

## The Path of the Sun

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## Outline

- Review radiation properties for solar collectors
- Orientation of earth and sun
- Earth-based solar path
  - Definition of angles
  - Calculation of path
- Optimum tilt angles for solar collectors
- Solar air mass

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TABLE 12-3 From last lecture			TABLE 12-3		
Comparison of the solar absorptivity $\alpha_s$ of some surfaces with their emissivity $\epsilon$ at room temperature			Comparison of the solar absorptivity $\alpha_s$ of some surfaces with their emissivity $\epsilon$ at room temperature		
Surface	$\alpha_s$	$\epsilon$	Surface	$\alpha_s$	$\epsilon$
Aluminum			Plated metals		
Polished	0.09	0.03	Black nickel oxide	0.92	0.08
Anodized	0.14	0.84	Black chrome	0.87	0.09
Foil	0.15	0.05	Concrete	0.60	0.88
Copper			White marble	0.46	0.95
Polished	0.18	0.03	Red brick	0.63	0.93
Tarnished	0.65	0.75	Asphalt	0.90	0.90
Stainless steel			Black paint	0.97	0.97
Polished	0.37	0.60	White paint	0.14	0.93
Dull	0.50	0.21	Snow	0.28	0.97
			Human skin (Caucasian)	0.62	0.97

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## Emissivity and Absorptivity

- Data on previous chart are due to difference in radiation properties associated with radiation temperature
  - For solar collector incoming radiation has source temperature about 5800 K
  - Outgoing radiation from collector has source temperature just over 300K
  - Kirchoff's Law says  $\alpha_\lambda = \epsilon_\lambda$ , but this is true only at individual wavelengths
  - What is average over all wavelengths?

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## Average Radiation Properties

- Integrated average properties over all wavelengths

$$\bar{\epsilon} = \frac{1}{\sigma T^4} \int_0^\lambda \epsilon_\lambda E_{b\lambda} d\lambda' \quad \bar{\alpha} = \frac{1}{\sigma T^4} \int_0^\lambda \alpha_\lambda E_{b\lambda} d\lambda'$$

- Look at simple example where  $\epsilon_\lambda = \epsilon_1$  for  $\lambda < \lambda_1$  and  $\epsilon_\lambda = \epsilon_2$  for  $\lambda > \lambda_1$

$$\bar{\epsilon} = \frac{1}{\sigma T^4} \int_0^\lambda \epsilon_1 E_{b\lambda} d\lambda' + \frac{1}{\sigma T^4} \int_\lambda^\infty \epsilon_2 E_{b\lambda} d\lambda'$$

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## Average Radiation Properties II

- Rearrange to get  $f_\lambda$ , the fraction of black body radiation between 0 and  $\lambda$

$$\bar{\epsilon} = \frac{\epsilon_1}{\sigma T^4} \int_0^\lambda E_{b\lambda} d\lambda' + \frac{\epsilon_2}{\sigma T^4} \int_\lambda^\infty E_{b\lambda} d\lambda' = \epsilon_1 f_\lambda + \epsilon_2 (1 - f_\lambda)$$

- Similar equation for absorptivity ( $\alpha_\lambda = \epsilon_\lambda$ )

$$\bar{\alpha} = \frac{\alpha_1}{\sigma T^4} \int_0^\lambda E_{b\lambda} d\lambda' + \frac{\alpha_2}{\sigma T^4} \int_\lambda^\infty E_{b\lambda} d\lambda' = \alpha_1 f_\lambda + \alpha_2 (1 - f_\lambda)$$

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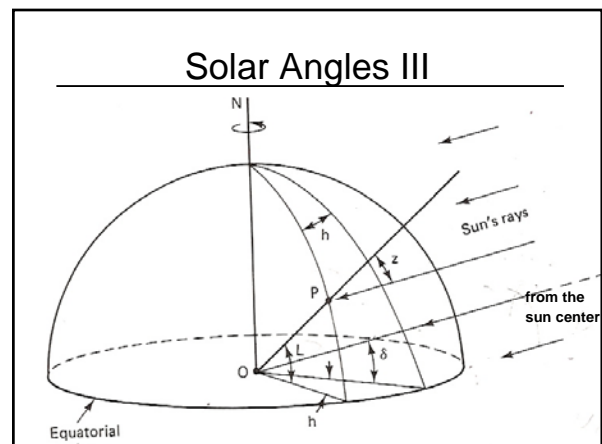
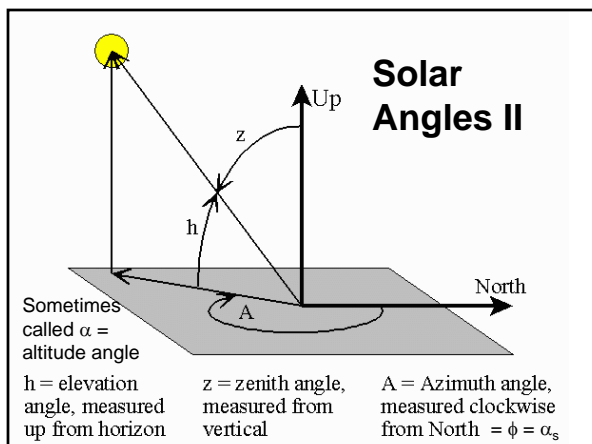
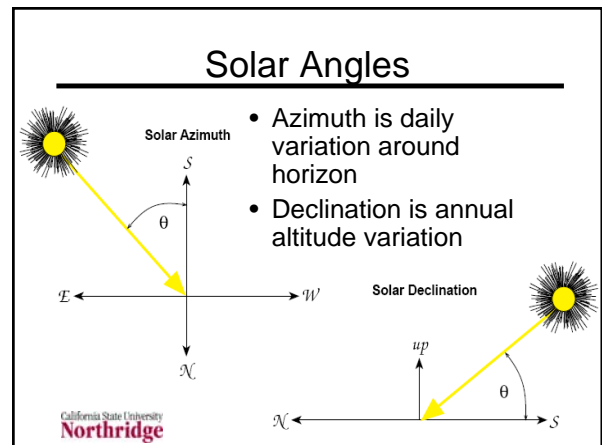
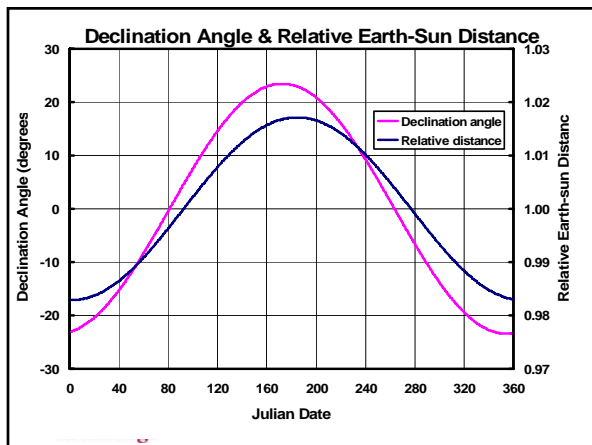
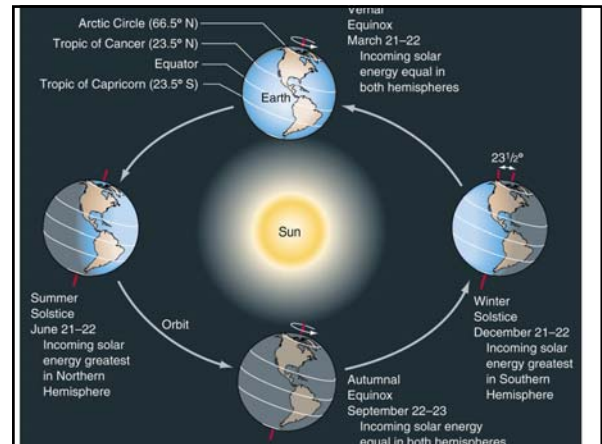
### Example

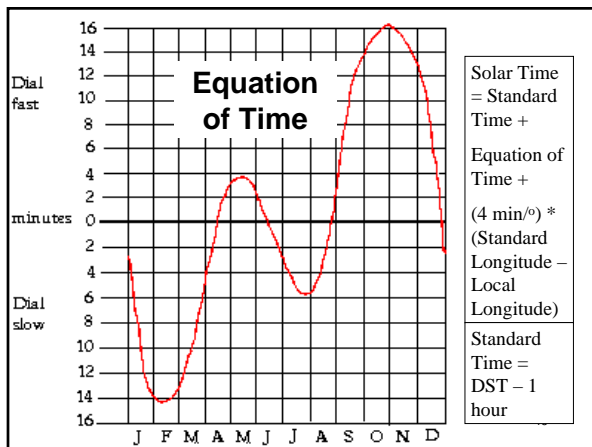
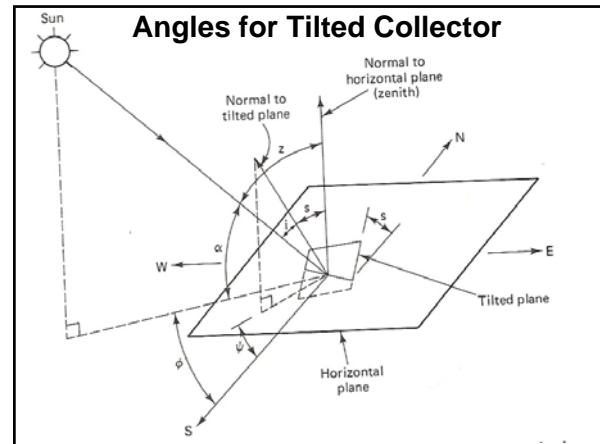
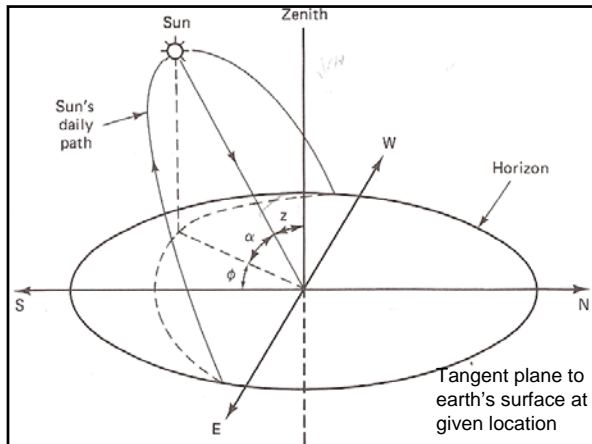
- Data:  $\epsilon_\lambda = 0.9$  for  $\lambda < 3 \mu\text{m}$  and  $\epsilon_\lambda = 0.2$  for  $\lambda > 3$ 
  - Solar T = 5800 K,  $\lambda T = 17,400 \mu\text{m}\cdot\text{K}$ ,  $f_\lambda(17,400 \mu\text{m}\cdot\text{K}) = 0.980155$ , find  $\alpha$ 
    - Use  $\alpha_\lambda = \epsilon_\lambda$
  - Earth T = 300 K,  $\lambda T = 900 \mu\text{m}\cdot\text{K}$ ,  $f_\lambda(17,400 \mu\text{m}\cdot\text{K}) = 0.001$ , find  $\epsilon$

$$\bar{\alpha}_{5800K} = \alpha_1 f_{\lambda_1} + \alpha_2 (1 - f_{\lambda_1}) = 0.9(0.980) + 0.2(1 - 0.980) = 0.886$$

$$\bar{\epsilon}_{300K} = \epsilon_1 f_{\lambda_1} + \epsilon_2 (1 - f_{\lambda_1}) = 0.9(0.001) + 0.2(1 - 0.001) = 0.201$$

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### Computing the Sun Path

- Input data: Latitude, L, date, hour h
- Find declination from serial date, n
 
$$\delta = (23.45^\circ) \sin \left[ \frac{360}{365} (284 + n) \frac{\pi}{180} \right] \quad (\delta \text{ in degrees})$$
- Two angles: altitude ( $\alpha$ ) and azimuth ( $\phi$ )
  - $\sin(\alpha) = \sin(L) \sin(\delta) + \cos(L) \cos(\delta) \cos(h)$
  - $\sin(\alpha_s) = \sin(\phi) = \cos(\delta) \sin(h) / \cos(\alpha)$
  - Sun path is plot of  $\alpha$  vs.  $\phi = \alpha_s$  for one day
  - Plot is symmetric about solar noon

Typically plot data for 21<sup>st</sup> of month

### Path Calculation Problem

- Angles given as  $\sin(\text{angle}) = x$  require arcsin function calculation
- Typical arcsine function returns angle between  $-90^\circ$  and  $90^\circ$ 
  - Limits correspond to range for sine between  $-1$  and  $+1$
  - Special calculation for hour angle limit
    - $h_{\text{limit}} = \pm \tan(\delta) / \tan(L)$
    - $\phi = \pm[\pi - \arcsin(\sin \phi)]$  for  $|h| > |h_{\text{limit}}|$

