First Midterm Review

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Mechanical Engineering 483

Alternative Energy Engineering II

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California State University Northridge

Midterm Exam

- Open book and notes
  - No books other than course text
  - No homework solutions or in-class exercise solutions
- Will be problems similar to those on homework and in-class exercise
- More credit for indicating correct approach to solution than for details of algebra or arithmetic

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What is energy

- Dictionary definition
- Capacity to do work
- Energy resources
- Energy and power (energy/time) units
  - Energy units: joules (J), kilowatt·hours (kWh), British thermal units (Btu)
    - 1 Btu = 1055.056 J
  - Power units: watts (W), Btu/hr
    - 1 W = 1 J/s = 3.412 Btu/hr

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Energy Units and Use

- Energy units: 1 Btu = 1055.056 J; 1 W = 1 J/s = 3.412 Btu/hr, 1 quad = 10^{15} Btu
- Fuel equivalencies: 1 ft^3 natural gas ≈ 1000 Btu; 1 bbl crude = 5.8 MMBtu; 1 Mtoe oil = 41.868×10^{15} J = 0.0387 quads
- World energy production (2006) is 466×10^{15} Btu = 466 quads (quadrillion Btu) = 491×10^{18} J = 491 exajoules
- World electricity generation (2006) is 18,930 TWh (terawatt hours)

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Energy Information Administration Data on "Heat Rates"

Coal MMBtu/ton	**Petroleum Products MMBtu/bbl**
Production: 21.070	Motor Gasoline: 5.204
Consumption: 20.753	Jet Fuel: 5.670
Coke 27.426	Distillate Fuel Oil: 5.825
Industrial: 22.489	Residual Fuel Oil: 6.287
Residential and Commercial: 23.880	Liquefied Petroleum Gas (LPG): 3.603
Electric Utilities: 20.401	Kerosene: 5.670
Crude Oil MMBtu/bbl	**Natural Gas Btu/ft^3**
Production: 5.800	Production, Dry: 1,027
Imports: 5.948	Consumption: 1,027
Electricity Consumption: .	Non-electric Utilities: 1,028
Btu per kilowatt-hour 3,412	Electric Utilities: 1,019
	Imports: Btu per cubic foot 1,022
	Exports: Btu per cubic foot 1,006

Energy Information Administration (EIA), *Annual Energy Review 2000*, DOE/EIA-0384(2000) (Washington, DC, August 2001)

Energy Costs

- Home costs (San Fernando Valley 2008)
  - Electricity: \$0.115/kWh = \$32/GJ
    - Increase from \$0.11/kWh to \$0.12/kWh
  - Natural gas: \$1.07/therm = \$11/GJ
    - One therm = 10^5 Btu is approximately the energy in 100 standard cubic feet of natural gas
    - Range was \$0.69 to \$1.22 per therm
  - Gasoline at \$3.00 per gallon (including taxes) costs \$26/GJ
    - Assumes energy content of gasoline is 5.204 MMBtu per (42 gallon) barrel
    - \$100/bbl oil costs \$6.20/GJ (5.80 MMBtu/bbl)
  - Energy cost without California gasoline taxes (\$0.585/gallon) is \$21/GJ

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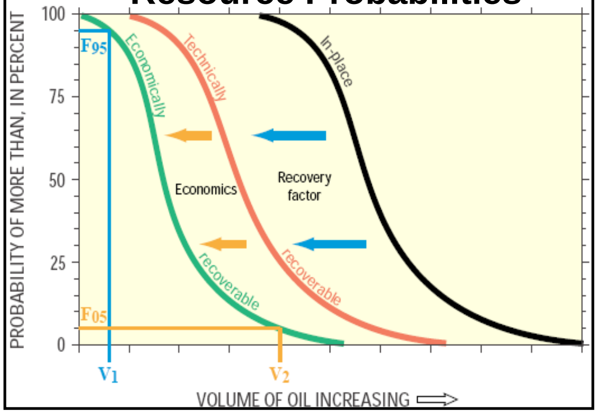
Resources *vs.* Reserves

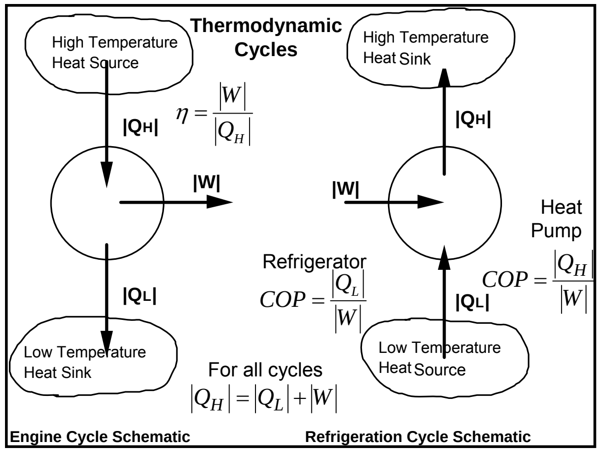
	Known	Unknown
Economical to Recover	Reserves	Resources
Not economical to recover	Resources	Resources

Resource Probabilities



Hubbert Peak

- Analysis due to M. King Hubbert
- Main publications in 1949 and 1956
- Correctly predicted peak in US oil production in early 1970s
- Not so accurate in other predictions
- Some recent applications show world oil production peak in next ten years
- Many other studies show later peak



Some Cycles

- Rankine cycle – steam power plant
- Brayton cycle – gas turbine engines
- Combined cycle – combination of Brayton and Rankine cycle
- Otto and Diesel cycles for reciprocating engines
- Air standard cycles *versus* consideration of heat addition from fuel
- Refrigerator *versus* heat pump

Basic Combustion Analysis

- General fuel formula: $C_xH_yS_zO_wN_v$
- x, y, z, w, and v from ultimate analysis or analysis of gas mixtures
- Ultimate analyses:
  - x = wt%C/12.0107, y = wt%H/1.00794, z = t%S/32.065, w = wt%O/16.0004, v = wt%N/14.0067, m_{fuel} = 100
  - M_{fuel} = 12.0107x + 1.00794y + 32.065z + 15.9994w + 14.0067v = m_{fuel}(1 – %MM)
- For mixture of compounds (ω_k = mole fraction)

$$x = \sum_{species} \omega_k x_k \quad y = \sum_{species} \omega_k y_k \quad M_{fuel} = \sum_{species} \omega_k M_k$$

Combustion Air

- A = x + y/4 + z – w/2 = stoichiometric moles O_2/mole fuel
- Need input data on Actual O_2/Stoichiometric O_2 = Relative air/fuel ratio = λ
- Air/fuel ratio = m_{air}/m_{fuel} = $138.28\lambda A/m_{fuel}$
- $C_xH_yS_zO_wN_v + \lambda A(O_2 + 3.77\ N_2) \rightarrow xCO_2 + (y/2)H_2O + zSO_2 + (\lambda - 1)AO_2 + 3.77\lambda A + v/2)N_2$

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Exhaust Oxygen and λ

- Can relate these two quantities with fuel properties
- Can compute theoretical $\%O_2$ for given λ
- Dry exhaust has water removed to protect chemical analyzers

$$\frac{\%O_2|_{dry}}{100} = \frac{(\lambda - 1)A}{x + 4.77\lambda\ A - A + z + \frac{v}{2}}$$

$$\lambda = \frac{A + \frac{\%O_2|_{dry}}{100}\left[x - A + z + \frac{v}{2}\right]}{A\left(1 - 4.77\frac{\%O_2|_{dry}}{100}\right)}$$

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Emission Rates

- Often stated as pollutant mass per unit heat input from fuel
- Equation used: $E_i = \rho_{i,d} F_d \frac{20.9}{20.9 - \%O_{2,d}}$
- Compute $\rho_{i,d} = y_{i,d}M_iP_{std}/R_uT_{std}$
- F_d is dry exhaust volume/heat input
  - Use default values or compute by equation
    - Feb 3 notes have values of K's and default F_d's

$$F_d = \frac{K(K_C\%C + K_H\%H + K_O\%O + K_S\%S + K_N\%N)}{Q_c}$$

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Other Equations

- Pollutant mass per unit heat input

$$\frac{m_{CO_2}}{Q_{fuel}} = \frac{3.6642}{Q_c}\frac{wt\%C}{100} \qquad \frac{m_{SO_2}}{Q_{fuel}} = \frac{1.9979}{Q_c}\frac{wt\%S}{100}$$

- Combustion Efficiency (definitions on next slide)

$$\eta_{comb} = \frac{|q|}{|q|_{\max}} = 1 - \frac{\left[1 + \frac{Air}{Fuel}\right]}{Q_c}\int_{T_{in}}^{T_{out}} c_{p_{Air}}\,dT' - \frac{xf\Delta h_{CO}}{M_{fuel}Q_c}$$

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Combustion Efficiency

- Air/fuel is the air to fuel (mass) ratio
- $C_{p,air}$ = 0.24 Btu/lb_m▪R = 1.005 kJ/kg▪K
- f = molar exhaust ratio CO/(CO + CO_2)
- x = carbon atoms in fuel formula, C_xH_y…
- Q_c = heat of combustion (Btu/lb_m or kJ/kg)
  - Use lower heating value for water vapor (usual case)
- Δh_{CO} 282,990 kJ/kgmol = 121,665 Btu/lbmol
- M_{fuel} is combustible fuel molar mass lb_m/lbmol or kg/kmol

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Energy Economics

- Look at balance between initial cost and ongoing costs
  - Uses interest rate to consider time value of money
- Key formula relates equivalence between initial cost, P (present value), and ongoing payment stream, A (annual cost)

$$A/P = \frac{i}{1 - (1+i)^{-n}} \qquad P/A = \frac{1 - (1+i)^{-n}}{i}$$

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Using the A/P formula

- Formula applies to any time period so long as i is interest rate per time period
- *E. g.*, for monthly costs with i = 6%/yr = 0.5%/month for N months $A/P = \frac{0.5}{1-(1+0.5)^{-N}}$
- Need trial-and-error solution (or financial calculator) to find i, given n and A/P
- Can find n for given i and A/P

$$n = -\frac{\ln\left(1-\frac{Pi}{A}\right)}{\ln(1+i)}$$

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Two Approaches

- Determine present worth of ongoing costs and add to initial costs
  - Total present worth = Initial Cost + (Ongoing Annual Costs)(P/A)
- Determine annualized equivalent of initial cost and add to ongoing costs
  - Total annual costs = Ongoing Annual Costs + (Initial Cost)(A/P)
- Power plant homework example
  - Initial C \$/kW, Fuel F \$/MMBtu, O&M M \$/kWh/yr, CF = capacity factor, HR = heat rate MMBtu/kWh, E = income \$/kWh

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Electricity Cost Present Worth

- Assume size S in kW; annual power production, PP = 8766 S(CF) kWh
  - Initial capital cost = CS
  - O&M present worth = M(PP)(P/A)
  - Fuel present worth F(PP)(HR)(P/A)
  - Electricity sales present worth = E(PP)(P/A)
  - For desired return i in P/A formula
    - CS + M(PP)(P/A) + F(PP)(P/A)(HR) = E(PP)(P/A)

$$E = \frac{CS}{8766S(CF)(P/A)} + M + F(HR) = \frac{C(A/P)}{8766(CF)} + M + F(HR)$$

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Annualized Electricity Cost

- Assume size S in kW; annual power production, PP = 8766 S(CF) kWh
  - Annualized capital cost = CS(A/P)
  - Annual O&M cost = M(PP)
  - Annual fuel cost = F(PP)(HR)
  - Annual electricity sales = E(PP)
  - For desired return i in P/A formula
    - CS(A/P) + M(PP) + F(PP)(HR) = E(PP)

$$E = \frac{CS(A/P)}{8766S(CF)} + M + F(HR) = \frac{C(A/P)}{8766(CF)} + M + F(HR)$$

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Subtle Point

- In the A/P formula for N years
  - The payment A is done at year end
- For homework problem comparing incandescent and compact fluorescent
  - Compact fluorescent lifetime is 9 years
  - Incandescent lifetime is 1 year
  - Present worth comparison over 9 years
    - IC = initial cost; AEC = annual electricity cost

$$P_I = IC_I\left(1+\frac{1-(1+i)^{-8}}{i}\right) + AEC_I\frac{1-(1+i)^{-9}}{i} \qquad P_{CF} = IC_{CF} + AEC_{CF}\frac{1-(1+i)^{-9}}{i}$$

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Payback Period

- Defined as initial cost in dollars divided by annual savings (or income) in dollars per year
- Simplified analysis of fixed cost versus annual savings
- Engineering economics texts recommend **not** using this measure
- But – it is easier to understand than the discount rate

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Environmental Aspects

- Multimedia impacts from energy use, development, production, refining
- Air pollution
  - Troposphere: O_3, CO, NO_x, SO_x, toxics
  - Stratosphere: CFC's reduce ozone layer
  - Global warming: CO_2, N_2O, CH_4, *etc*.
- Carbon capture and sequestration
  - Remove CO_2 from energy use and bury it underground

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Energy Storage Measures

- Energy per unit mass (kJ/kg; Btu/lb_m)
- Energy per unit volume (kJ/m^3; Btu/ft^3)
- Rate of delivery of energy to and from storage (kW/kg; Btu/hr·lb_m)
- Efficiency (energy out/energy in)
- Life cycles – how many times can the storage device be used
  - Particularly important for batteries

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Fuel Energy

- Volumetric energy storage in Btu/gallon
  - Gasoline: 109,000 to 125,000
  - Diesel fuel: 128,000 to 130,000
  - Biodiesel: 117,000 to 120,000
  - Natural gas: 33,000 to 38,000 at 3,000 psi, 38,000 to 44,000 at 3,600 psi, and ~73,500 as liquefied natural gas (LNG)
  - 85% ethanol in gasoline: ~80,000
  - 85% methanol in gasoline: 56,000 to 66,000
  - Hydrogen: ~6,500 at 3,000 psi, ~16,000 at 10,000 psi, and ~30,500 as liquid
  - Liquefied petroleum gas (LPG): ~84,000

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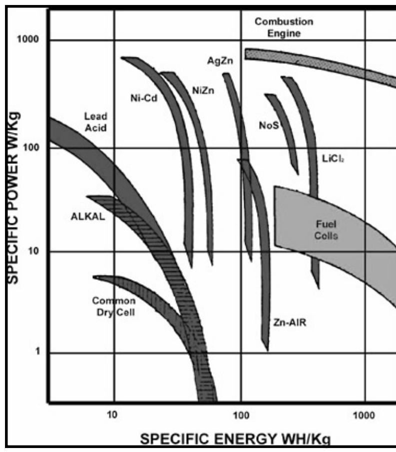
http://www.eere.energy.gov/afdc/altfuel/fuel_comp.html



Compare

Batteries versus other motive power

http://www.nap.edu/books/0309092612/html/40.html

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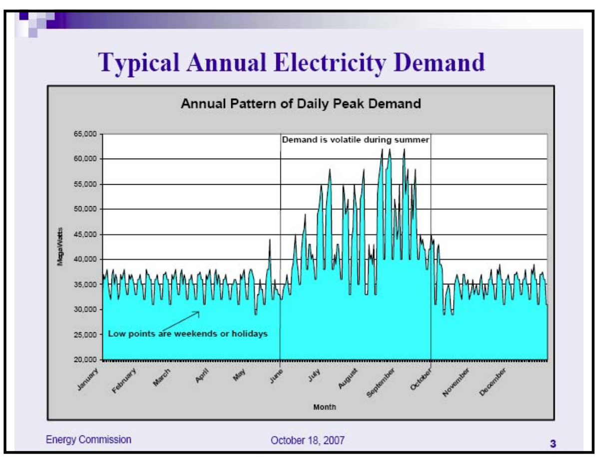

Storage Technologies	Main Advantages (relative)	Disadvantages (Relative)	Power Application	Energy Application
Pumped Storage	High Capacity, Low Cost	Special Site Requirement		●
CAES	High Capacity, Low Cost	Special Site Requirement, Need Gas Fuel		●
Flow Batteries: PSB VRB ZnBr	High Capacity, Independent Power and Energy Ratings	Low Energy Density	◐	●
Metal-Air	Very High Energy Density	Electric Charging is Difficult		●
NaS	High Power & Energy Densities, High Efficiency	Production Cost, Safety Concerns (addressed in design)	●	●
Li-ion	High Power & Energy Densities, High Efficiency	High Production Cost, Requires Special Charging Circuit	●	○
Ni-Cd	High Power & Energy Densities, Efficiency		●	◐
Other Advanced Batteries	High Power & Energy Densities, High Efficiency	High Production Cost	●	○
Lead-Acid	Low Capital Cost	Limited Cycle Life when Deeply Discharged	●	○
Flywheels	High Power	Low Energy density	●	○
SMES, DSMES	High Power	Low Energy Density, High Production Cost	●	
E.C. Capacitors	Long Cycle Life, High Efficiency	Low Energy Density	●	◐

Store

- Symbols
  - –fully capable
  - –reasonable
  - –feasible, but not quite…
  - –not feasible

•http://www.electricitystorage.org/tech/technologies_comparisons.htm

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Renewable/Alternative

- Alternative or renewable resources
  - Solar energy
  - Wind energy
  - Ocean energy (tides, waves and temperature gradients)
  - Geothermal energy
  - Hydropower especially small hydro
  - Biomass fuels
  - Conservation as an alternative resource
    - Reduced usage and improved efficiencies including vehicle fuel economy

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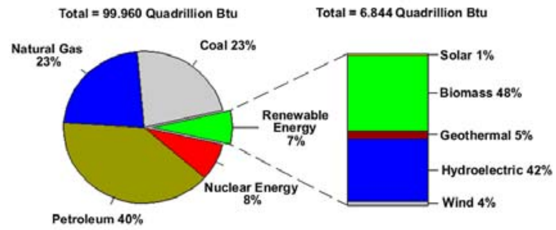
US Renewable Use 2006



- Solar + geothermal + wind = 10% of renewables = 0.7% of total energy

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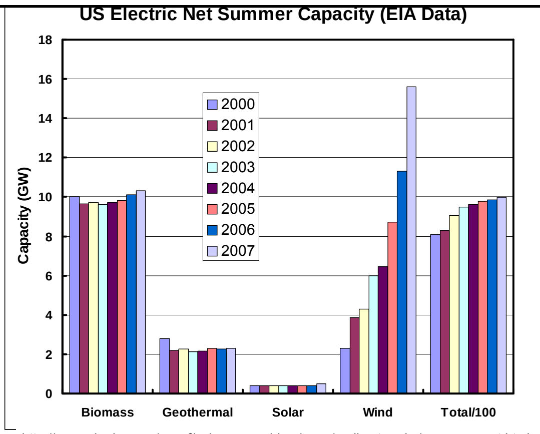
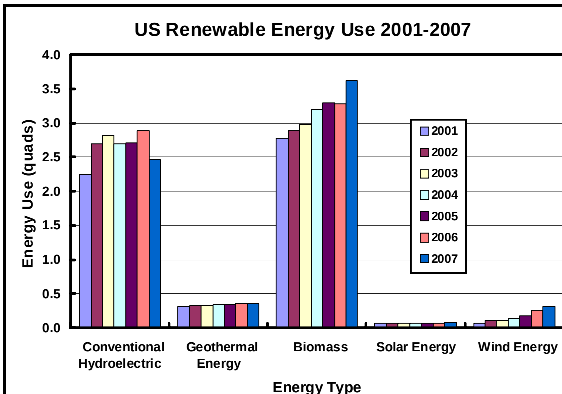


http://www.eia.doe.gov/cneaf/solar.renewables /page/prelim_trends /rea_prereport.html



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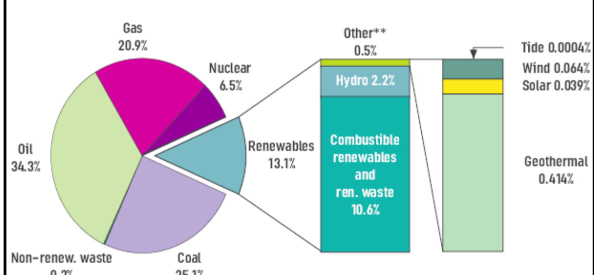
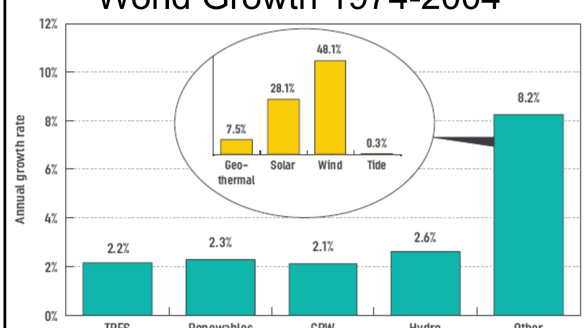
World Renewable Use 2004



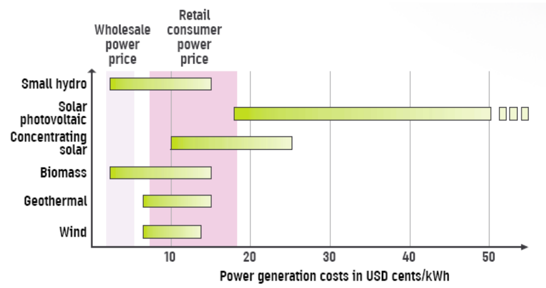
California State University Northridge http://www.iea.org/textbase/papers/2006/renewable_factsheet.pdf 35

World Growth 1974-2004



California State University Northridge http://www.iea.org/textbase/papers/2006/renewable_factsheet.pdf 36

Power Generation Costs



http://www.iea.org/textbase/papers/2006/renewable_factsheet.pdf

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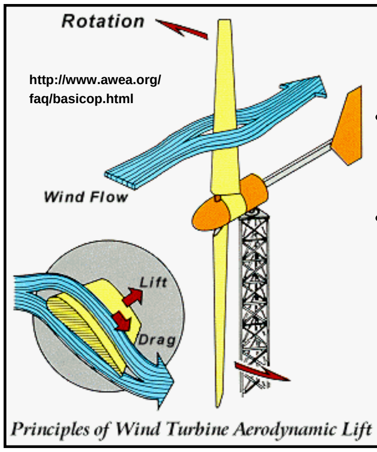

Blades

- Turbine blades as airfoils
- “Lift” becomes force in direction of rotation

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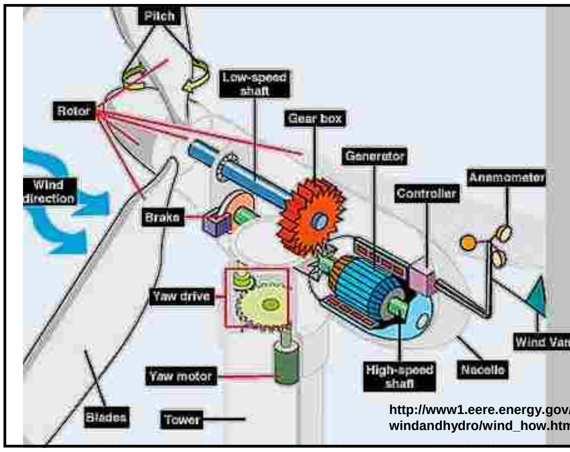

Wind Power and Betz Limit

- Power in incoming air = $\dot{m}e = \dot{m}V^2/2 = (\rho VA)V^2/2 = \rho AV^3/2 = P_0$
  - Air density, $\rho \approx 1.2$ kg/m^3
  - A = swept area of rotor = $\pi(D_{rotor})^2/4$
  - V = wind velocity
- c_p = power coefficient = turbine power divided by power in wind
  - Alternative: (generator power) / (wind power)
- Betz Limit: Maximum theoretical c_p = 16/27 ≈ 0.593

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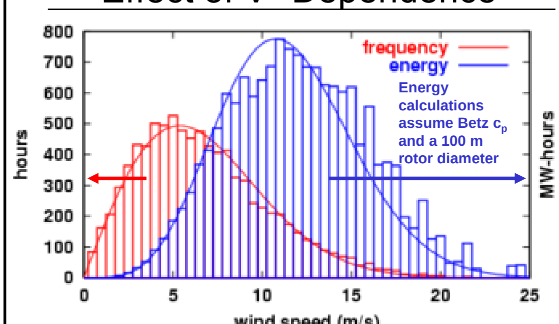
Effect of V^3 Dependence



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http://www.sandia.gov/wind/other/LeeRanchData-2002.pdf and http://en.wikipedia.org/wiki/Wind_power for plot
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Roughness Parameters

- Open water (r = 0.0002 m)
- Completely open terrain with smooth surface (r = 0.0024 m)
- Agricultural area varying amounts of fences, hedgerows, buildings (r = 0.03 m, 0.055 m, 0.1 m, 0.2 m)
- Small villages (r = 0.4 m)
- Larger cities with tall buildings (r = 0.8 m)
- Very large cities/skyscrapers (r = 1.6 m)

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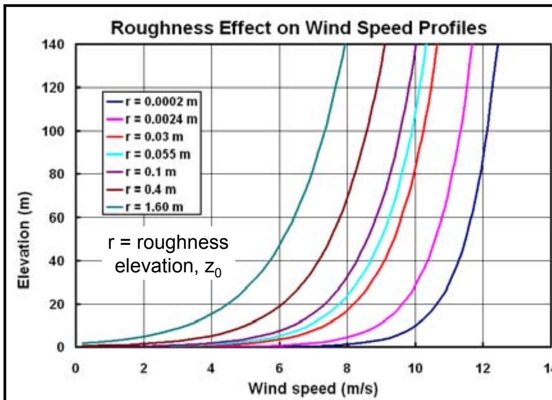

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Wind Classes (10 m)

Class	power/area(W/m²)		Speed(m/s)/(mph)	
	mim	max	min	max
1	0	100	0	4.4/9.8
2	100	150	4.4/9.8	5.1/11.5
3	150	200	5.1/11.5	5.6/12.5
4	200	250	5.6/12.5	6.0/13.4
5	250	300	6.0/13.4	6.4/14.3
6	300	400	6.4/14.3	7.0/15.7
7	400	1000	7.0/15.7	9.4/21.1

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Wind Classes (50 m)

Class	power/area(W/m²)		Speed(m/s)/(mph)	
	min	max	min	max
1	0	200	0	5.6/12.5
2	200	300	5.6/12.5	6.4/14.3
3	300	400	6.4/14.3	7.0/15.7
4	400	500	7.0/15.7	7.5/16.8
5	500	600	7.5/16.8	8.0/17.9
6	600	800	8.0/17.9	8.8/19.7
7	800	2000	8.8/19.7	11.9/26.6

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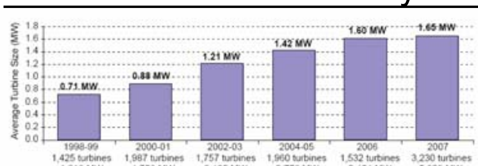
Turbine Size History



Turbine Size Range	1998-99	2000-01	2002-03	2004-05	2006	2007
	1,018 MW	1,758 MW	2,125 MW	2,776 MW	2,454 MW	5,329 MW
	1,425 turbines	1,987 turbines	1,757 turbines	1,960 turbines	1,532 turbines	3,230 turbines
0.05-0.5 MW	1.3%	0.4%	0.5%	1.8%	0.7%	0.0%
0.51-1.0 MW	98.5%	73.9%	43.4%	18.5%	10.7%	11.0%
1.01-1.5 MW	0.0%	25.4%	43.5%	56.0%	54.2%	48.6%
1.51-2.0 MW	0.3%	0.4%	12.5%	23.6%	17.6%	24.1%
2.01-2.5 MW	0.0%	0.0%	0.0%	0.1%	16.3%	15.0%
2.51-3.0 MW	0.0%	0.0%	0.1%	0.0%	0.5%	1.3%

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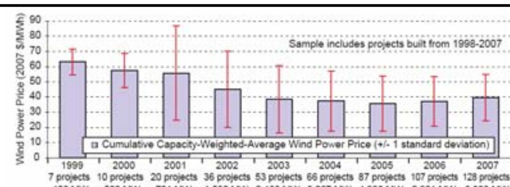
DOE Wind Annual 2007, http://www1.eere.energy.gov/windandhydro/pdfs/43025.pdf 47



Source: Berkeley Lab database.

Figure 13. Cumulative Capacity-Weighted-Average Wind Power Prices Over Time

Source: Berkeley Lab database.

DOE Wind Annual 2007, http://www1.eere.energy.gov/windandhydro/pdfs/43025.pdf

Figure 14. 2007 Wind Power Prices by Commercial Operation Date (COD)

Probability Distributions

- Probability distribution function, f
  - Probability that the random variable x lies in a certain range a ≤ x ≤ b is integral of pdf

$$P(a \le x \le b) = \int_a^b f(x)dx \qquad \int_{x_{min}}^{x_{max}} f(x)dx = 1$$

- Cumulative distribution function, F

$$F(b) = P(x \le b) = \int_{x_{min}}^{b} f(x)dx$$

$$P(a \le x \le b) = F(b) - F(a)$$

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Rayleigh Distribution

- At least three variations are used

$$V_{mp} = \beta = \frac{c}{\sqrt{2}} \qquad f(V) = \frac{Ve^{-V^2/2\beta^2}}{\beta^2} \qquad 0 \le V < \infty$$

$$2\beta^2 = c^2 \qquad f(V) = \frac{2Ve^{-V^2/c^2}}{c^2} \qquad 0 \le V < \infty$$

$$\bar{V} = \beta\sqrt{\frac{\pi}{2}} = \frac{c}{2}\sqrt{\pi} \qquad f(V) = \frac{\pi V e^{-\pi V^2/4\bar{V}^2}}{2\bar{V}} \qquad 0 \le V < \infty$$

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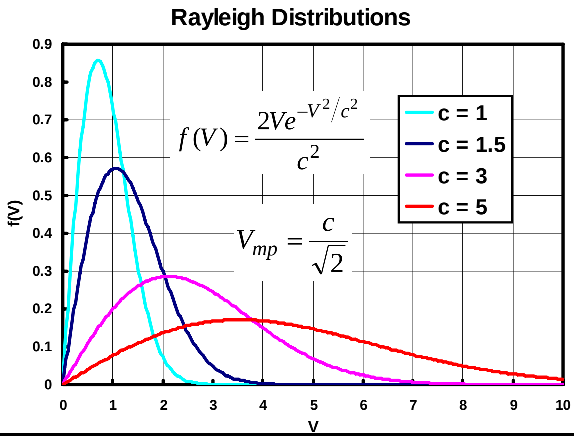

Weibull Distribution

- A two-parameter distribution with shape parameter, k, and scale parameter, c
- Rayleigh distribution is Weibull distribution with k = 2
- Mean = $c\Gamma(1 + k^{-1})$
- Variance = $c^2[\Gamma(1 + 2k^{-1}) - \Gamma^2(1 + k^{-1})]$

Γ is the gamma function

$$f(V) = \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} e^{-(V/c)^k} \qquad 0 \le V < \infty$$

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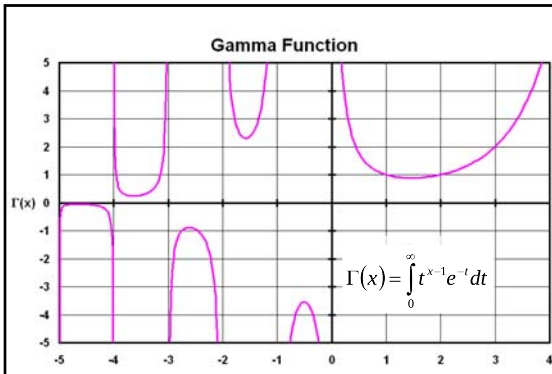
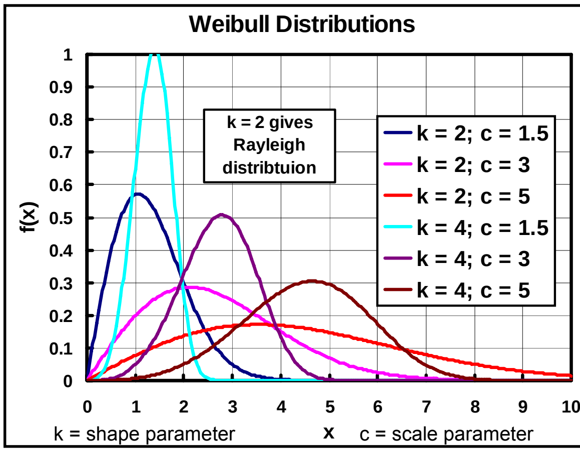


$$\Gamma(x+1) = x\Gamma(x) \qquad \Gamma(1) = \Gamma(2) = 1 \qquad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$



Wind Power

- Instantaneous wind power: $P_0 = \rho V^3 A/2$
- Total or average wind power: $\overline{P}_0 = \frac{\rho A \overline{V^3}}{2} = \frac{\rho A}{2}\int_0^\infty V^3 f(V)dV$
- Total or average turbine power: $\overline{P}_{total} = c_p \overline{P}_0 = c_p \rho A \overline{V^3}/2$

$$\left(\overline{V^3}\right)_{Weibull} = c^3\Gamma\left(\frac{3}{k}+1\right)$$

$$\left(\overline{V^3}\right)_{Rayleigh} = c^3\Gamma\left(\frac{3}{2}+1\right) = c^3\frac{3}{2}\Gamma\left(\frac{3}{2}\right) = c^3\frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = c^3\frac{3\sqrt{\pi}}{4}$$

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Wind Power Distribution

- Wind power between V_1 and V_2
  - Weibull (Set k = 2 for Rayleigh)

$$\text{Wind } \overline{P} \text{ between } V_1 \text{ and } V_2 = \frac{\rho A c^3}{2}\int_{(V_1/c)^k}^{(V_2/c)^k} y^{\frac{3}{k}} e^{-y} dy$$

  - Found by numerical integration with results in tables

$$\left[\begin{matrix}\text{Wind } \overline{P} \text{ between} \\ V_1 \text{ and } V_2\end{matrix}\right] = \left[f_P(V_2) - f_P(V_1)\right]\frac{\rho A c^3}{2}\Gamma\left(\frac{3}{k}+1\right)$$

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Wind Turbine Operation

- No operation until wind velocity reaches a minimum called the cut-in velocity
- Then operate at full turbine output power until turbine output is greater than generator can accept
- Limit turbine output power to full generator power at high wind speeds
- No operation above maximum velocity called cut-out velocity

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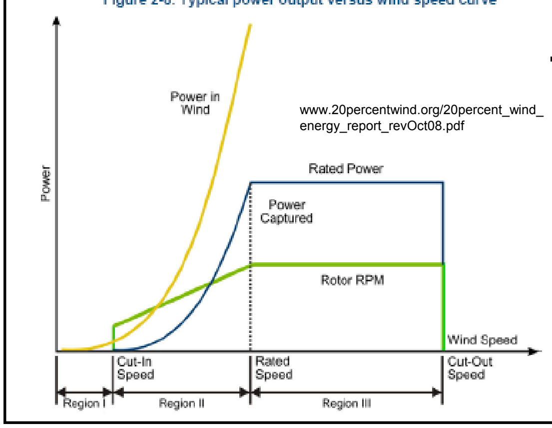
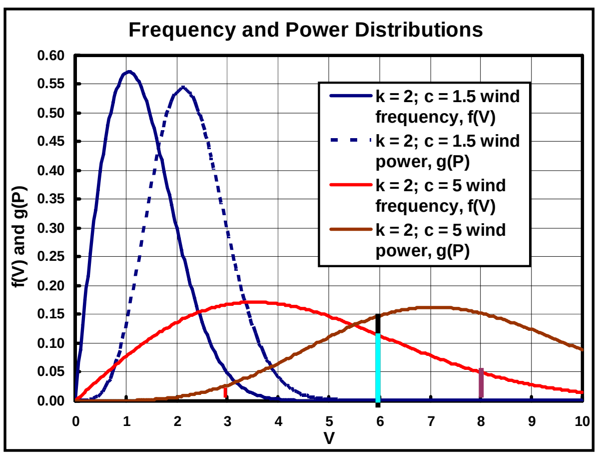
Figure 2-8. Typical power output versus wind speed curve





Average Operating Power

- Generator uses turbine power between $V_{cut\text{-}in}$ and rated (maximum power) velocity, $V_{Pmax} = [2P_{max}/(c_p\rho A)]^{1/3}$
  - Power coefficient c_p = generator power divided by wind power
- Between V_{Pmax} and $V_{cut\text{-}out}$ operate at maximum power

$$\overline{P}_{operation} = \int_{V_{cut-in}}^{V_{P\max}} \frac{c_p \rho A V^3}{2} f(V)dV + \int_{V_{P\max}}^{V_{cut-out}} P_{\max} f(V)dV$$

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Average Operating Power II

- Using power fraction table

$$\int_{V_{cut-in}}^{V_{P\max}} \frac{c_p \rho A V^3}{2} f(V)dV = \left[f_P(V_{cut-in}) - f_P(V_{P\max})\right]\frac{c_p \rho A c^3}{2}\Gamma\left(\frac{3}{k}+1\right)$$

- Using cumulative distribution

$$\int_{V_{P\max}}^{V_{cut-out}} P_{\max} f(V)dV = P_{\max}\left[\left(1-e^{-(V_{cut-out}/c)^k}\right)-\left(1-e^{-(V_{P\max}/c)^k}\right)\right]$$

$$\overline{P}_{operation} = \left[f_P(V_{cut-in}) - f_P(V_{P\max})\right]\frac{c_p \rho A c^3}{2}\Gamma\left(\frac{3}{k}+1\right) + P_{\max}\left(e^{-(V_{P\max}/c)^k} - e^{-(V_{cut-out}/c)^k}\right)$$

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Example Problem

- Data: k = 1.6, c = 10 m/s, V_{cut-in} = 5 m/s, V_{Pmax} = 15 m/s, $V_{cut-out}$ = 25 m/s
- V_{cut-in}/c = (5 m/s) / (10 /m/s) = 0.5
- V_{Pmax}/c = (25 m/s) / (10 /m/s) = 1.5
- $f_P(V_{Pmax}) - f_P(V_{cut-in})$ = 0.328893 – 0.006295 = 0.322598 (table next chart)

$$\Gamma\left(\frac{3}{k}+1\right) = \Gamma\left(\frac{3}{1.6}+1\right) = \Gamma(2.875) = (1.875)\Gamma(1.875) = (1.875)(0.875)\Gamma(0.875)$$

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$\Gamma(2.875)$ = 1.789 (chart after next) 62

Use of Power Fraction Table

Fraction of Wind Power Between V = 0 and Given V

V/c	Fraction for Following Values of k					
	k = 1.4	**k = 1.6**	k = 1.8	k = 2	k = 2.2	k = 2.4
0.50	0.004948	**0.006295**	0.007273	0.007877	0.008149	0.008156
0.60	0.010169	0.013427	0.016118	0.018147	0.019530	0.020338
0.70	0.018369	0.024959	0.030868	0.035837	0.039800	0.042794
0.80	0.030166	0.041893	0.053010	0.063024	0.071728	0.079088
0.90	0.046036	0.064971	0.083615	0.101180	0.117277	0.131769
1.00	0.066279	0.094589	0.123163	0.150855	0.177061	0.201517
1.10	0.091003	0.130760	0.171457	0.211508	0.250044	0.286651
1.20	0.120122	0.173115	0.227629	0.281520	0.333573	0.383162
1.30	0.153376	0.220943	0.290233	0.358382	0.423715	0.485322
1.40	0.190353	0.273260	0.357401	0.439009	0.515849	0.586691
1.50	0.230519	**0.328893**	0.427029	0.520117	0.605340	0.681244

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Gamma Function Table

Abridged Table of Gamma Functions

x	Γ(x)	x	Γ(x)	x	Γ(x)
0.01	99.43259	0.16	5.81127	0.40	2.21816
0.02	49.44221	0.18	5.13182	0.45	1.968136
0.03	32.78500	0.20	4.59084	0.50	1.772454
0.04	24.46096	0.22	4.15048	0.55	1.616124
0.05	19.47009	0.24	3.78550	0.60	1.489192
0.06	16.14573	0.26	3.47845	0.65	1.384795
0.07	13.77360	0.28	3.21685	0.70	1.298055
0.08	11.99657	0.30	2.99157	0.75	1.225417
0.09	10.61622	0.32	2.79575	0.80	1.16423
0.10	9.51351	0.34	2.62416	**0.85**	**1.112484**
0.11	8.61269	0.36	2.47273	**0.90**	**1.068629**
0.12	7.86325	0.38	2.33826	0.95	1.031453
0.13	7.23024	For x outside the range (0, 1) you use the relationship that Γ(x+1) = xΓ(x)			
0.14	6.68869				

Interpolate for $\Gamma(0.875)$ = 1.0905565

$\Gamma(2.875)$ = (1.875)(0.875) $\Gamma(0.875)$ = (1.875)(0.875) (1.0905565) = 1.789

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Example II

- Computations below rated speed

$$\frac{1}{A}\left[f_P(V_{P\max}) - f_P(V_{cut-in})\right]\frac{c_p \rho A c^3}{2}\Gamma\left(\frac{3}{k}+1\right) = [0.322598]$$

$$\frac{0.48}{2}\frac{1.225\,kg}{m^3}\left(\frac{10\,m}{s}\right)^3(1.789)\frac{1W\cdot s^3}{kg\cdot m^2} = \frac{526.0W}{m^2}$$

- Computations above rated speed ($V_{cut-out}/c$ = 2.5) give 0.1345 P_{max}

$$P_{\max}\left(e^{-(V_{P\max}/c)^k} - e^{-(V_{cut-out}/c)^k}\right) = P_{\max}\left[e^{-(1.5)^{1.6}} - e^{-(2.5)^{1.6}}\right]$$

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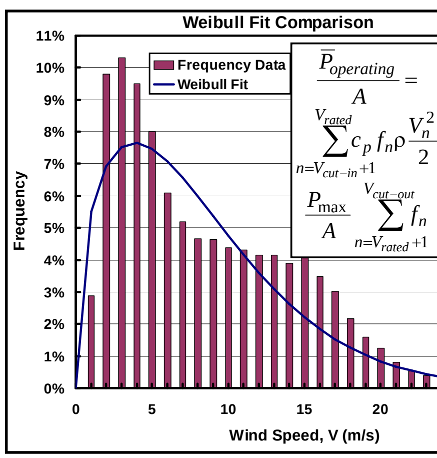
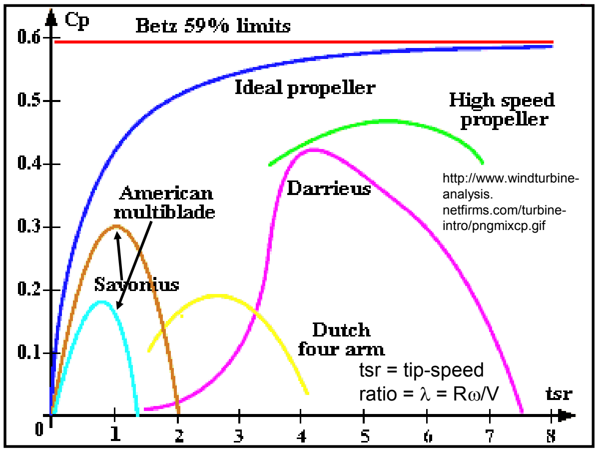


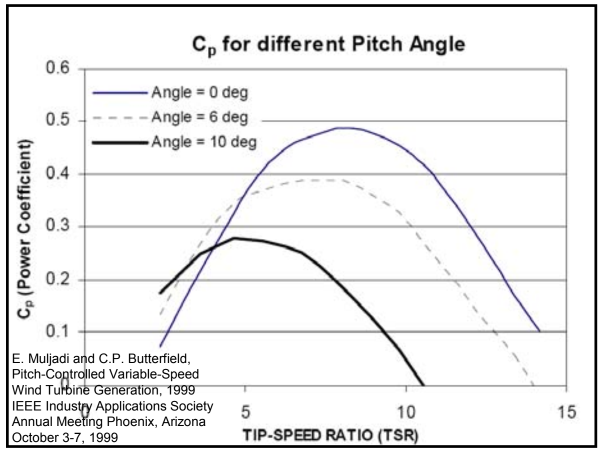
$$\frac{\overline{P}_{operating}}{A} = \sum_{n=V_{cut-in}+1}^{V_{rated}} c_p f_n \rho \frac{V_n^2}{2} \qquad \frac{P_{\max}}{A}\sum_{n=V_{rated}+1}^{V_{cut-out}} f_n$$

Analyze discrete data when there is not a good fit

Find power in each velocity band above cut-in and up to rated wind speed

Find frequencies between rated and cut out wind speed





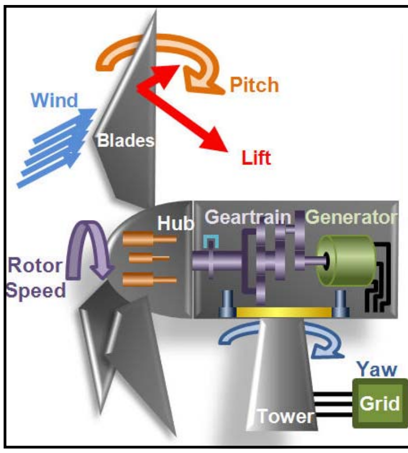


Illustration of changes in lift and pitch that are used to control wind power accepted by wind turbines.

http://www.mathworks.com/matlabcentral/fx_files/25752/3/Model_Wind_Turbine_With_MW_Tools.jpg

Turbine Power Controls

- Pitch controls – adjusts the angle of the movable blades (pitch)
  - Reduces fraction of power extracted from wind, keeping generator power constant
- Passive stall controls design the angle of the fixed rotor blades to reduce lift force at higher wind speeds
  - Maintains constant force on rotor shaft as wind speed increases above maximum

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Turbine Power Controls II

- Passive stall control
  - uses basic rotor design to ensure proper control
  - no changes in rotor blades' position during operation
- Active stall control
  - Changes blade pitch to increase stall force in a manner similar to pitch control

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Environmental Impacts

- Visual impact
- Noise
- Effect on birds (avian impacts)
- Electromagnetic interference
- Bats
- Not discussed
  - Environmental benefits in reduction of fossil-fuel generated pollutants, including greenhouse gas CO_2

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