is heating demand J

f Equations

- For water heating: $f = 1.029Y - 0.065X - 0.245Y^2 + 0.0018X^2 + 0.0215Y^3$
  - Adjustments required
    - Adjust X for hot water supply only and storage capacity different from standard
    - Adjust Y for load heat exchanger capacity
- For air heating: $f = 1.040Y - 0.065X - 0.159Y^2 + 0.00187X^2 - 0.0095Y^3$
  - Solar collectors heating air have no heat exchanger so $F'_R = F_R$

Adjustments

- Adjust X for storage capacity, M, in L/m^2 X' = X(75/M)$^{1/4}$
- Adjustment for water heating only
  - See f-chart notes for details
- Adjust Y for load heat exchanger factor, Z: Y' = Y(0.39 + 0.65e$^{-0.139/Z}$)
  - ε_C = heat exchanger effectiveness

$$Z = \varepsilon_C\left(\dot{m}c_p\right)_{\min}/(UA)_L$$

NREL 1961-1990 LAX Average

SOLAR RADIATION FOR FLAT-PLATE COLLECTORS FACING SOUTH AT A FIXED-TILT (kWh/m2/day) Percentage Uncertainty = 9

Tilt(deg)		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
0	Average	2.8	3.6	4.8	6.1	6.4	6.6	7.1	6.5	5.3	4.2	3.2	2.6	4.9
	Minimum	2.3	3.0	4.0	5.5	5.7	5.6	6.4	6.1	4.4	3.8	2.7	2.1	4.7
	Maximum	3.3	4.4	5.6	6.8	7.2	7.7	8.0	7.0	5.8	4.5	3.6	3.0	5.1
Lat - 15	Average	3.8	4.5	5.5	6.4	6.4	6.4	7.1	6.8	5.9	5.0	4.2	3.6	5.5
	Minimum	2.9	3.6	4.5	5.8	5.7	5.4	6.3	6.3	4.7	4.4	3.4	2.7	5.2
	Maximum	4.6	5.7	6.4	7.3	7.3	7.3	7.9	7.2	6.6	5.6	4.9	4.3	5.7
Lat	Average	4.4	5.0	5.7	6.3	6.1	6.0	6.6	6.6	6.0	5.4	4.7	4.2	5.6
	Minimum	3.3	3.8	4.7	5.6	5.4	5.0	5.9	6.1	4.8	4.7	3.7	3.0	5.3
	Maximum	5.4	6.4	6.7	7.2	6.8	6.7	7.3	7.0	6.7	6.0	5.6	5.0	5.9
Lat + 15	Average	4.7	5.1	5.6	5.9	5.4	5.2	5.8	6.0	5.7	5.5	5.0	4.5	5.4
	Minimum	3.4	3.8	4.5	5.2	4.8	4.4	5.2	5.5	4.5	4.7	3.9	3.1	5.1
	Maximum	5.9	6.6	6.6	6.7	6.1	5.8	6.3	6.4	6.5	6.1	6.0	5.4	5.7
90	Average	4.1	4.1	3.8	3.3	2.5	2.2	2.4	3.0	3.6	4.2	4.3	4.1	3.5
	Minimum	2.9	3.0	3.1	2.9	2.3	2.1	2.3	2.8	2.9	3.5	3.2	2.7	3.3
	Maximum	5.2	5.4	4.5	3.6	2.7	2.3	2.5	3.2	4.1	4.7	5.2	5.0	3.7