MATH 250 | Calculus III | Spring 2008

Review Sheet for Midterm Exam I

The first midterm exam will take place on Feb. 21st during regular class time at our usual location, JD1553. You are allowed to bring one page (one side of an 8.5×11 in. sheet of paper) with notes. No calculators.

Problem 1. Determine whether the following statements are True or False. Justify your answer.

(a) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3 $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

(b) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3 $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$

(c) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3 $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{v} \times \mathbf{u}\|$

(d) The cross product of two unit vectors is a unit vector

(e) The linear equation $Ax + By + Cz = D$ represents a line in space

(f) If $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{u} = 0$ or $\mathbf{v} = 0$

(g) If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{u} = 0$ or $\mathbf{v} = 0$

(h) If $\mathbf{u} \cdot \mathbf{v} = 0$, then $\mathbf{u} = 0$ or $\mathbf{v} = 0$

(i) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3 $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = 0$

(j) If $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are differentiable vector functions, then

$$\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t)) = \mathbf{u}'(t) \times \mathbf{v}'(t)$$

(k) If $\|\mathbf{r}(t)\| = 1$ for all t, then $\|\mathbf{r}'(t)\| = 0$

(l) If $\|\mathbf{r}(t)\| = 1$ for all t, then $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are perpendicular

(m) If $\mathbf{r}(t)$ is a differentiable vector functions, then

$$\frac{d}{dt}\|\mathbf{r}(t)\| = \|\mathbf{r}'(t)\|$$

(n) The lines with parametric equations

$$\ell_1 : \begin{cases} x = 1 + 2t \\ y = -2 + t \\ z = 1 - t \end{cases} \quad \text{and} \quad \ell_2 : \begin{cases} x = -1 + t \\ y = 2 - t \\ z = 1 + t \end{cases}$$

are parallel.

Problem 2. Find the equation of the sphere with center $(-3, 2, 4)$ and radius 5.

Problem 3. A river flows South with a current of 5 mph. A motorboat travels at a speed of 25 mph in still water, In what direction must the boat be headed in order to move directly east across the river?

Problem 4. Find the angle between the vectors $\mathbf{u} = \langle \sqrt{3}, -1 \rangle$ and $\mathbf{v} = \langle -1, \sqrt{3} \rangle$.

Problem 5. Find the equation of the plane through the points $(1, 2, -3)$, $(3, 1, 0)$, and $(2, 3, -7)$.

Problem 6. Find a vector with length 3 that is perpendicular to both $\mathbf{u} = \langle 2, -1, 3 \rangle$ and $\mathbf{v} = \langle 1, -2, -1 \rangle$.

Problem 7. Find expressions for the unit tangent vector, $\mathbf{T}$, and the principal normal unit vector, $\mathbf{N}$, to the curve described by $\mathbf{r}(t) = \langle 2\cos 6t, 2\sin 6t, 3t \rangle$.

Problem 8. Find the curvature of the curve described by $\mathbf{r}(t) = \langle 3\cos 2t, \sin 2t \rangle$ at the point $(0, 1)$.

Problem 9. If $\mathbf{r}(t) = \langle 2t^2, \frac{3}{2}t^2, 5t \rangle$, find the tangential and normal components of the acceleration.

Problem 10. Write the equation $x^2 - y^2 = 1$ in cylindrical coordinates.

Problem 11. Write the equation $z = 5$ in spherical coordinates.

Problem 12. Write the equation $x = 6$ in cylindrical coordinates.

Problems from the text:

Section 11.6, pg. 592: 22, 24, 28, 31, 32.

Section 11.7, pg. 602: 64, 65