Review for Second Midterm

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Mechanical Engineering 483

Alternative Energy Engineering II

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Outline

- Black-body and solar radiation
- Emissivity and absorptivity
- Path of the sun
- Solar collectors
  - Basic analysis
    - Useful gain = Absorbed solar – Heat Loss
    - Overall loss coefficient, U_c
    - Effectiveness terms and factors: F, F', F_R, F'_R
  - f-chart method

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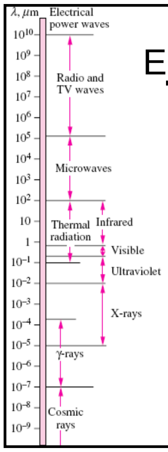
Electromagnetic Radiation



- Radiation heat transfer by electromagnetic radiation
  - Part of much larger spectrum
  - Thermal radiation transfers heat without contact
    - Use of fire or electric resistance heating are best examples
    - Thermal radiation lies in infrared and visible part of spectrum (with some in ultraviolet)

Figure 12-3 from Çengel, *Heat and Mass Transfer*

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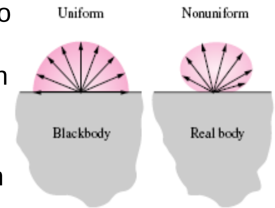
Black-body Radiation

- Perfect emitter – no surface can emit more radiation than a black body
- Diffuse emitter – radiation is uniform in all directions



- Perfect absorber – all radiation striking a black body is absorbed

Figure 12-7 from Çengel, *Heat and Mass Transfer*

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Black-Body Radiation II

- Basic black body equation: $E_b = \sigma T^4$
  - E_b is total black-body radiation energy flux W/m^2 or Btu/hr·ft^2
  - σ is the Stefan-Boltzmann constant
    - $\sigma = = 2\pi^5k^4/(15h^3c^2) = 5.670\times10^{-8}$ W/m^2·K^4 = 0.1714×10^{-8} Btu/hr·ft^2·R^4
    - k = Boltzmann's constant = 1.38065×10^{-23} J/K (molecular gas constant) = $R_u/N_{Avagadro}$
    - h = Planck's constant = 6.62607×10^{-34} J·s
    - c = 299,792,458 m/s = speed of light in a vacuum

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Black-body Radiation Spectrum

- Energy (W/m^2) emitted varies with wavelength and temperature
- $E_{b\lambda}$ is spectral radiation
  - Units are W/(m^2·μm)
  - $E_{b\lambda}d\lambda$ is fraction of black body radiation in range dλ about wavelength λ
- Maximum occurs at λT = 2897.8 μm·K
  - T increase shifts peak shift to lower λ
- Diagram on next chart

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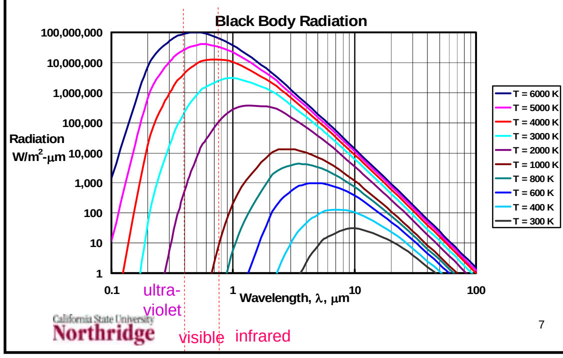
Spectral $E_{b\lambda}$



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Spectral Black-body Energy

- $E_{b\lambda}d\lambda$ = black-body emissive power in a wavelength range $d\lambda$ about λ
  - Typical units for $E_{b\lambda}$ are W/m²·μm or Btu/hr·ft²·μm

$$E_{b\lambda}d\lambda = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}d\lambda$$

- $C_1 = 2\pi hc^2 = 3.74177$ W·μm⁴/m²
- $C_2 = hc/k = 14387.8$ μm/K
  - h = Planck's constant, c = speed of light in vacuum, k = Boltzmann's constant

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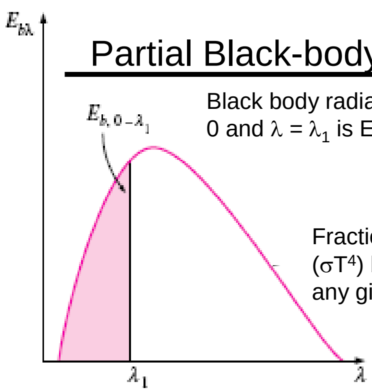
Partial Black-body Power



Black body radiation between λ = 0 and $\lambda = \lambda_1$ is $E_{b,0-\lambda 1}$

$$E_{b,0-\lambda_1} = \int_0^{\lambda_1} E_{b\lambda}d\lambda$$

Fraction of total radiation (σT^4) between λ = 0 and any given λ is f_λ

$$f_\lambda = \frac{1}{\sigma T^4}\int_0^{\lambda} E_{b\lambda}d\lambda'$$

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Figure 12-13 from Çengel, *Heat and Mass Transfer*

Radiation Tables

- Can show that f_λ is function of λT

$$f_\lambda = \frac{1}{\sigma T^4}\int_0^{\lambda} E_{b\lambda}d\lambda = \frac{1}{\sigma T^4}\int_0^{\lambda}\frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}d\lambda = \frac{1}{\sigma}\int_0^{\lambda T}\frac{C_1}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)}d(\lambda T)$$

- Radiation tables give f_λ versus λT
  - See table 12-2, page 118 in Hodge
  - Extract from similar table shown at right

Blackbody radiation functions f_λ

λT, μm · K	f_λ
200	0.000000
400	0.000000
600	0.000000
800	0.000016
1000	0.000321
1200	0.002134
1400	0.007790
1600	0.019718
1800	0.039341
2000	0.066728

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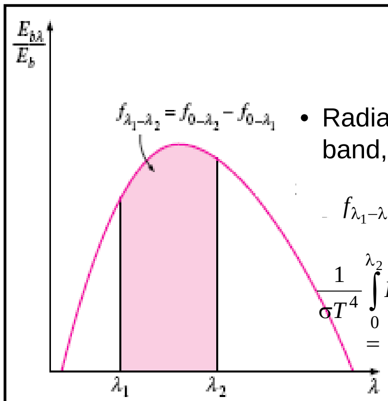
$\Delta\lambda$



- Radiation in finite band, $\Delta\lambda$

$$f_{\lambda_1-\lambda_2} = \frac{1}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{b\lambda}d\lambda = \frac{1}{\sigma T^4}\int_0^{\lambda_2} E_{b\lambda}d\lambda - \frac{1}{\sigma T^4}\int_0^{\lambda_1} E_{b\lambda}d\lambda = f(\lambda_2 T) - f(\lambda_1 T)$$

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Figure 12-14 from Çengel, *Heat and Mass Transfer*

Emissivity

- Emissivity, ε, is ratio of actual emissive power to black body emissive power
  - May be defined on a directional and wavelength basis, $\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) = I_{\lambda,e}(\lambda,\theta,\phi,T)/I_{b\lambda}(\lambda,T)$, called spectral, directional emissivity
  - Total directional emissivity, average over all wavelengths, $\varepsilon_\theta(\theta,\phi,T) = I_e(\theta,\phi,T)/I_b(T)$
  - Spectral hemispherical emissivity average over directions, $\varepsilon_\lambda(\lambda,T) = I_\lambda(\lambda,T)/I_{b\lambda}(\lambda,T)$
  - Total hemispheric emissivity = $E(T)/E_b(T)$

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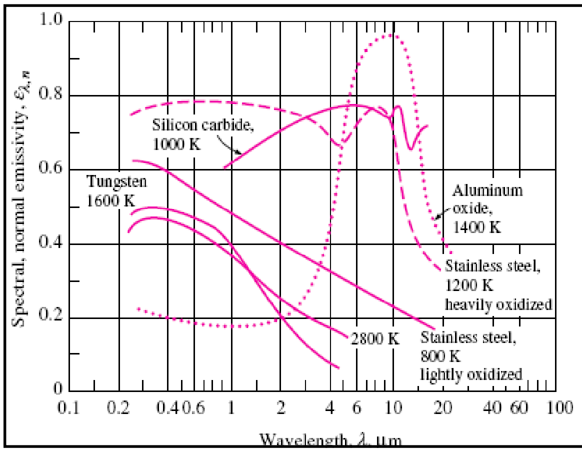
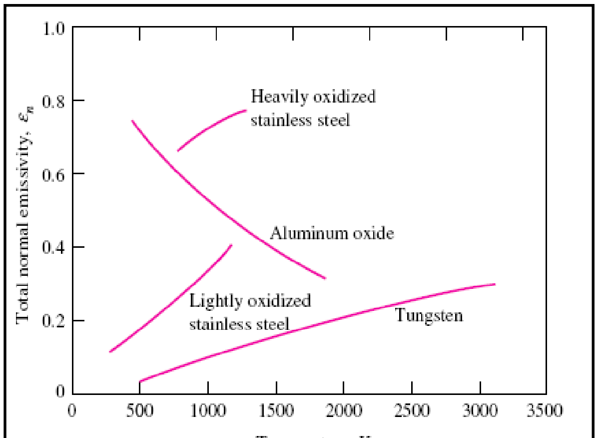
Emissivity Assumptions

- Diffuse surface – emissivity does not depend on direction
- Gray surface – emissivity does not depend on wavelength
- Gray, diffuse surface – emissivity is the does not depend on direction or wavelength
  - Simplest surface to handle and often used in radiation calculations





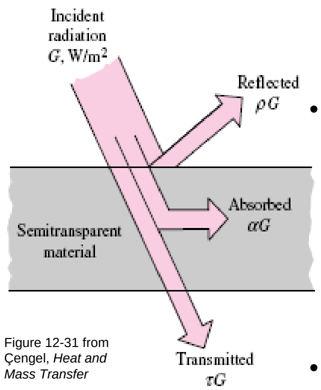


Figure 12-31 from Çengel, *Heat and Mass Transfer*

Properties

- When radiation, G, hits a surface a fraction ρG is reflected; another fraction, αG is absorbed, a third fraction τG is transmitted
- Energy balance: $\rho + \alpha + \tau = 1$

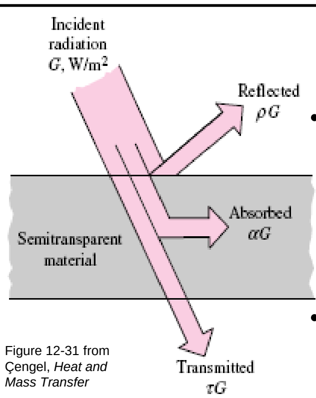


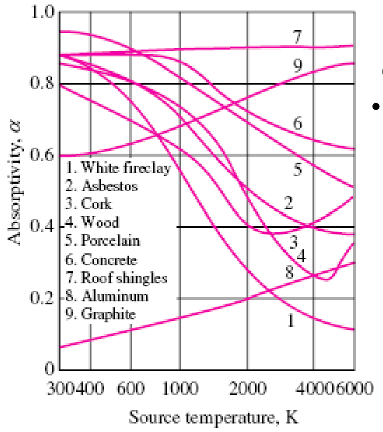
Figure 12-31 from Çengel, *Heat and Mass Transfer*

Properties II

- Fractions on previous chart are properties
  - Reflectivity, ρ
  - Absorptivity, α
  - Transmissivity, τ
- Energy balance: $\rho + \alpha + \tau = 1$

Properties III

- As with emissivity, α, ρ, and τ may be defined on a spectral and directional basis
  - Can also take averages over wavelength, direction or both as with emissivity
  - Simplest case is no dependence on either wavelength or direction
  - Reflectivity may be diffuse or have angle of reflection equal angle of incidence



α Data

- Solar radiation has effective source temperature of about 5800 K

Figure 12-33 from Çengel, *Heat and Mass Transfer*

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Kirchoff's Law

- Absorptivity equals emissivity (at the same temperature) $\alpha_\lambda = \varepsilon_\lambda$
- True only for values in a given direction and wavelength
- Assuming total hemispherical values of α and ε are the same simplifies radiation heat transfer calculations, but is not always a good assumption

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Effect of Temperature

- Emissivity, ε, depends on surface temperature
- Absorptivity, α, depends on source temperature (e.g. $T_{sun} \approx 5800$ K)
- For surfaces exposed to solar radiation
  - high α and low ε will keep surface warm
  - low α and high ε will keep surface cool
  - Does not violate Kirchoff's law since source and surface temperatures differ

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TABLE 12–3

Comparison of the solar absorptivity α_s of some surfaces with their emissivity ε at room temperature

Surface	α_s	ε
Aluminum		
Polished	0.09	0.03
Anodized	0.14	0.84
Foil	0.15	0.05
Copper		
Polished	0.18	0.03
Tarnished	0.65	0.75
Stainless steel		
Polished	0.37	0.60
Dull	0.50	0.21
Plated metals		
Black nickel oxide	0.92	0.08
Black chrome	0.87	0.09
Concrete	0.60	0.88
White marble	0.46	0.95
Red brick	0.63	0.93
Asphalt	0.90	0.90
Black paint	0.97	0.97
White paint	0.14	0.93
Snow	0.28	0.97
Human skin (Caucasian)	0.62	0.97

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From Çengel, *Heat and Mass Transfer*

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Average Radiation Properties

- Integrated average properties over all wavelengths

$$\bar{\varepsilon} = \frac{1}{\sigma T^4}\int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda \qquad \bar{\alpha} = \frac{1}{\sigma T^4}\int_0^\infty \alpha_\lambda E_{b\lambda} d\lambda$$

- Look at simple example where $\varepsilon_\lambda = \varepsilon_1$ for $\lambda < \lambda_1$ and $\varepsilon_\lambda = \varepsilon_2$ for $\lambda > \lambda_1$

$$\bar{\varepsilon} = \frac{1}{\sigma T^4}\int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda = \frac{1}{\sigma T^4}\int_0^{\lambda_1} \varepsilon_1 E_{b\lambda} d\lambda + \frac{1}{\sigma T^4}\int_{\lambda_1}^\infty \varepsilon_2 E_{b\lambda} d\lambda$$

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Average Radiation Properties II

- Rearrange to get f_λ, the fraction of black body radiation between 0 and λ

$$\bar{\varepsilon} = \frac{\varepsilon_1}{\sigma T^4}\int_0^{\lambda_1} E_{b\lambda} d\lambda' + \frac{\varepsilon_2}{\sigma T^4}\int_{\lambda_1}^\infty E_{b\lambda} d\lambda' = \varepsilon_1 f_{\lambda_1} + \varepsilon_2\left(1 - f_{\lambda_1}\right)$$

- Similar equation for absorptivity ($\alpha_\lambda = \varepsilon_\lambda$)

$$\bar{\alpha} = \frac{\alpha_1}{\sigma T^4}\int_0^{\lambda_1} E_{b\lambda} d\lambda' + \frac{\alpha_2}{\sigma T^4}\int_{\lambda_1}^\infty E_{b\lambda} d\lambda' = \alpha_1 f_{\lambda_1} + \alpha_2\left(1 - f_{\lambda_1}\right)$$

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Example

- Data: ε_λ = 0.9 for λ < 3 μm and ε_λ = 0.2 for λ > 3
  - Solar T = 5800 K, λT = 17,400 μm·K, f_λ(17,400 μm·K) = 0.980155, find α
    - Use $\alpha_\lambda = \varepsilon_\lambda$

$$\overline{\alpha}_{5800\,K} = \alpha_1 f_{\lambda_1} + \alpha_2\left(1 - f_{\lambda_1}\right) = 0.9(0.980) + 0.2(1 - 0.980) = 0.886$$

  - Earth T = 300 K, λT = 900 μm·K, f_λ(17,400 μm·K) = 0.001, find ε

$$\overline{\varepsilon}_{300\,K} = \varepsilon_1 f_{\lambda_1} + \varepsilon_2\left(1 - f_{\lambda_1}\right) = 0.9(0.001) + 0.2(1 - 0.001) = 0.201$$

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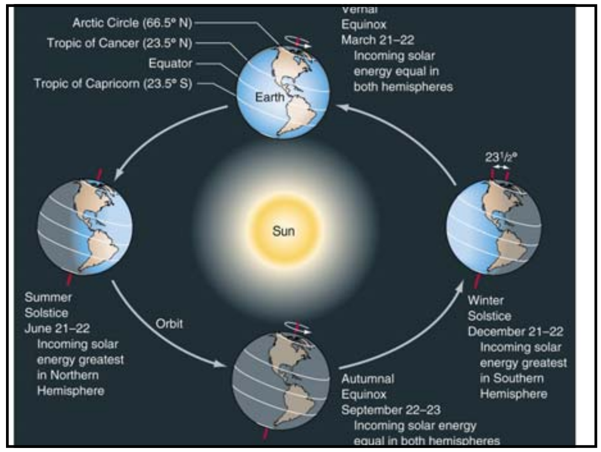
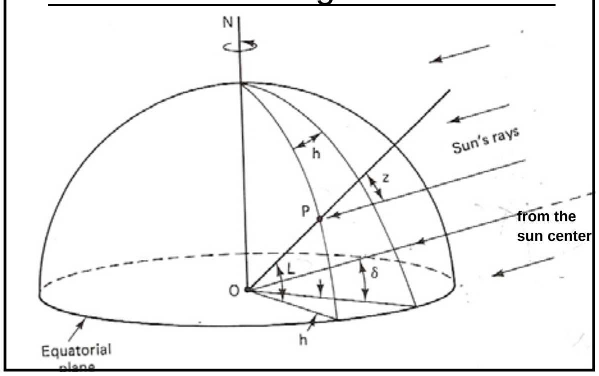
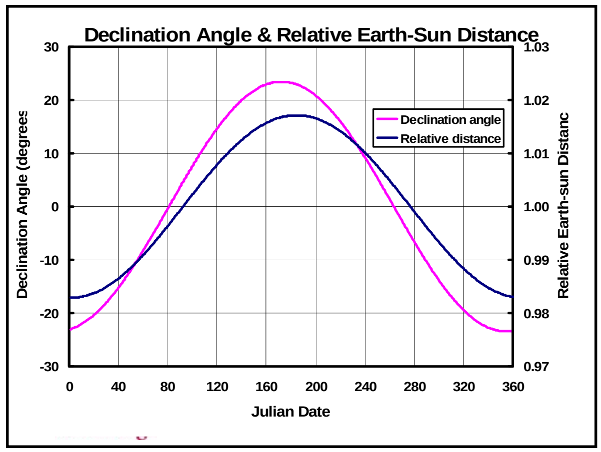
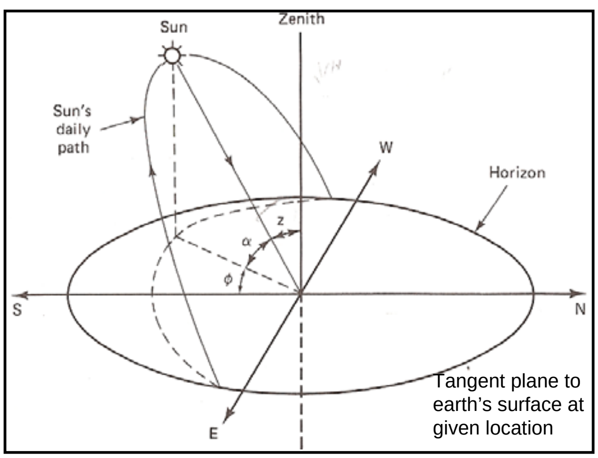


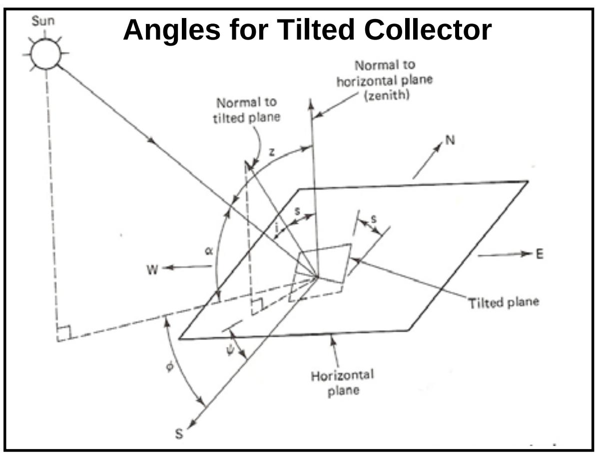
Solar Angles III

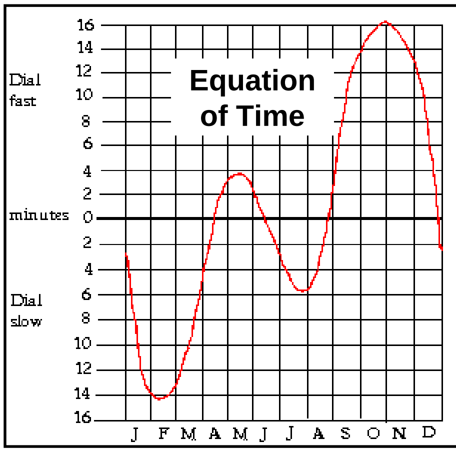






Angles for Tilted Collector





Solar Time = Standard Time + Equation of Time + (4 min/°) * (Standard Longitude – Local Longitude)

Standard Time = DST – 1 hour

Computing the Sun Path

- Input data: Latitude, L, date, hour h
- Find declination from serial date, n

$$\delta = \left(23.45^o\right)\sin\left[\frac{360}{365}(284+n)\frac{\pi}{180}\right] \qquad (\delta \textbf{ in degrees})$$

- Two angles: altitude (α) and azimuth (ϕ)
  - $\sin(\alpha) = \sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(h)$
  - $\sin(\alpha_s) = \sin(\phi) = \cos(\delta)\sin(h) / \cos(\alpha)$
  - Sun path is plot of α *vs.* $\phi = \alpha_s$ for one day
  - Plot is symmetric about solar noon
  - Typically plot data for 21st of month

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Path Calculation Problem

- Angles given as sin(angle) = x require arcsin function calculation
- Typical arcsine function returns angle between –90º and 90º
  - Limits correspond to range for sine between –1 and +1
  - Special calculation for hour angle limit
    - $h_{limit} = \pm\tan(\delta) / \tan(L)$
    - $\phi = \pm[\pi - \arcsin(\sin\phi)]$ for $|h| > |h_{limit}|$

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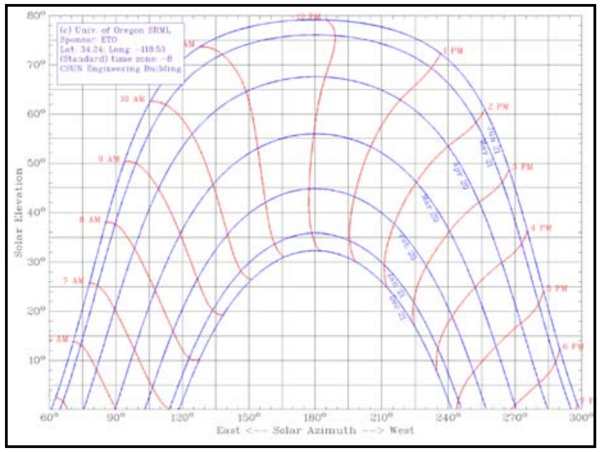

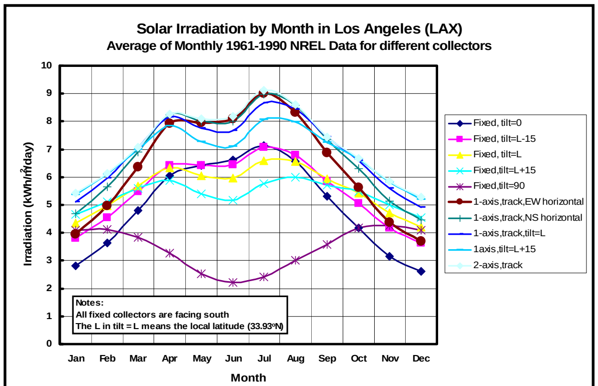

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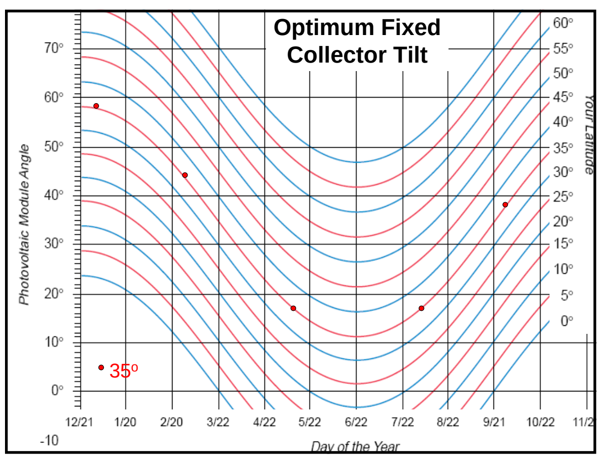

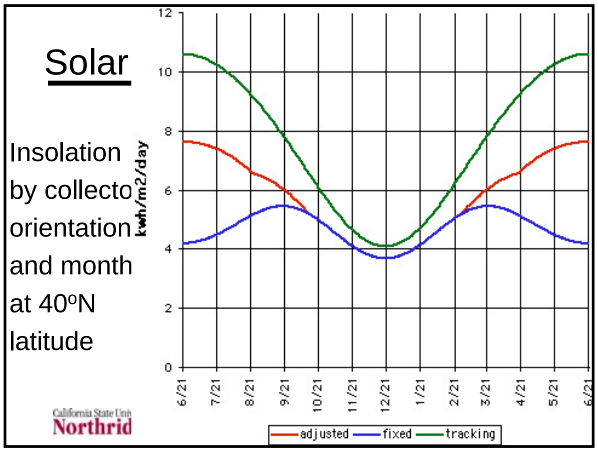
Solar
Insolation by collector orientation and month at 40ºN latitude
kwh/m2/day
12
10
8
6
4
2
0
6/21
7/21
8/21
9/21
10/21
11/21
12/21
1/21
2/21
3/21
4/21
5/21
6/21
adjusted
fixed
tracking
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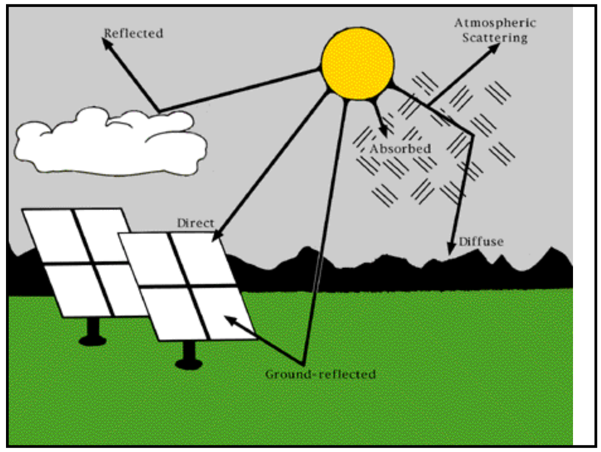
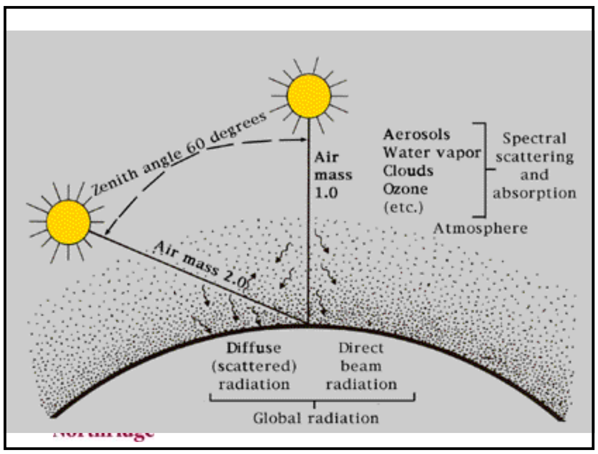
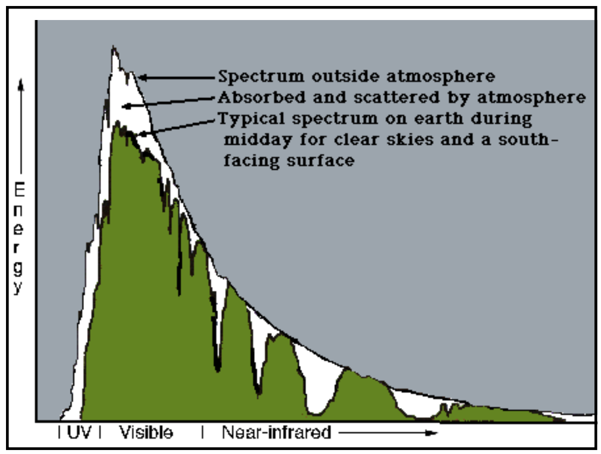
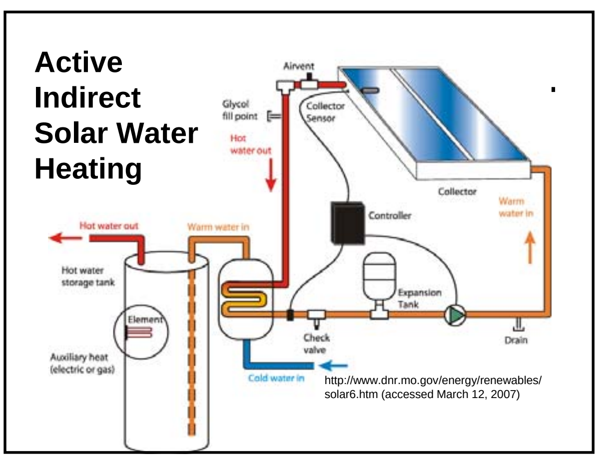
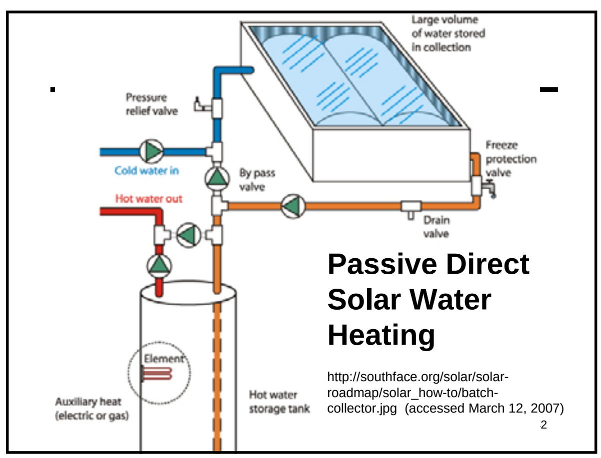

Reflected
Atmospheric Scattering
Absorbed
Direct
Diffuse
Ground-reflected


Zenith angle 60 degrees
Air mass 1.0
Aerosols
Water vapor
Clouds
Ozone
(etc.)
Spectral scattering and absorption
Atmosphere
Air mass 2.0
Diffuse (scattered) radiation
Direct beam radiation
Global radiation


Spectrum outside atmosphere
Absorbed and scattered by atmosphere
Typical spectrum on earth during midday for clear skies and a south-facing surface
Energy
UV
Visible
Near-infrared


Active Indirect Solar Water Heating
Airvent
Glycol fill point
Collector Sensor
Hot water out
Collector
Controller
Warm water in
Hot water out
Warm water in
Hot water storage tank
Element
Auxiliary heat (electric or gas)
Expansion Tank
Check valve
Drain
Cold water in
http://www.dnr.mo.gov/energy/renewables/solar6.htm (accessed March 12, 2007)


Large volume of water stored in collection
Pressure relief valve
Cold water in
By pass valve
Freeze protection valve
Hot water out
Drain valve
Passive Direct Solar Water Heating
Element
Auxiliary heat (electric or gas)
Hot water storage tank
http://southface.org/solar/solar-roadmap/solar_how-to/batch-collector.jpg (accessed March 12, 2007)

Basic Solar Collector Analysis

- Overall heat balance
  - Incoming solar radiation
  - Heat loss from collector to environment
  - Useful energy gain = Incoming Solar Radiation – Environmental Heat Loss
- Environmental heat loss proportional to $\Delta T = T_{collector} - T_{ambient}$
  - Applications that require high collector temperatures will have more heat loss

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Useful Heat Transfer

- Heat is added to a collector fluid
  - Typically collector fluid is water or water and anti-freeze solution
  - Air is also used as collector fluid for home heating
- Energy added from simple first law for open system with consant pressure heat addition

$$\dot{Q}_u = \dot{m} c_p \left(T_{f,out} - T_{f,in}\right)$$

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Solar to Useful Energy

- Solar transmission through glass covers provides absorbed radiation, $H_a = H_i \tau\alpha$
- Consider three losses
  - Conduction through bottom of solar collector box
  - Conduction through edge of box
  - Loss through top
    - Convection between absorber plate and glass covers with conduction through glass
    - Convection from top glass cover to ambient

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Loss Through Top

- In steady state the following heat rates will be the same
  - Between absorber plate and bottom glass
  - From bottom glass to top glass
    - Consider two-plate collector
  - From top glass to ambient
  - Look at exchange between absorber plate at temperature T_P and bottom glass at temperature T_{g2}
  - Have convection plus radiation

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Loss from Absorber Plate

$$Q_{top} = h_{p-g2} A_c \left(T_P - T_{g2}\right) + \frac{A_c \sigma \left(T_P^4 - T_{g2}^4\right)}{\frac{1}{\varepsilon_P} + \frac{1}{\varepsilon_{g2}} - 1}$$

$$\frac{A_c \sigma \left(T_P^4 - T_{g2}^4\right)}{\frac{1}{\varepsilon_P} + \frac{1}{\varepsilon_{g2}} - 1} = \frac{A_c \sigma \left(T_P^2 + T_{g2}^2\right)\left(T_P + T_{g2}\right)\left(T_P - T_{g2}\right)}{\frac{1}{\varepsilon_P} + \frac{1}{\varepsilon_{g2}} - 1} = h_{r,p-g2}\left(T_P - T_{g2}\right)$$

$$Q_{top} = \left(h_{p-g2} + h_{r,p-g2}\right) A_c \left(T_P - T_{g2}\right) = \frac{T_P - T_{g2}}{R_{p-g2}}$$

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Remaining Top Loss Path

- Between glass plates

$$Q_{top} = \left(h_{g2-g1} + h_{r,g2-g1}\right) A_c \left(T_{g2} - T_{g1}\right) = \frac{T_{g2} - T_{g1}}{R_{g2-g1}}$$

$$h_{r,g2-g1} = \frac{A_c \sigma \left(T_{g1}^2 + T_{g2}^2\right)\left(T_{g1} + T_{g2}\right)}{\frac{1}{\varepsilon_{g1}} + \frac{1}{\varepsilon_{g2}} - 1}$$

- Top plate to ambient

$$Q_{top} = \left(h_{g1-a} + h_{r,g1-a}\right) A_c \left(T_{g1} - T_a\right) = \frac{T_{g1} - T_a}{R_{g2-g1}}$$

$$h_{r,g1-a} = \frac{A_c \sigma \left(T_{g1}^2 + T_{sky}^2\right)\left(T_{g1} + T_{sky}\right)}{\frac{1}{\varepsilon_{g1}} + \frac{1}{\varepsilon_{g2}} - 1} \frac{T_{g1} - T_{sky}}{T_{g1} - T_a}$$

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Loss Through Top/Bottom

- Combine three resistances in series to get $R_{top} = R_{P\text{-}g2} + R_{g2\text{-}g1} + R_{g1\text{-}a}$
  - $Q_{top} = (T_P - T_a)/R_{top} = U_{top}A_c(T_P - T_a)$
- Loss through bottom is conduction through insulation (k_{ins}, Δx_{ins}) in series with convection to ambient with $h_{b\text{-}a}$

$$Q_{bottom} = \frac{T_P - T_a}{R_{ins} + R_{conv}} = \frac{T_P - T_a}{\frac{k_{ins}}{\Delta x_{ins} A_c} + \frac{1}{h_{b-a} A_c}} = U_{bottom} A_c (T_P - T_a)$$

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Total Loss

- $Q_{sides} = U'_{side}A_{side}(T_P - T_a)$
  - Can estimate $U'_{side} = 0.5$ W/m²·K
  - Use $U_{side}A_c = U'_{side}A_{side}$ for common area
  - $Q_{sides} = U_{side}A_c(T_P - T_a) = (T_P - T_a)/A_{side}$
- Total is sum of individual losses
- $Q_{loss} = U_cA_c(T_P - T_a) = (T_P - T_a)/R_c$
- Overall conductance and resistance
- $U_c = U_{top} + U_{bottom} + U_{sides}$

$$\frac{1}{R_c} = \frac{1}{R_{top}} + \frac{1}{R_{bottom}} + \frac{1}{R_{side}}$$

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Approximate U_{top} Equation

$$U_{top} = \frac{1}{\frac{A'}{T_p}\left(\frac{T_p - T_a}{N + B}\right)^{0.33} + \frac{1}{h_w}} + \frac{\sigma(T_p + T_a)(T_p^2 + T_a^2)}{\frac{1}{\varepsilon_p + 0.05N(1 - \varepsilon_p)} + \left(\frac{2N + B - 1}{\varepsilon_g}\right) - N}$$

N = number of glass covers

A’ = 250[1 – 0.0044(s – 90)]

s = tilt angle (degrees)

B = $(1 - 0.04h_w + 0.0005h_w^2)(1 + 0.091N)$

h_w = heat transfer coefficient from top to ambient

Other symbols have previous definitions

Equation uses SI units: U_c and h in W/m²·K, T in K, $\sigma = 5.670\times10^{-8}$ W/m²·K⁴, ε_g is same for all glass covers

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Absorber Plate Analysis

- Three analysis steps for solar energy to heat fluid (Hottel-Whillier-Bliss equation)
  - Solar energy into plate flows across plate to location of tubes at some line on plate
  - At same line heat flow into collector fluid from plate is determined
  - Integrate heat flow into fluid from inlet to exit to get total useful heat transfer to fluid

$$\dot{Q}_u = F_R A_c \left[H_a - U_c\left(T_{f,in} - T_a\right)\right]$$

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Flat Plate Collector

k = absorber thermal conductivity

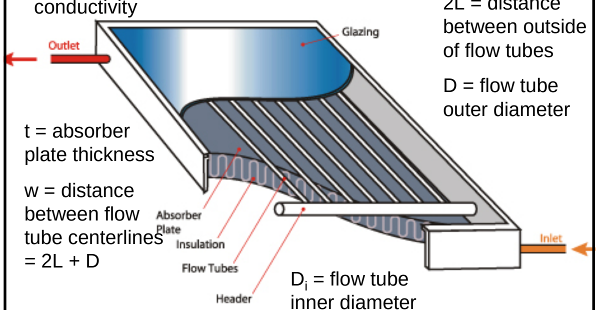
2L = distance between outside of flow tubes

D = flow tube outer diameter

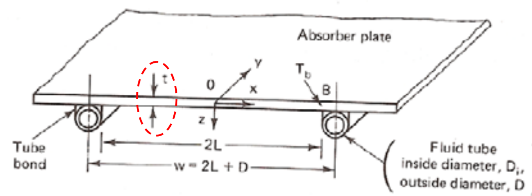
t = absorber plate thickness

w = distance between flow tube centerlines = 2L + D

D_i = flow tube inner diameter



Absorber Plate Analysis



- Define $\mathbf{m^2 = U_c/(tk_{plate})}$
- Effectiveness factor, **F = tanh(mL) / (mL)**
- Total (useful) heat transfer per unit length of tube

$$q_{total} = (2LF + D)\left[H_a - U_c(T_b - T_a)\right] = q_u'$$

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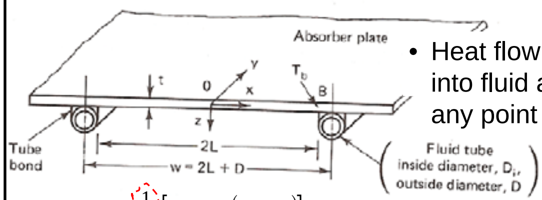
Absorber Plate Analysis II



- Heat flow into fluid at any point

$$q_u' = \frac{\frac{1}{U_c}\left[H_a - U_c\left(T_f - T_a\right)\right]}{\left[\frac{1}{(2LF + D)U_c} + \left(\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}\right)\right]} = wF'\left[H_a - U_c\left(T_f - T_a\right)\right]$$

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Factors, F' and F_R

- Collector efficiency factor, F'

$$F' = \frac{\text{Thermal Resistance Between Plate and Ambient}}{\text{Thermal Resistance Between Fluid and Ambient}}$$

$$F' = \frac{1/U_c}{w\left[1/\left[(2LF + D)U_c\right] + \left(1/C_B + 1/\left(h_{c,i}\pi D_i\right)\right)\right]}$$

- Heat removal factor, F_R

$$\frac{F_R}{F'} = \frac{\dot{m}c_p}{U_c A_c F'}\left(1 - e^{-\frac{U_c A_c F'}{\dot{m}c_p}}\right) = \frac{1}{a}\left(1 - e^{-a}\right) \quad a = \frac{U_c A_c F'}{\dot{m}c_p}$$

$F_R/F' \to [1, 1/a]$ as $a \to [0, \infty]$

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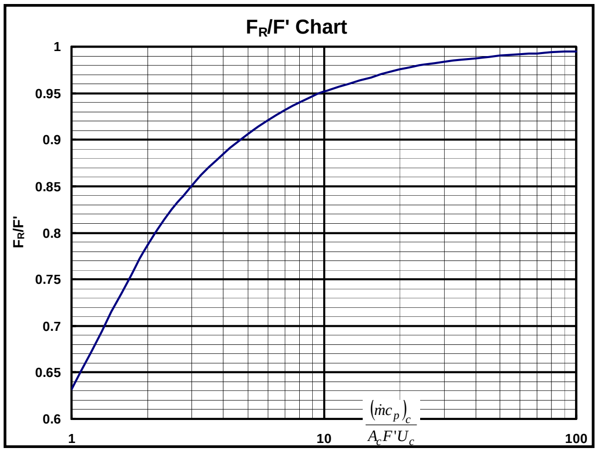

Summary of Results

- Q_u = useful heat transfer to working fluid

$$F' = \frac{1}{\frac{w}{U_c}\left[\frac{1}{U_c(2LF + D)} + \frac{1}{C_B} + \frac{1}{\pi D_i h_i}\right]}$$

$$H_a = H_i\left(\tau\alpha\right) \qquad F_R = \frac{\dot{m}c_p}{U_c A}\left[1 - e^{-\frac{U_c A F'}{\dot{m}c_p}}\right]$$

$$Q_u = AF_R\left[H_a - U_c\left(T_{f,in} - T_a\right)\right] \qquad T_{f,out} = T_{f,in} + \frac{Q_u}{\dot{m}c_p}$$

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Collector Efficiency, $\eta_c = Q_u/A_cH_i$

- Start with Hottel-Whillier-Bliss Equation

$$Q_u = F_R A_c\left[H_a - U_c\left(T_{f.in} - T_a\right)\right]$$

- Replace H_a by $H_i\tau\alpha$

$$Q_u = F_R A_c\left[H_i\tau\alpha - U_c\left(T_{f.in} - T_a\right)\right]$$

- Substitute into efficiency equation

$$\eta_c = \frac{Q_u}{A_c H_i} = \frac{F_R A_c\left[H_i\tau\alpha - U_c\left(T_{f.in} - T_a\right)\right]}{A_c H_i} = F_R\tau\alpha - \frac{F_R U_c\left(T_{f.in} - T_a\right)}{H_i}$$

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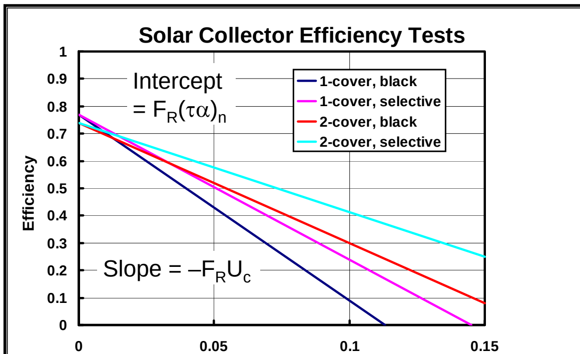

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Sample Rating Sheet

SOLAR COLLECTOR CERTIFICATION AND RATING	CERTIFIED SOLAR COLLECTOR	
	SUPPLIER:	Heliodyne, Inc. 4910 Seaport Avenue Richmond, CA 94804
SRCC OG-100	MODEL:	Heliodyne Gobi 408
	COLLECTOR TYPE:	Glazed Flat-Plate
	CERTIFICATION #:	100-1981-085A

COLLECTOR THERMAL PERFORMANCE RATING

Megajoules Per Panel Per Day				Thousands of Btu Per Panel Per Day			
CATEGORY (Ti-Ta)	CLEAR DAY 23 MJ/m²·d	MILDLY CLOUDY 17 MJ/m²·d	CLOUDY DAY 11 MJ/m²·d	CATEGORY (Ti-Ta)	CLEAR DAY 2000 Btu/ft²·d	MILDLY CLOUDY 1500 Btu/ft²·d	CLOUDY DAY 1000 Btu/ft²·d
A (-5°C)	49	37	25	A (-9°F)	46	35	24
B (5°C)	45	33	21	B (9°F)	43	32	20
C (20°C)	39	27	15	C (36°F)	37	25	14
D (50°C)	24	14	4	D (90°F)	23	13	4
E (80°C)	10	2		E (144°F)	10	2	

A-Pool Heating (Warm Climate) B-Pool Heating (Cool Climate) C-Water Heating (Warm Climate) D-Water Heating (Cool Climate) E-Air Conditioning

Original Certification Date: August 1, 1983

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Sample Rating Sheet II

COLLECTOR SPECIFICATIONS

Gross Area:	2.996	m²	32.25	ft²		Net Aperture Area:	2.771	m²	29.83	ft²
Dry Weight:	60.382	kg	133	lb		Fluid Capacity:	3.0	l	0.8	gal
Test Pressure:	1034	kPa	150	psig						

COLLECTOR MATERIALS

Frame:	Aluminum Extrusion
Cover (Outer):	Low Iron Tempered Glass
Cover (Inner):	None
Absorber Material:	Tube - Copper / Plate - Copper
Absorber Coating:	Black Chrome
Insulation (Side):	Isocyanurate Foam
Insulation (Back):	Isocyanurate Foam & Fiberglass

PRESSURE DROP

Flow		ΔP	
ml/s	gpm	Pa	in H_2O

slope = $-F_R U_c$

intercept = $F_R(\tau\alpha)_n$

TECHNICAL INFORMATION

Efficiency Equation [NOTE: Based on gross area and (P) = Ti-Ta]

						Y Intercept	Slope	
S I Units:	η = 0.725	-3.2000	(P)/I	-0.0220	(P)²/I	0.737	-4.57	W/m²·°C
I P Units:	η = 0.725	-0.5639	(P)/I	-0.0022	(P)²/I	0.737	-0.805	Btu/hr·ft²·°F

Incident Angle Modifier [(S) = 1/cos θ - 1, 0°≤ θ ≤60°]

$K_{ατ}$ = 1.0 -0.0900 (S) 0.0000 (S)²

$K_{ατ}$ = 1.0 -0.09 (S) (Linear Fit)

Model Tested: Gobi 408

Test Fluid: Water

Test Flow Rate: 56 ml/s 0.89 gpm

REMARKS:

November, 2006

F-chart Water Heating Only

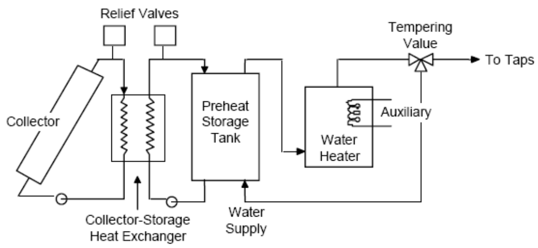

Figure 4.1: Two-Tank Domestic Water Heating System

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System with Solar Air Heating

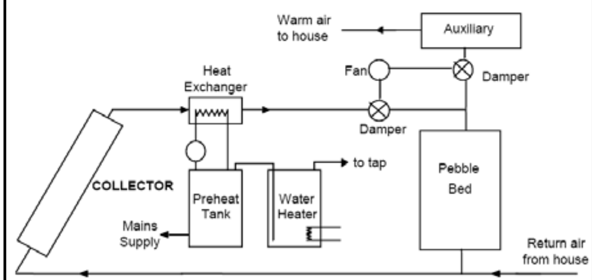

Figure 4.2: Pebble Bed Storage Space and Water Heating System

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F-Chart Water and Space Heating

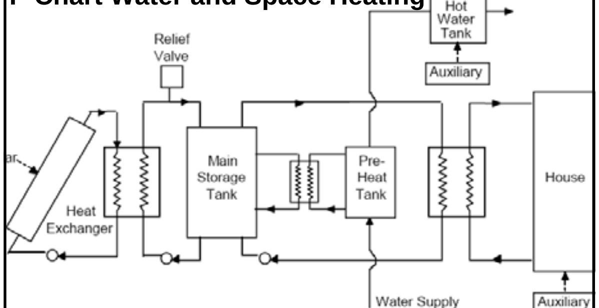

Figure 4.3: Water Storage Space and Water Heating System

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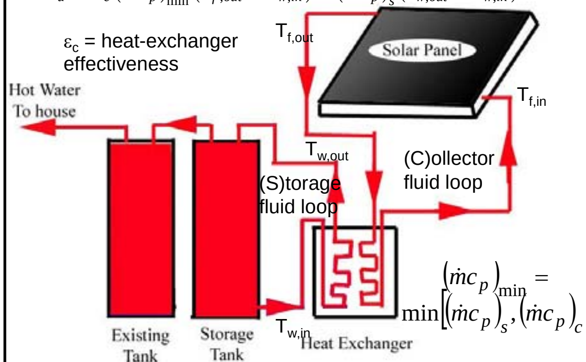
$$Q_u = \varepsilon_c(\dot{m}c_p)_{min}(T_{f,out} - T_{w,in}) = (\dot{m}c_p)_s(T_{w,out} - T_{w,in})$$

ε_c = heat-exchanger effectiveness



$T_{f,out}$, $T_{f,in}$, $T_{w,out}$, $T_{w,in}$

$$(\dot{m}c_p)_{min} = \min[(\dot{m}c_p)_s, (\dot{m}c_p)_c]$$

http://starfiresolar.com/db2/00144/starfiresolar.com/_uimages/solardiagramallred.jpg

Heat Exchanger

- Start with Hottel-Whillier-Bliss equation
  - Replace $T_{f,in}$ by $T_{w,in}$

$$Q_u\left[1-\frac{A_cF_RU_c}{(\dot{m}c_p)_c}+\frac{A_cF_RU_c}{\varepsilon_c(\dot{m}c_p)_{min}}\right]=A_cF_RH_a-A_cF_RU_c(T_{w,in}-T_a)$$

$$Q_u=A_cF_R'\left[H_a-U_c(T_{w,in}-T_a)\right]$$

$$F_R'=F_R\left[1-\frac{A_cF_RU_c}{(\dot{m}c_p)_c}+\frac{A_cF_RU_c}{\varepsilon_c(\dot{m}c_p)_{min}}\right]^{-1}=F_R\left[1+\frac{A_cF_RU_c}{(\dot{m}c_p)_c}\left(\frac{(\dot{m}c_p)_c}{\varepsilon_c(\dot{m}c_p)_{min}}-1\right)\right]^{-1}$$

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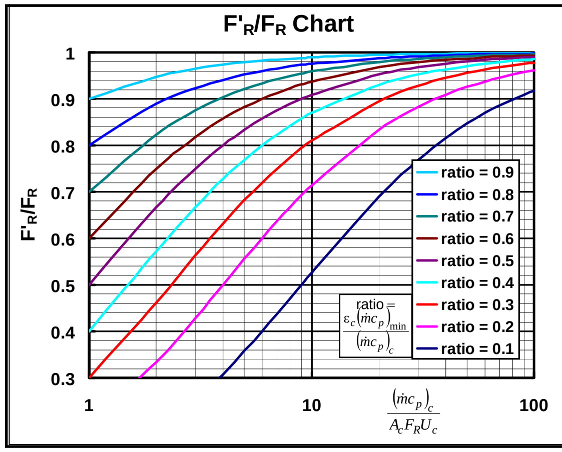

f-chart Method

- Predicts fraction of demand over a time period (usually monthly) than can be supplied by solar
- Two empirical pamameters, X and Y
  - X is ratio of reference collector loss to total heating load
  - Y is ratio of absorbed solar energy to total heating load

$$X=\frac{A_cF_R'U_c}{D}\left(T_{ref}-\overline{T}_a\right)\qquad Y=\frac{A_cF_R'\overline{\tau\alpha}}{D}H_{i,total}$$

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Computing X (dimensionless)

$$X=A_c\left[F_RU_c\frac{F_R'}{F_R}\frac{\Delta t}{D}\left(T_{ref}-\overline{T}_a\right)\right]$$

- A_c = collector area (m^2)
- F_RU_c ($W/m^2\cdot K$) from slope of collector test data
- F'_R/F_R computed or assumed = 0.97
- Usual averaging period, Δt = 1 month, converted to seconds
- D = heating demand for averaging period (J)
- Tref = 100ºC; $\overline{T}_a$ from NREL data

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Computing Y (dimensionless)

$$Y=A_c\left[F_R(\tau\alpha)_n\frac{F_R'}{F_R}\frac{\overline{\tau\alpha}}{(\tau\alpha)_n}\frac{H_{i,total}}{D}\right]$$

- A_c = collector area (m^2)
- $F_R(\tau\alpha)_n$ from intercept of collector test
- F'_R/F_R computed or assumed = 0.97
- Ratio $\overline{\tau\alpha}/(\tau\alpha)_n$ = 0.94 (October – March), = 0.90 (April – September) or computed
- $H_{i,total}$ is available from NREL data for Δt = 1 month (convert to J/m^2)
- D is heating demand J

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f Equations

- For water heating: $f = 1.029Y – 0.065X – 0.245Y^2 + 0.0018X^2 + 0.0215Y^3$
  - Adjustments required
    - Adjust X for hot water supply only and storage capacity different from standard
    - Adjust Y for load heat exchanger capacity
- For air heating: $f = 1.040Y – 0.065X – 0.159Y^2 +0.00187X^2 – 0.0095Y^3$
  - Solar collectors heating air have no heat exchanger so $F'_R = F_R$

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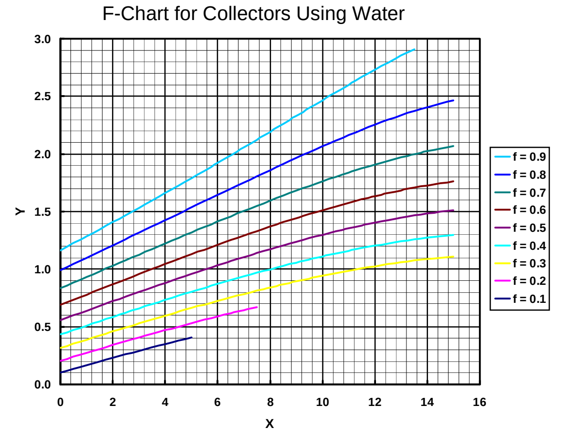

Adjustments

- Adjust X for storage capacity, M, in L/m^2
  X' = X(75/M)$^{1/4}$
- Adjust Y for load heat exchanger factor, Z: Y' = Y(0.39 + 0.65e$^{-0.139/Z}$)
  - ε_L = heat exchanger effectiveness
  - mass flow times heat capacity and UA factors defined previously

$$Z = \varepsilon_L\left(\dot{m}c_p\right)_{\min} / (UA)_L$$

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Another Adjustment

- For systems with only water heating
  - T_w = water temperature to household
  - T_m = cold water supply temperature
  - T_a = monthly average ambient temperature
- Multiply X by correction factor, CF, below

$$CF = \frac{11.6 + 1.18T_w + 3.86T_m - 2.32\overline{T}_a}{100 - \overline{T}_a}$$

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NREL Data

- National Renewable Energy Laboratory
- Collector data for 1961-1990 for 360 individual months and monthly averages
  - Available for variety of collectors
    - Flat plate collector data for several angles
- TMY3 data: Typical Meteorological Year
  - Hourly data on radiation components
  - Compute resultant for given collector geometry

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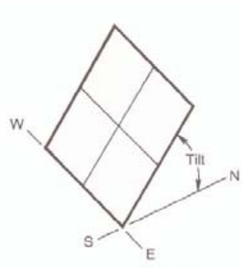
NREL Collector Types '61-'90

- Data available at different tilt levels for flat-plate collectors facing south
  - Horizontal (0º)
  - Latitude – 15º
  - Latitude
  - Latitude + 15º
  - Vertical (90º)



Flat-plate collector facing south at fixed tilt

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NREL 1961-1990 LAX Average

SOLAR RADIATION FOR FLAT-PLATE COLLECTORS FACING SOUTH AT A FIXED-TILT (kWh/m2/day) Percentage Uncertainty = 9

Tilt(deg)		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
0	Average	2.8	3.6	4.8	6.1	6.4	6.6	7.1	6.5	5.3	4.2	3.2	2.6	4.9
	Minimum	2.3	3.0	4.0	5.5	5.7	5.6	6.4	6.1	4.4	3.8	2.7	2.1	4.7
	Maximum	3.3	4.4	5.6	6.8	7.2	7.7	8.0	7.0	5.8	4.5	3.6	3.0	5.1
Lat - 15	Average	3.8	4.5	5.5	6.4	6.4	6.4	7.1	6.8	5.9	5.0	4.2	3.6	5.5
	Minimum	2.9	3.6	4.5	5.8	5.7	5.4	6.3	6.3	4.7	4.4	3.4	2.7	5.2
	Maximum	4.6	5.7	6.4	7.3	7.3	7.3	7.9	7.2	6.6	5.6	4.9	4.3	5.7
Lat	Average	4.4	5.0	5.7	6.3	6.1	6.0	6.6	6.6	6.0	5.4	4.7	4.2	5.6
	Minimum	3.3	3.8	4.7	5.6	5.4	5.0	5.9	6.1	4.8	4.7	3.7	3.0	5.3
	Maximum	5.4	6.4	6.7	7.2	6.8	6.7	7.3	7.0	6.7	6.0	5.6	5.0	5.9
Lat + 15	Average	4.7	5.1	5.6	5.9	5.4	5.2	5.8	6.0	5.7	5.5	5.0	4.5	5.4
	Minimum	3.4	3.8	4.5	5.2	4.8	4.4	5.2	5.5	4.5	4.7	3.9	3.1	5.1
	Maximum	5.9	6.6	6.6	6.7	6.1	5.8	6.3	6.4	6.5	6.1	6.0	5.4	5.7
90	Average	4.1	4.1	3.8	3.3	2.5	2.2	2.4	3.0	3.6	4.2	4.3	4.1	3.5
	Minimum	2.9	3.0	3.1	2.9	2.3	2.1	2.3	2.8	2.9	3.5	3.2	2.7	3.3
	Maximum	5.2	5.4	4.5	3.6	2.7	2.3	2.5	3.2	4.1	4.7	5.2	5.0	3.7

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