MATH 255 **Applied Honors Calculus III** **Winter 2005**

Final Exam Review Sheet – Part II

1. True or False. Determine whether the following statements are true or false and provide an explanation for your answer (5 points each).

(a) Let $\mathbf{F}(x, y, z)$ be a vector field whose components have continuous second partial derivatives in $\mathbb{R}^3$, then

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0$$

(b) If $\mathbf{F} = \langle P, Q \rangle$ and $P_y = Q_x$ in an open region D, then $\mathbf{F}$ is conservative.

(c) If f has continuous partial derivatives on $\mathbb{R}^3$ and C is any circle, then $\oint_C \nabla f \cdot d\mathbf{r} = 0$.

(d)

$$\int_{-C} f(x, y)\ ds = -\int_C f(x, y)\ ds.$$

(e) If S is a sphere and $\mathbf{F}$ is a constant vector field, then $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$.

(f) Suppose a solid E and its boundary surface S satisfy the conditions of the divergence theorem, then

$$\iint_S (f\nabla g - g\nabla f) \cdot \mathbf{n}\ dS = \iiint_E (f\nabla^2 g - g\nabla^2 f)\ dV$$

(g)

$$\int_{-C} f(x, y)\ dx = -\int_C f(x, y)\ dx.$$

(h) The integral

$$\int_0^{2\pi} \int_0^{2\pi} \int_r^2 dz\ dr\ d\theta$$

represents the volume enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$.

(i) If C is a simple closed curve in $\mathbb{R}^2$, $\mathbf{n}$ its unit normal vector, D the region it encloses, and $\mathbf{F}$ a vector field whose component functions have continuous first partial derivatives, then

$$\oint_C \mathbf{F} \cdot \mathbf{n}\ ds = \iint_D \nabla \cdot \mathbf{F}\ dA.$$

2. (10 points) Evaluate $\oint_C P\ dx + Q\ dy$ where $P(x, y) = \tan y$,$Q(x, y) = 3x + x\sec^2 y$ and C is the circle $(x - 2)^2 + (y - 5)^2 = 4$.

3. (10 points) Find the area of the portion of the plane $x + 2y + 3z = 1$ that is inside the cylinder $x^2 + y^2 = 2$

4. (10 points) Find the area of the parametric surface given by

$$\mathbf{r}(u, v) = \langle u\cos v, u\sin v, u \rangle, \quad 0 \le u \le 1, \quad 0 \le v \le 2\pi,$$

and describe the surface.

5. (10 points) Find the area of the parametric surface given by

$$\mathbf{r}(u, v) = \langle \sin u \cos v, \sin u \sin v, \sin u \rangle, \quad 0 \leq u \leq \frac{\pi}{4}, \quad 0 \leq v \leq 2\pi,$$

and describe the surface.

6. (15 points) Use the divergence theorem to compute the flux across S of the vector field given by $\mathbf{F} = \langle x, y, z \rangle$, where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$ (without the bottom). **Caution**: the divergence theorem is only part of what you need, you will still need to calculate a surface integral!

Problems from the text, page 1172: 11, 13, 15, 17, 19, 27, 29, 32, 37, 38, and 39