Name: ___________________________________

College of Engineering and Computer Science
Mechanical Engineering Department

California State University Northridge

Mechanical Engineering 309
Numerical Analysis of Engineering Systems

Spring 2014 Number: 15237 Instructor: Larry Caretto

Solution to Eighth Quiz, April 30, 2014

The modified Euler method for the solution of ordinary differential equations is given by the algorithm shown at the right.

$$y_{i+1/2} = y_i + \left[\frac{h_{i+1}}{2}\right] f(x_i, y_i) \qquad x_{i+1/2} = x_i + \frac{h_{i+1}}{2}$$

$$y_{i+1} = y_i + h_{i+1} f(x_{i+1/2}, y_{i+1/2})$$

Convert the differential equation $d^2y/dx^2 - 4\ dy/dx = 1$ into a set of two first order equations and obtain a solution at x = 0.3 using the initial conditions that y(0) = 0 and $dy/dt|_{x=0} = 1$ and a step size, h = 0.1

Define $v = f_y = dy/dx$ as one differential equation and the second differential equation becomes dv/dx = 1 + 4v. From our definition of the derivative as $dy_k/dx = f_k$ we see that the y derivative equation is $f_y = v$ and the v derivative equation is $f_v = 1 + 4v$.

At x = 0 we have v = 1 and y = 0; this gives the following derivatives at x = 0: $f_v = 1 + 4v = 1 + 4(1) = 5$ and $f_y = v = 1$.
Applying the algorithm gives the following results for the midpoint $x_{1/2} = h/2 = 0.05$:
$y_{1/2} = y_0 + 0.05f_y = 0 + 0.05(1) = 0.05$
$v_{1/2} = v_0 + 0.05f_v = 1 + 0.05(5) = 1.25$.
With these midpoint values we find the following midpoint derivative values that will be applied to the full step:
$f_v = 1 - y + 4v = 1 + 4(1.25) = 6$ and $f_y = v = 1.25$.
Applying these derivative values for the full step from 0 to 0.1 gives
$y_1 = y_0 + 0.1f_y = 0 + 0.1(1.25) = 0.125$ and $v_1 = v_0 + 0.1f_v = 1 + 0.1(6) = 1.6$.

Continuing the algorithm for the second step requires the derivatives at x = 1. These are $f_v = 1 + 4v = 1 + 4(1.6) = 7.4$ and $f_y = v = 1.6$.

With these derivatives we find the following results for the midpoint:

$x_{3/2} = x_1 + h/2 = 0.15$; $y_{3/2} = y_1 + 0.05f_y = 0.125 + 0.05(1.6) = 0.205$ and $v_{3/2} = v_1 + 0.05f_v = 1.6 + 0.05(7.4) = 1.97$.

With these midpoint values we find the following midpoint derivatives: $f_v = 1 + 4v = 1 + 4(1.97) = 8.88$ and $f_y = v = 1.97$.

Applying these derivative values for the full step from 0.1 to 0.2 gives $y_2 = y_1 + 0.1f_y = 0.125 + 0.1(1.97) = 0.322$ and $v_2 = v_1 + 0.1f_v = 1.6 + 0.1(8.88) = 2.488$.

The final step is a repetition of the second step. We first get the derivatives at x = 2. These are $f_v = 1 + 4v = 1 + 4(2.488) = 10.952$ and $f_y = v = 2.488$. With these derivatives we find the following results for the midpoint $x_{5/2} = x_5 + h/2 = 0.25$: $y_{5/2} = y_2 + 0.05f_y = 0.322 + 0.05(2.488) = 0.4464$ and $v_{5/2} = v_2 + 0.05f_v = 2.488 + 0.05(10.952) = 3.0356$. With these midpoint values we find the following derivative values: $f_v = 1 + 4v = 1 + 4(3.0356) = 13.1424$ and $f_y = v = 3.0356$. Applying these derivative values for the full step from 0.2 to 0.3 gives $y_3 = y_2 + 0.1f_y = 0.322 + 0.1(3.0356) = 0.62556$ and $v_3 = v_2 + 0.1f_v = 2.488 + 0.1(13.1424) = 3.80224$.

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