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THE YOUNG EINSTEIN

The advent of relativity

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1 *Einstein's education: mathematics and the laws of nature*

The misperceived legacy of the Luitpold Gymnasium

Einstein received formal, advanced schooling during the years 1888–1900. Over the past five generations there has never been more attention focused on the reform of science and mathematics teaching than in this thirteen-year period. The ferment spread throughout German-speaking. Europe—from Königsberg on the Baltic to Berne in the foothills of the" Alps, from German enclaves in Romania to the Hanseatic cities on the North Sea. Writers of the time spoke of it as the *Schulkrieg*, the war over the schools. It was a struggle between proponents of the classical values associated with education in Latin and Greek and supporters of instruction in modern languages and natural sciences. The question was asked over and again: which offered better preparation for life, abstract and impractical sensitivity to noble values or concrete training in practical arts? To characterise in a simple way the course of this complex struggle would be deceiving. It involved the professional and aesthetic goals of engineers, government officials, university professors, and secondary-schoolteachers. Kaiser Wilhelm II of Germany took a personal interest in the matter. Passions ran high as the new curricula and certifying procedures were debated, for the future of many professions hung in the balance.[1]

Two passages indicate the main lines of the great educational debate in late nineteenth-century central Europe. The first is from an **1886** address by the physicist Ernst Mach, a liberal-minded thinker whose writings produced a deep impression on the young Einstein. Mach was entirely opposed to the monopoly of classical philology on preparing pupils for university study. It was silly, he believed, to require familiarity with Greek and Latin of future doctors and scientists: 'In ***modern*** times the Greeks and Romans are simply two objects, among others, for archaeological and historical research.' Exclusive instruction in mathematics and natural sciences, he continued, would in fact provide a better general education than would instruction limited to the philological specialities. Only mathematics and natural sciences, Mach argued, showed how to elucidate 'the economical organisation and organic association' of new concepts.[2] The second passage comes from a petition of 1888 circulated by the reactionary *Gymnasium* professor at Heidelberg, Gustav Uhlig. The petition attracted more than 4000 signatures, including 500 of the 1500 German university *Dozenten*. Uhlig's petition emphasised 'that the German nation has every reason to be grateful for that which the *Gymnasien* have attained and will attain'. It urged that the *Gymnasien* remain bastions of instruction in Greek and Latin. The signers would accept only minor changes in the educational system. They feared that the *Gymnasien*, 'a national blessing of the first order', would be weakened by 'the zeal of their opponents'.[3] Uhlig's and Mach's points of view defined the limits of debate for more than a decade.

The reform struggle centred on Germany, and within Germany on the kingdom of Prussia. Control of educational policy in the German Empire from 1870 to 1918 was retained by the individual German states, even though in practice the model set by Prussia, the largest and most populous state, was imitated widely. Throughout Germany the entire pattern of secondary education changed dramatically over the period 1890–1900. Before this period, graduates of the *Gymnasien* (the classical schools offering Greek and Latin) retained many official privileges not extended to graduates of the *Realgymnasien* (the semi-classical schools offering Latin but not Greek) and the *Oberrealschulen* (the non-classical schools offering neither Greek nor Latin). Although in the early years of the German Empire the Prussian *Realgymnasien* were awarded the right to send students to the universities to study modern languages and natural sciences and in 1882 the same privilege was extended to the *Oberrealschulen*, until 1900 most university programmes and many government positions remained closed to students trained in these *Realanstalten*. Only in 1901 did the Prussian educational authorities grant the *Realanstalten* virtual parity with the *Gymnasien*. The other German states soon followed the lead of Prussia.

Many more pupils attended *Gymnasium* in Prussia than in all the other states of Germany combined. In 1895 enrolment at Prussian *Gymnasien* exceeded 80 000, while in the state with the second largest school system, Bavaria, it reached only 18 000; in third place came Württemberg with 6400 pupils.[4] Einstein attended *Gymnasium* in Munich, capital of Bavaria. The smaller secondary-school system of nineteenth-century Bavaria differed from that of Prussia in several respects. The former was split for the most part between classical *Gymnasien* offering Latin and Greek and

Realschulen offering neither. In addition, Bavarian *Gymnasien* devoted more time to geography, history, and German than did their northern German counterparts.[5]

What can we say about Einstein's personal experience with Bavarian secondary-school education at his institution of instruction, the Luitpold Gymnasium in Munich?[6] By German standards it was a large *Gymnasium*. During the years of Einstein's attendance the school grew from 684 pupils in 1888 to 1330 pupils in the autumn of 1894.[7] Around the middle of the 1890s the school instructed seven per cent of all *Gymnasium* pupils in Bavaria. Most of the pupils were Catholic; only five per cent were Jewish, somewhat greater than the proportion of Jewish residents in the city of Munich.[8] In Einstein's time the Luitpold Gymnasium had a reputation as an enlightened school. The mathematician Abraham A Fraenkel, who entered the Luitpold twelve years after Einstein did, characterised the rector there, Dr Wolfgang Markhausen, as a kind and generous man who had established a liberal atmosphere at the school during the late 1880s and 1890s. Fraenkel contrasted Markhausen with his replacement in 1902, a slightly ridiculous martinet who invoked a curfew for his charges and forbade them to see many Munich plays, including some by Lessing and Schiller.[9]

At Markhausen's Luitpold Gymnasium Einstein enjoyed what passed for progressive instruction in the exact sciences. During his last year and a half he used as a text one of the later editions of Viennese school inspector Josef Krist's *Essentials of Natural Science*. Topics from the text included the distinction between chemistry and physics, the rigid and the fluid body, gases and gas laws, and heat.[10] In the 1864 preface to the first edition of his *Essentials* Krist was clear that experiments formed the basis for natural laws. Taking issue with the dominant view of the time in Prussia, where physics was taught as a branch of mathematics, Krist insisted that physics remained 'a science of experience' which had to be taught by appealing to intuition. By the fifth edition of 1872 Krist emphasised that he made less use of 'rigorous scientific systems' than of pedagogical and didactic principles. No dry prose could replace, in his view, the living word of a teacher. His book could only help the teacher instruct the pupil to distinguish 'through experiment the true from the untrue', and 'to draw the correct consequences from observations and thus formulate the correct laws in words'. Krist expressed the traditional hope that his pupils would obtain from physical instruction not only a strong dose of inductive logic but also the ability to express their thoughts precisely in words.[11] Though it differed from the usual *Gymnasium* fare, Krist's was an empiricist compilation. In the hands of an uninspiring teacher it could easily have become a burden for the pupils. We can only guess about how Krist's book was presented to Einstein by his mathematics and physics teachers Gottlieb Effert and Joseph Ducrue.[12]

We do know, nevertheless, that Einstein was privileged from 1888 to 1891 to have Adolf Sickenberger as a mathematics teacher. Sickenberger successfully completed *Gymnasium* at Aschaffenburg and in 1869 at the age of twenty-one became certified to teach mathematics and physics. He passed through various posts before arriving at the Luitpold Gymnasium. Sickenberger rose rapidly as the private tutor of crown prince Rupprecht, and in 1881 he became a member of the Bavarian chamber of deputies. A vocal partisan of school reform, he sat as director of the Realschulmännerverein, the German association that demanded reform of formal, sterile, and impractical instruction in the secondary schools. In 1891 Sickenberger was called to become the first rector of the Luitpold–Kreis Realschule in Munich.[13]

Throughout Einstein's five and a half years at the Luitpold Gymnasium, he was taught mathematics from one or another edition of the separately published parts of Sickenberger's *Textbook of Elementary Mathematics*. When it first appeared in 1888 the book constituted a major contribution to reform pedagogy. Sickenberger based his book on twenty years of experience that in his view necessarily took precedence over 'theoretical doubts and systematic scruples'. At the same time Sickenberger made much use of the recent pedagogical literature, especially that published in the pages of Immanuel Carl Volkmar Hoffmann's *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, the leading pedagogical mathematics journal of the day. Following in the tradition of the reform movement, he sought to present everything in the simplest, most intuitive way possible. He opposed introducing scientific rigour and higher approaches in an elementary text. He emphasised that he would follow neither the synthesis of Euclidean geometry nor the so-called analytical–genetic approach. He opted for a great deal of freedom in the form of presentation because he believed that a textbook was no more than a crutch for oral instruction. The spoken word, in Sickenberger's view, could infuse life into the dead forms of the printed text. Too often, he insisted in the preface to his text, mathematics was seen and valued 'as the pure science of reason'. In reality, he continued, mathematics was also 'an essential tool for daily work'. In view of the practical dimension of mathematics Sickenberger sought most of all to present basic propositions clearly rather than to arrive at formal conciseness. Numerous examples took the place of long, complicated, and boring generalities.14 In addition to the usual rules of arithmetic Sickenberger introduced diophantine equations. To solve three linear, homogeneous, first-order equations with three unknowns he specified determinants and determinant algebra. Then he went on to quadratic equations and logarithms. In the second part of his book, Sickenberger treated plane geometry.

According to a biography of Einstein written by his step-son-in-law, Rudolf Kayser—one that the theoretical physicist described as 'duly

accurate'—when he was twelve years old Einstein fell into possession of the 'small geometry book' used in the Luitpold Gymnasium before this subject was formally presented to him.[15] Einstein corroborated Kayser's passage in autobiographical notes of 1949, when he described how at the age of twelve 'a little book dealing with Euclidean plane geometry' came into his hands 'at the beginning of a school year'. The 'lucidity and certainty' of plane geometry according to this 'holy geometry booklet' made, Einstein wrote, 'an indescribable impression on me'. Einstein saw here what he found in other texts that he enjoyed: it was 'not too particular' in logical rigour but 'made up for this by permitting the main thoughts to stand out clearly and synoptically'.[16] Upon working his way through this text, Einstein was then presented with one of the many editions of Theodor Spieker's geometry by Max Talmey, a medical student at the University of Munich who dined with the Einsteins and who was young Einstein's friend when Einstein was between the ages of ten and fifteen. We can only infer from Einstein's retrospective judgment that the first geometry book exerted an impact greater 'than that produced by Spieker's treatment, 'by the popular science expositions of Aaron Bernstein and Ludwig Büchner also given to him by Talmey, or by the texts of Heinrich Borchert Lübsen from which Einstein had by the age of fourteen taught himself differential and integral calculus.[17]

Which text constituted the 'holy geometry booklet'? In his will Einstein gave 'all his books' to his long-time secretary Helen Dukas. Present in this collection are three bearing the signature 'J Einstein': a logarithmic and trigonometric handbook, a textbook on analysis, and an introduction to infinitesimal calculus. The signature is that of Einstein's father's brother Jakob, a business partner and member of Einstein's household in Ulm and Munich. He presented the books to his nephew Albert. A fourth book in Miss Dukas's collection, which does not bear Jakob Einstein's name, is the second part of a textbook on geometry, a work of astronomer Eduard Heis's which was rewritten after his death by the Cologne schoolteacher Thomas Joseph Eschweiler.[18] Without offering reasons for his choice Banesh Hoffmann has recently identified Heis and Eschweiler's text as the geometry book that made such an impression on Einstein.[19] Yet, assuming that Kayser's unambiguous reporting is correct, it is far more likely that the geometrical part of Sickenberger's text was what Einstein referred to in his autobiographical notes. Sickenberger's exposition was published seven years after that of Heis and Eschweiler, and unlike the latter it appeared with a Munich press. Because it was used in the Luitpold Gymnasium, copies would have been readily available to Uncle Jakob or to whoever first acquainted Einstein with Euclidean geometry.

The Luitpold Gymnasium was, in the early 1890s, a good school of its type; and Einstein could have received a fine education there. His grades across the board were, in fact, excellent, in mathematics and German as well as in Latin and Greek.[20] Yet we know that the young man found it inflexible and stultifying. In an essay written shortly before his death Einstein elaborated on his passage through the 'authoritarian' Munich school. He contrasted instruction at the Luitpold Gymnasium with that which he would later receive in democratically inclined Switzerland. The Munich system, he noted, was based on 'drill, external authority, and ambition'. In the text of his essay there then follows an unusual sentence. 'True democracy', Einstein wrote, 'is no empty illusion.'[21] By these words he meant to emphasise the democratic quality of Swiss education and to indicate that it was something extremely precious. The sentence may be taken as a direct indictment of his Munich education in a second way, as well. In the course syllabus for Einstein's German class during his last year at Munich virtually an identical wording appears as the theme set for one of the pupil essays. There the sentence reads: 'The truth is no empty illusion.' Pupils were expected to comment on this thought by referring to a play written by the nineteenth-century poet, free thinker, and revolutionary Johann Ludwig Uhland: ***Ernst,*** Duke *of Swabia*.[22] Einstein's appeal to the Gymnasium essay—if, indeed, that is what he intended—is an unequivocal rejection of his years at the progressive Luitpold Gymnasium. We can only speculate on what irreparable damage might have been inflicted on the young genius had Einstein attended a more traditionally inclined secondary school.

Einstein's black picture of the Luitpold Gymnasium gives a misleading impression that the school offered reactionary and inhumane instruction. His retrospective vision was certainly coloured by intense feelings directed against a Germany that had allowed the rise of fascism. Recognition of this later antipathy must not obscure our knowledge that even as a youth Einstein was hostile to some of the dominant values of southern Germany. One suspects that at least a part of this hostility was directed against authority of any kind. In December 1894 Einstein left the Luitpold Gymnasium during the middle of his sixth year. He parted using a friendly physician's note citing the necessity of leave due to his 'nervous exhaustion', although Philipp Frank reports that before Einstein could use his fabricated medical excuse he was asked to go by one of his teachers because his disruptive presence disturbed the other pupils.[23] Einstein took with him a letter from his mathematics teacher attesting to his abilities, but he knew that without the final certificate called the Abitur he would be ineligible to become a higher-level secondary-school teacher; excluded would be higher positions in the military, postal, mining, and railway services and in all other government operations. Einstein placed himself outside the intellectual life of a society that valued culture and formal education highly. He wanted none of it.

Personal factors sped Einstein's decision to leave the Luitpold. For the preceding six months fifteen-year-old Einstein had been boarding at the home of an older woman. After some years of success the electrical manufacturing business of his father and his uncle Jakob had failed, and the extended family decided to relocate and set up shop in Milan. Einstein missed his family; letters that he received from Italy portrayed a happy life.. Young Einstein liked to ramble in the countryside around Munich, but he had never been on a long trip.[24] He could not resist the temptation to flee south.

Italy, in the middle 1890s, was collecting its breath after having plunged into the Second Industrial Revolution. At the end of 1894 the kingdom had just come through a lustrum of financial austerity caused by rapid expansion, during the 1880s, in manufacturing, railways, and public ventures. Gross fiscal malfeasance on the part of the federal cabinet coupled with a state of apprehended insurrection forced the second Crispi administration to curb industrial expansion. Liquid capital was not readily available among Italian investors. An entrepreneur from northern Europe, bringing his own assets, could reasonably hope to set up a profitable venture. In looking south to Lombardy and the Piedmont, Hermann and Jakob Einstein focused on the most dynamic part of the industrialising nation. In the two provinces lay most of Italy's chemical and manufacturing plants. A rail grid connected the interior to domestic and foreign markets and to port facilities around Genoa and Venice. Electricity ran the northern economy. Hydroelectrical generating stations dotted the southern foothills of the Alps at the same time that smaller, coal-burning stations proliferated.[25]

Albert Einstein spent most of 1895 in Pavia, whence his family had moved from Milan in the hope of finding a better business climate. The Pavian site of the company Einstein and Garrone was typical of late nineteenth-century, northern Italian industrial architecture. The brothers Hermann and Jakob established separate households in the relatively small university town.[26] There, facing few responsibilities and no pressures in a serene, foreign country, Albert definitively rejected his homeland. Even before leaving Munich Einstein had thought about giving up German allegiance, but apparently he took this resolve soon after his arrival in Italy while on a hike to visit cousins in Genoa.[27] No longer a citizen of Germany, the young man avoided the opprobrium of spending three years in the German army as an ordinary soldier.[28] He was accordingly free to travel through Germany without risking arrest as a draft dodger.

The pleasure of Einstein's new Italian home was tempered by economic realities. **As** the family business continued to flounder, Einstein was persuaded by his parents to follow the solid career of electrical engineering, a known quantity in the Einstein household. To become an engineer required more education. Without the Abitur Einstein could not hope to matriculate at a German university, or even at a German institute of technology. The one path open to him in German-speaking Europe lay in nearby Switzerland.

'In 1895 at the age of sixteen I came to Zurich from Italy', Einstein wrote a lifetime later. His purpose in going to Zurich was to study at the famous Federal Institute of Technology, or Polytechnic. He knew that he could attend the Polytechnic if he passed an entrance examination. Beyond this clear goal, however, he had few thoughts. Let us listen to Einstein again: 'I was a conscientious but unassuming young man who had acquired his meagre store of pertinent knowledge of the essentials through self-study. Eager for deeper understanding but endowed with few prerequisites and burdened with a poor memory', formal study was hard for him. 'With a feeling of well-founded uncertainty', Einstein, apparently through the intercession of his mother, was allowed to sit for the entrance examination in the engineering section even though he was two years younger than the regulations specified.[29] He did very well in mathematics and physics but failed modern languages, zoology, and botany. Einstein remembered that his examiners were 'patient and understanding'. The rector of the Polytechnic, professor of engineering mechanics Albin Herzog, advised the young man to attend the technical division of the nearby cantonal secondary school in Aarau. In a year he could graduate and in this way satisfy the entrance requirements at Zurich.[30]

We can obtain some idea of the extent to which he would have impressed Herzog from an essay written by Einstein bearing the title, 'On the Investigation of the State of the Ether in the Magnetic Field'. Einstein sent the essay to his mother's brother, Cäsar Koch, a successful broker in Brussels of whom he was fond. Accompanying the undated manuscript was an undated letter, where Einstein explained that he was soon supposed to study at the Zurich Polytechnic. This course of action carried 'significant difficulties', for Einstein was then at least two years younger than he should have been to matriculate; Einstein added that he would write to Koch about the outcome. Because the entrance age at the Polytechnic was eighteen, from Einstein's remark it is certain that his letter and the accompanying manuscript were written after March 1895 and before the early autumn of the same year, the latest date when he could have taken the Polytechnic entrance examination.[31]

Although the essay was, in his words, more a 'programme' than a real contribution, it furnished his respected uncle with evidence of Einstein's having mastered much of Heinrich Hertz's electrodynamics. Einstein began by observing how an electric current set up a potential state in the ether, a state known as the magnetic field. He emphasised how Hertz—the only authority cited—had revealed the dynamic nature of electromagnetic phenomena. The programme that he wanted to follow, but at the moment could not see how to carry out, involved determining the potential state of the ether by measuring its elastic deformation, a phenomenon that could be related directly to changes in the velocity of ether waves. Most revealing in the essay is Einstein's methodological orientation. Near the end of his five-page text he remarked: 'Quantitative research on the absolute values of the density and elastic force of the ether can, so I believe, only begin if qualitative results exist which are connected with certain ideas.' Experiment formed the basis of physical knowledge, in his view, but the physicist had to be guided in the first instance by a clear, one might interpose an intuitive, apprehension of nature's laws.

The impact of the cantonal school at Aarau

With these views of physics in mind Einstein spent most of an academic year at the Aargau cantonal school in Aarau, capital of the fertile, northern canton that encompasses the area around the confluence where the Limmat and Reuss run together to form the river Aare. Aargau was a pastoral canton, supporting tobacco-growing, silk ribbon-weaving, and straw-plaiting occupations. At the end of the nineteenth century Aargau comprised 200 000 people, slightly less than seven per cent of the population of Switzerland. Nearly all residents spoke German, and somewhat more than half were Protestant. The 1885 constitution of Aargau reflected the increasingly democratic movement that had spread across late nineteenth-century Switzerland. The legislature that sat at Aarau contained one representative for every 1100 inhabitants, and all laws that it enacted had to be approved by an obligatory referendum. In 1900 around 7000 inhabitants lived in the capital and largest town in the canton. From Aarau Zurich was about half an hour distant by train.[32]

Einstein would have found Aarau congenial because of its historically progressive attitude towards Jewish residents. When in 1798 the French invaders proclaimed a Swiss republic, they preserved tradition by denying civil rights to Jews, who were treated as foreigners. Jews remained in Switzerland at the pleasure of municipal magistrates and their councils. The largest Jewish population in early nineteenth-century Switzerland established itself at Aarau, and in 1824 the far-sighted citizens of Aargau granted Jews the privilege of citizenship and guaranteed their schools and institutions. The Christian burghers did not see fit, however, to allow Jews free movement from one town to another. That right came a generation later in federal legislation following the ferment of 1848. Berne gave citizenship to all Swiss residents without regard to religion, although it, like other federal governments of the period, did not initially presume to dictate how its constituent members, the cantons, enfranchised voters for local elections. Aargau Jews became totally emancipated in 1863. Three years later Berne guaranteed freedom of belief to all Swiss citizens.[33]

Throughout Switzerland cantonal governments supported higher secondary schools as well as the eight Swiss universities, academies, or faculties. In the rich canton of Aargau, which had no university, the secondary school formed the apex of the educational pyramid. Founded by a public subscription of 6982 Swiss francs in 1802, the school originally offered instruction in the modern languages and natural knowledge. The city of Aarau placed it in a three-storey hospital building constructed during the 1780s and into the twentieth century still used for the school chemistry laboratory and a girls' school. In 1804 the private school split into two divisions. A technical division trained pupils for the world of commerce, and a philological division emphasised classical languages. Nine years later the school was sanctioned and partially funded by the canton of Aargau. At a time when senior teachers received around 1500 francs per year, the canton promised to provide annual support of 10 000 francs, and the city of Aarau donated 22 000 francs for a physical cabinet, library, mathematical instruments, and mineralogical collection.[34]

Over the next two decades the school acquired professors of mathematics, physics and chemistry, and natural history. After receiving private donations totalling 150 000 francs, in 1826 the technical division was elevated to form a three-class *Gewerbeschule*, or trade school. During the late 1820s and early 1830s pupils in the trade school were expected to spend a quarter of their time on physical sciences and a fifth on mathematics, which included the elements of differential and integral calculus. These requirements coincided with those at the most advanced *Realschulen* in German-speaking Europe at the time. Striking to a modern observer is the regulation of 1832 that pupils in the trade school spend one afternoon per week in the school chemistry laboratory.[35] This regulation is more remarkable when it is recalled that the first teaching laboratory in chemistry at a German university was that founded in 1832 by Johann Wolfgang Döbereiner at Leipzig.[36] Because of its consistently high standards and secure financial base, pupils and their parents expressed unwavering confidence in the school. From 1830 to 1900 attendance generally ran at over one hundred regular pupils.

With the establishment in 1855 of the federal Polytechnic in Zurich, the two highest classes of the trade school took on the character of a preparatory school. In 1860 the Polytechnic recognised the leaving certificate of the trade school at Aarau, along with that of schools at Frauenfeld, Berne, and Geneva, as an entry card that released its holder from the obligation to sit for an otherwise mandatory qualifying examination.[37] Because the school year at Aarau ended in April and in Zurich began in the autumn, the trade-school course of study was extended by a half-year to three-and-a-half years' duration. Among the first class entering the Polytechnic under the new arrangement was Friedrich Mühlberg, later Einstein's geology professor at Aarau. By the middle 1880s most Swiss higher secondary schools enjoyed the privilege of sending graduates directly to the Polytechnic, although the federal government seems to have held the Aarau institution in especially high regard.[38] When in 1885 it affirmed the *modus vivendi* of the Aarau–Polytechnic entente, the educational council of the federal Interior Department clarified how in principle it reserved the right to elect examiners for the leaving examination, or *Maturitätsprüfung*. In practice this reservation meant that several professors from the Polytechnic assisted in administering the examination at the trade school.[39]

Einstein found at Aarau, then, an unusual secondary school. It had three divisions. The oldest was a *Gymnasium*, offering instruction comparable to that achieved a decade later in the four highest years of progressive German *Reformgymnasien*. There Latin was mandatory and Greek optional. The second division was the trade school, where Einstein registered. It offered a curriculum like that in the highest years of a German *Oberrealschule*; because Greek and Latin were elective subjects, Einstein was freed from the grip of dead languages which had dominated his education at Munich. The third division was a two-year commercial school, just created that year for both girls and boys. As an annexe the cantonal school had a five-year *Progymnasium*, channelling pupils into the *Gymnasium* or, if they wished, into the trade or commercial school.

In his centenary retrospective of 1902 Einstein's physics teacher August Tuchschmid described Aarau in the 1890s as a *Reformschule* where there was free movement among *Gymnasium*, *Realgymnasium*, and *Oberrealschule* tracks. Tuchschmid was proud that at Aarau gymnasiasts could prepare for the Polytechnic and that technicians could take Latin in anticipation of medical or dental careers. In this multiplicity was unity, he offered. Healthy education lay neither in greater decentralisation and specialisation nor in stereotyped uniformity, 'but rather in an organisation by which individual talents—those generally determining the future direction of youth—are taken into account'.[40] This philosophy was precisely that outlined during the middle 1890s by the most radical German school reformers.

Einstein's fellow pupils were a homogeneous lot. In 1895/96 the cantonal school taught 163 regular pupils and 5 *Hospitanten*, pupils not in a diploma programme. The *Gymnasium* hosted 62 pupils, the trade school 65, and the commercial school 36. Of the 163 regular pupils only six were Jewish, two registered in the trade school. The six Jews constituted three per cent of the Jewish population of Aarau. Although many pupils came from Swiss cantons beyond Aargau, only five were foreigners, four of these in the trade school. At Aarau Einstein was seen by his fellow pupils as an unusual young man. He distinguished himself further by his stated career plans. In the autumn of 1896 eight of the nine pupils graduating from the *Gymnasium* sought to study law or medicine at a university; the ninth indicated chemistry as his choice, presumably at a university. Among Einstein's eight fellow graduates of the trade school most wanted to pursue a liberal profession at the Polytechnic; one went there to become a secondary-school teacher. Only Einstein was unconcerned with a profession. No longer did he want to become an engineer. Instead, he went to the Polytechnic as a student of 'mathematics and physics'.[41]

At Aarau Einstein lived close to one of the teachers in the cantonal school, and the experience furnished him with the measure of a teacher's independence. He boarded at the home of the professor of history Jost Winteler, at the professor's insistence.[42] Winteler was a distinguished philologist and an avid, amateur naturalist. Einstein became a member of the family, a brother to the young Winteler brood. He maintained ties with the family throughout his life. When in 1899 a Winteler daughter sought guidance about a marriage proposal, she wrote to Einstein. Following his urging, Einstein's younger sister Maja studied for three years at Aarau. She found a warm welcome at the Winteler household, for in 1910 she married Jost's son Paul. It seems that just at the time that Einstein's parents sent him out on a path leading to a profession, the young man discovered for himself a surrogate family.[43] Here I cannot dwell on Einstein's personal relationship with the Wintelers. Rather, I consider in some detail Einstein's science education at Aarau.

Einstein's passage was to some extent an unorthodox one. He arrived late in October 1895 for the third quarter of the term and sat in the third class of the trade school. There he was a year or more the junior of his classmates. The grades that he received for industry and mastery of his course material were at first uniformly poor. For the final quarter, ending in April 1896, he showed little improvement except in Italian, where he earned five from a possible six points. In arithmetic and algebra he received the lowest grade, one out of six. At his own initiative or at that of his teachers Einstein took private instruction in French, chemistry, and natural history. After Einstein registered for the final half-year at Aarau, his performance in the exact sciences improved dramatically. During the first quarter of the new year—May and June of 1896—he received a six in .arithmetic and algebra, and in physics a six for industry and a five for mastery of the material.[44] Einstein.remained at Aarau through early July, when he took part in the annual youth festival, and into early September.[45] On the fifth of the latter month the rector recorded Einstein's final course grades along with those of the other pupils in the fourth, two-quarter class at the trade school. The grades reveal that Einstein had made only some progress towards filling in the gaps revealed by his abortive Polytechnic entrance examination. Among his class he was the best in algebra (a grade of 6), geometry (6), physics (5/6), and in German (4/5); he was among the best in history (5) and natural history (5); he was middling in descriptive geometry *(5)* and chemistry (5); he was among the worst in freehand drawing (4) and technical drawing (4); he was absolutely the worst pupil in French (3/4) and geography (4).[46]

One of the principal attractions at Aarau during the summer of 1896 was a newly completed school building. When Einstein arrived late in 1895 he would have found himself in the midst of great excitement over its construction, for the physical plant—still in the eighteenth-century hospital—had remained unchanged for generations. The rooms were low, narrow, and dark, and the benches and desks carved with an intaglio of students' initials.[47] During the 1880s and the early 1890s school directors along with the Aargau Scientific Society urged the city and canton to provide more space, especially for instruction in natural sciences. The governments claimed poverty and refused. Finally, the Cultural Society of Aarau offered 100 000 francs for a new building provided that construction began at the latest during July 1894. The Society's gift was 250 000 francs short of the sum calculated by the city for erecting a new building—not counting purchase of the requisite land. Just at this time the city was able to acquire, with the help of a few local citizens, the estate of a politician for only 150 000 francs. The city reserved an existing structure for a trade museum and gave the eastern portion of land along with 50 000 francs for the cantonal school. The canton was then pressurised to come through with the remaining money.[48]

The new building opened in 1896. It was designed by Karl Moser, the architect who also drafted plans for the new quarters of the University of Zurich. The Aargau structure was a thoroughly modern, four-storey building capped with a small clock tower. By far its favoured laboratory was that housing the 'physical cabinet'. In place were a two-horsepower alternating-current motor with accessories, a lathe, a milling machine, a joiner's bench, a work bench, a grindstone, various tools, a gear transmission, a small dynamo with accessories for physical experiments, an accumulator battery with switchboard and Edelmann galvanometer, and a projector.[49] These provisions were nothing short of extraordinary for a combined *Gymnasium–Realanstalt* of the period. In very few other places across German-speaking Europe would a pupil have been in such close proximity to so fine a collection of physical apparatus.[50] It was, in the words of one former pupil, a physical institute that a small university could envy.[51] Einstein was among the first young men turned loose in this magnificent instrument collection.

The structure was inaugurated **on** a beautiful, sunny, spring day, 26 April 1896.[52] Einstein assembled with the rest of the pupils and teachers to witness and take part in the public events, one of which was awarding an honorary doctorate from the University of Zurich to the old Aarau philologist and historian Jakob Hunziker. If he were attentive, Einstein would have noted the reactionary Heidelberg *Gymnasium* professor Gustav Uhlig among the invited dignitaries; Uhlig, who opposed Mach over the issue of instruction in the classics, had earlier taught Latin and Greek at Aarau. At the opening ceremonies Einstein would have heard the address of the rector, his physics teacher August Tuchschmid. The school was to be a strong bulwark for higher principles in life, Tuchschmid emphasised. As the struggle for existence intensified in the world outside, so the school had to become increasingly stronger in pointing youth 'towards the love of the true, the courage to be right, to awaken and nurse the sense for the noble and good, and to struggle against egoism'. The cantonal school educated the individual to know the value of general learning, enabling him to lead a rewarding existence. At the same time, Tuchschmid continued, the school had always placed great value on useful learning.53 This formula succeeded well. Einstein was not the only pupil of foreign origin at Aarau around the turn of the century who would later win a Nobel Prize. Also a graduate of the school was Paul Karrer, winner of the chemistry laurels in 1937.

Whereas throughout the nineteenth century most rectors at Aarau served for only a few years at a time, Tuchschmid held this position from 1889 throughout the First World War. His was the guiding spirit behind the remarkable expansion of the school. Upon arriving as physics teacher in 1882 he transformed his discipline from the most poorly served science to the most respected. Tuchschmid was, quite simply, one of Switzerland's most gifted native sons. He had come through the Thurgau cantonal school and, after a period of private study, passed the ***Abitur*** in 1873. From the beginning he wanted to become a secondary-schoolteacher of mathematics and sciences, and before he reached the age of twenty he was installed in such a school in the Bernese Jura, teaching in both German and French. After three years he moved to Zurich, to study at the Polytechnic and to teach at the Zurich Craft School as well. Upon receiving a diploma from the Polytechnic in 1880 he became assistant to the same physics professor who later taught Einstein, Heinrich F Weber. He remained an assistant for two years, until he landed a position teaching physics at Aarau. When he arrived in 1882 he was shocked at the small, dirty, and inadequate facilities reserved for physics. Fourteen years later he had one of the finest physical laboratories in Switzerland?

In his inspired teaching Tuchschmid aimed for depth rather than breadth. He insisted before his classes that the concepts of the infinitesimal and the differential were keys to all advanced physics and mathematics. A former pupil of his recalled that Tuchschmid's introduction to electric current was a shining piece of experimental physics which equalled university lectures. In all his courses Tuchschmid strove for clarity, intelligibility, and precision. He omitted those parts of physics not based on rigorous scientific foundations or those areas giving play to theoretical speculations. He never engaged in chatter on physical subjects and spared no effort to convey fundamental laws and phenomena. As might be expected from his choice of physics apparatus in 1896, Tuchschmid's principal interest at

Aarau lay in the burgeoning field of electrotechnology. In 1888 the city of Aarau chose him as a member of its advisory commission on electrification. In part as a reward for this service, in 1918 the city made him an honorary citizen, the first teacher at the cantonal school to obtain such a distinction?

Supporting Tuchschmid's pedagogical activity was the geologist Friedrich Mühlberg, also a teacher of Einstein's. At the school's centenary in 1902 Mühlberg observed how the harmony of nature was captured in the organisation of his school: just as the living organism depended on mutual interdependence among its organs, so in human affairs the highest aim was for the individual to serve his fellow man. If Tuchschmid was an educator–technologist, Mühlberg was an educator–scientist. By 1900 Mühlberg was widely recognised as a leader in the school movement. At the same time, scientific research dominated his restless activity. He wrote a definitive flora of Aargau, a treatise on glacial geology in the canton, and descriptions of the geology of the Jura mountains. By 1895 Mühlberg had well over 1300 pages in print, an output comparing quite favourably with that of university professors of the time. He attended around 450 sessions of the Aargau Natural History Society, delivering 90 addresses of which the majority communicated original research in geology. It was natural that he should have edited the Society's ***Mitteilungen.*** In 1888, at the age of forty-eight, he received an honorary doctorate from the University of Basle.[56]

When Einstein took Mühlberg's class in geology and physical geography, he was taught by one of Switzerland's greatest geologists, a kindly and distinguished man of fifty-five. The young man made an impression on his teacher. Mühlberg, who died in 1915, remembered Einstein as a clever pupil.[57] Einstein came to know Mühlberg in a personal way, too. It was the custom at Aarau to organise excursions every year for the pupils. In June 1896 twenty pupils in the highest classes at the trade school and the ***Gymnasium*** enjoyed a three-day walking tour led by Mühlberg. The group travelled to the eastern Swiss canton of Appenzell. There Mühlberg directed a climb of the 2700-metre peak Säntis. He and his fellow tourists crossed snow fields despite inhospitable weather.[58] Potential tragedy dogged the expedition. Einstein suffered a bad fall and was saved by the quick action of one of his fellow pupils.[59]

Einstein was fortunate in a third science teacher at Aarau. More traditionally minded than either Mühlberg or Tuchschmid was **his** mathematics teacher, Heinrich Ganter. A native of Baden, Ganter attended the ***höhere Bürgerschule*** in Freiburg im Breisgau and then continued at a private school in Frankfurt am Main. After working in industry for several years he served as a lieutenant in the Franco–Prussian War. He then entered his father's mill. The business failed, and he was forced to follow another calling. He chose teaching. By 1877 Ganter found himself an instructor at the *Realgymnasium* in Karlsruhe. His advancement limited by an incomplete education, thirty-year-old Ganter decided to return to school. He spent three years studying mathematics at the universities of Berlin and Zurich. While a student at Zurich he worked as an assistant for higher mathematics at the Polytechnic and at the local Gymnasium. He received a doctorate from Zurich in 1884. Two years later he arrived at Aarau.[60]

Ganter was a good mathematician, but also a leading mathematics teacher. He told a colleague at Aarau that he was not really talented enough for a career in higher mathematics, but he could teach, 'something that many speculative gentlemen cannot do'.[61] Ganter, together with his friend the Polytechnic professor Ferdinand Rudio, wrote a school textbook on analytical geometry.[62] It was a text that Tuchschmid used when teaching mathematics in the Gymnasium at Aarau. The book surely reflected Ganter's instruction in mathematics. A particular achievement of Ganter's lay in presenting rigorous concepts to pupils in the trade school, those who, in the opinion of Tuchschmid, 'generally see mathematics quite differently from gymnasiasts'.[63] A former pupil of Ganter's wrote that he 'never treated us demeaningly, but taught us as men'. From the 1890s Ganter's favourite author was Friedrich Nietzsche. A colleague saw Nietzsche's thoughts reflected in Ganter's deep commitment to mould in his pupils a beautiful and strong character. His former pupil the Polytechnic professor Ernst Meissner remembered Ganter as a teacher who, far from transmitting mere practical information to prepare a pupil for a career, educated the heart and character and truly civilised his charges. If all teachers were like Ganter, Meissner insisted, there would be no need for school reform.[64]

Later in life Einstein remembered his congenial year at Aarau. In 1952 he wrote to Guido Fischer of the Aarau Art Collection that the school remained for him 'the most satisfying image of this kind of cultural institution'+ What struck Einstein most forcefully, he went on, was the freedom of instruction and learning that he had experienced there. It was a freedom traditional at Aarau and, despite superficial resemblances, one not emerging from the process of school reform in northern Germany. The atmosphere created by Einstein's science and mathematics teachers would have appealed to a great many pupils, but in Einstein it struck a particularly resonant chord. Einstein would have found in Mühlberg a practising scientist who unpretentiously and arduously dedicated himself to understanding natural events. Ganter would have presented algebra and geometry as clear and vibrant domains of learning. From Tuchschmid Einstein would have received a feeling for the practical applications of physical laws and a prejudice to avoid encyclopaedic compilation in favour of seeking fundamental theories of nature. These values were among the ones that Einstein carried to his later career.

Einstein's final examinations at Aarau

How the Aarau experience affected Einstein is revealed in the record of his trade-school leaving examinations, which he wrote on 18, 19 and 20 September and defended orally on 30 September. In registering for the examinations he followed all the usual procedures, since he submitted a short vita to the canton's instructional commission.[66] By the time that he applied Einstein knew which of his teachers would formulate the examination questions. Among other examiners Ganter was responsible for algebra and geometry, Tuchschmid for physics, Mühlberg for natural history, and the old philologist Hunziker for French. Other written and oral examinations were set in German language and literature and in chemistry. History and descriptive geometry carried only oral interrogations.[67] Conforming to the federal policy in such matters, the Polytechnic sent two observers to Aarau, presumably to attend the oral defences; one of the two, in the autumn of 1896, was Einstein's sympathetic adviser Albin Herzog.[68] Einstein did not disappoint the Zurich engineer. He received straight sixes—the highest possible grades—in his mathematics and physics examinations, a record setting him clearly above the heads of his fellows.[69]

The three mathematical problems that Einstein answered in his examination papers were similar to those set during this period in Prussian *Realgymnasien*. They were easier than questions asked, for example, in Hamburg *Oberrealschulen*.[70] One of the geometrical problems was in trigonometry, where Einstein calculated the angles of a triangle after having been given its sides. He used logarithmic tables to obtain numerical results. His written answer was evidently a fair copy, for in marginal comments on the paper Ganter noted an error in transcription (*Abschreiben*).[71] A second problem in analytical geometry involved solving a second-order, inhomogeneous equation. This time Einstein was less sure how to proceed, and his answer is complete with false starts. At one point he began to solve a quadratic equation correctly, only to cross out his calculations. Something farther along must have caught his eye.[72]

The final geometrical problem inscribed a circle in a triangle and gave the line segments connecting the median to each of the apices of the triangle; their ratio was 1:2:3. Einstein had to find the radius of the circle in terms of the smallest line segment. This he did by remembering a general equality for the three angles of a triangle, α, β, γ:

$$\sin^2(\alpha/2) + \sin^2(\beta/2) + \sin^2(\gamma/2) + 2\sin(\alpha/2)\sin(\beta/2)\sin(\gamma/2) = 1.$$

With substitutions this identity reduced to a cubic equation. Einstein found the three roots by invoking the general formula for solving such an equation and by then evaluating the square-root discriminant in the formula with the help of a trigonometric substitution and logarithmic tables. Though it depended on instant recall of complicated mathematical formulae, Einstein's solution was the very opposite of one based on brute-force calculations. He was careful to arrive at numerical values only after having made general observations on, among other things, the rationality of the roots of the cubic equation and on the geometrical requirements that a solution would have to satisfy.[73]

A third examination where Einstein demonstrated outstanding talent was physics. In his four-page essay he discussed the theory and design of the tangent compass and the galvanometer. It was a question that could have been posed at this time on the *Maturitätsprüfungen* of few other schools in German-speaking Europe; few had facilities comparable with those in Tuchschmid's laboratory for providing pupils with first-hand experience in making electrical measurements. Einstein knew exactly how to proceed. He began by noting that any electrical current is based on the observable effects of lines of force. Magnetic force is indirectly proportional to the distance from the conductor and directly proportional to the standard current strength in the conductor. Then he described the construction of the tangent compass. In this instrument, a magnetised needle is freely suspended from its midpoint by a non-conducting thread mounted at the apex inside a vertical ring of conducting wire; when an electric current passes through the ring, the needle is deflected and orients itself at an angle to the axis of the coil, an angle depending on the strength of the current. In his paper Einstein gave the tangent of the angle as equal to the magnetic force of the current (a quantity directly proportional to the current) divided by the component of force in this direction coming from the earth's magnetism. He wrote $\tan \phi = Ik/H$, where k was a constant, I the current strength, and H the contribution from terrestrial magnetism. One could then compare, Einstein observed, two currents I and I', because since k and H remained constant, the observed ratio $\tan \phi/\tan \phi'$ was precisely I/I'. He continued by emphasising that a galvanometer worked like a tangent compass, except that a coil replaced the needle. Einstein concluded by describing the uses of both instruments.[74]

In his physics essay, unlike the manuscript that he had earlier sent to his uncle Cäsar Koch, Einstein almost pointedly made no use of the concept of an electromagnetic field. Such a concept did not then belong to the accepted body of physical theory in German-speaking Europe, and Einstein no doubt sought not to antagonise his examiner Tuchschmid by mentioning it. In writing his answer Einstein emphasised how general physical phenomena could be harnessed to design measuring instruments. He stressed the basic theory of the apparatus and concluded by describing practical applications. Einstein also included some technical details. He mentioned how light rays reflected from a mirror mounted on the suspending filament or magnet could provide an accurate display of displacement, and in passing he referred to the way that a small telescope could be used

to observe the needle set into oscillation when current was introduced into the ring. To accompany his answer Einstein carefully and unnecessarily included a top and side view of the tangent compass, making use of his mechanical-drawing instruments to inscribe circles and draw straight lines; perhaps Einstein felt that, because of his poor performance during the year in mechanical drawing, it would be a good idea to demonstrate some facility in this area. Significantly, Einstein did not spend time discussing the mechanical parts of the apparatuses. Only implicitly did he introduce the notion of a restoring force for the needle in the tangent compass or the coil in a d'Arsonval-type galvanometer. He also avoided discussing the mathematical procedure for obtaining precise results by reducing multiple data readings with the help of the method of least squares. For him these were, and remained, far from fundamental questions.

In chemistry and natural history Einstein wrote good responses to simple questions. He set up a chemical equation for combining sulphuric acid with table salt, and he correctly calculated quantities of reagents and products using 'atomic weights'. Upon giving the desired solution Einstein set down a series of remarks on the origin and uses of the compounds that he considered.[75] In natural history Einstein wrote on the effect of previous glaciation on geological formations, the theme set by Mühlberg. In composing his answer Einstein would have been able to draw on first-hand experience gathered during Mühlberg's geological excursions.[76]

One is struck by the clear and unpretentious quality of Einstein's examination essays, strong points present as well in his later published writings. The essays begin with a statement of a generalisation of physical phenomena in terms of theoretical concepts, such as electromagnetic lines of force. Then the essays consider the problem at hand by omitting distracting albeit interesting detail in favour of the fundamentals. In view of his obvious literary talents it is not surprising that the seventeen-year-old Einstein was the best in his class in German. For his German essay Einstein wrote a summary of Goethe's play about the sixteenth-century Swabian military adventurer Götz von Berlichingen, a text in which Goethe recast the rough and unprincipled warrior into a moral and sympathetic thinker. In his summary, Einstein characterised the personalities and motivations of Goethe's characters much as a secondary-school essayist today. would discuss the actors in Shakespeare's ***Julius Caesar.*** Einstein's direct and complete treatment earned him a grade of 5.[77]

If Einstein was at his best in German, he was incontestably the worst pupil in French. In April **1896,** he passed into the fourth year at the trade school over the protest of his French instructor, a protest that continued into the summer.[78] His French essay for the *Maturitätsprüfung* was, in the view of the philologist Hunziker, only marginally acceptable. Amid misspellings and grammatical errors, nevertheless, Einstein elaborated a disarmingly cogent picture. The theme concerned his 'future plans'. It was, Einstein wrote at the beginning, natural for a 'serious young man' to be as precise as possible about his goals. If he were fortunate enough to succeed in his examinations, he would go to the Zurich Polytechnic. There he would spend four years 'studying mathematics and physics'. As a career, Einstein imagined becoming a teacher ***(professeur)***, specialising in the 'theoretical part' of the natural sciences. He was suited for this career, Einstein continued, because of his 'individual disposition towards abstract thoughts and mathematics', and his 'lack of imagination [*phantaisie*] and practical talents'. By the latter phrase Einstein probably meant to reject not only a career in the creative arts but also one in the world of human affairs and commerce, the very career that his father was then having such difficulty sustaining. Einstein concluded his short essay by noting that he looked forward very much to the 'independence' that characterised the learned profession.[79] With the models of Tuchschmid, Ganter, and Mühlberg before him, and living at the home of the congenial Jost Winteler, Einstein knew exactly what he wanted.

Uncongenial mathematics at Zurich

Several weeks after having acquitted himself with distinction in the most important of his leaving examinations at one of Switzerland's finest secondary schools, Einstein arrived at the Zurich Polytechnic. During the next four years he was a student there. Much has been written on Einstein at the Polytechnic, and for this reason I shall not provide details of Einstein's student years.[80] By 1896 the Polytechnic had achieved a reputation throughout German-speaking Europe as a leading centre of higher learning in mathematics and physics.[81] As the only federal educational institution in Switzerland, it benefited early in the 1890s when the country swelled with new revenues resulting from rising duties on imported goods. In 1896 it had an excellent physical laboratory, completed in 1890, and with it a first-rate teaching staff.

How did Einstein, with his formidable talents and with the special encouragement that he had received at Aarau, respond to the physicists and mathematicians at the Polytechnic? His love for the physical laboratory of Heinrich Friedrich Weber is well documented.[82] It was a passion that had been fanned, at the very least, under Tuchschmid's direction. Yet Einstein's achievements as a physicist lie not in experiment but in the realm of mathematised theory. For this reason I shall concentrate here not on the emotions that Einstein may have brought to experimental physics but rather on what he found in the mathematics lectures of Hermann Minkowski, his most distinguished teacher. Minkowski is remembered as the discoverer in 1907 of four-dimensional space–time, a formal representation for Einstein's special theory of relativity. He

proposed this theory as the first step in formulating a more general approach to energy and matter which, before his death in 1909, he was unable to elaborate. He believed that pure mathematics might successfully resolve the problems confronting physics.

Minkowski's optimistic faith in the explanatory power of pure mathematics was quite different from the view on these matters of his pupil Albert Einstein. Einstein has recalled that after a short time at the Polytechnic he came to believe that mathematics presented many separate domains, any one of which might easily absorb the energies of a lifetime. In physics, though not in mathematics, Einstein felt that he could intuitively sense those problems that were most important.[83] In his early published work in physics Einstein followed his nose, so to speak, in dealing with physical problems. Maurice Solovine, Einstein's close friend during the period just after he graduated from the Polytechnic, remembered that 'Einstein, who handled the mathematical instrument with incomparable dexterity, often spoke against the abusive use of mathematics in physics. Physics, he would say, is essentially a concrete and intuitive science. Mathematics serves only as a means of expressing the laws that govern phenomena.'[84] The young researcher Einstein viewed mathematics with suspicion. 'I do not believe in mathematics', Einstein is reported to have affirmed before 1910.[85]

Although Einstein's antipathy to mathematics is reflected in the modest level of mathematical exposition characterising his early scientific papers, during his student years he expressed a strong interest in mathematics. Einstein's matriculation record reveals that he registered for nearly as many mathematics courses as did his classmates Marcel Grossmann and Louis Kollros, both of whom were preparing for careers as mathematicians.[86] Particularly striking in Einstein's school record is the presence of courses by the newly appointed professor of mathematics Hermann Minkowski. Einstein took nine courses from Minkowski, more than from anyone else. To a greater extent than any other source, the content of Minkowski's courses reveals the sort of mathematics that Einstein found uncongenial.

Einstein's decision to take three algebraic courses with Minkowski was unusual for a physical scientist of the period, even for one expressing strong interest in mathematics. Traditional mathematical preparation for physicists consisted of analysis, geometry, and mechanics, each defining a large domain in applied mathematics. Together with arithmetic, algebra constituted one of the most important fields of pure mathematics at the end of the nineteenth century. Algebra applied elementary arithmetical operations to abstract mathematical entities. Complex analysis, particularly the characterisation of solutions to polynomial equations, was the central focus of much algebraic research. In the section on algebra in the first volume of the encyclopaedia of mathematical sciences edited by

Wilhelm Franz Meyer, only three of the nine articles did not have applications to the solution of equations as their principal aim.[87] In 1896 Eugen Netto devoted the first volume of his widely used text on algebra entirely to the roots of algebraic equations.[88] Group theory and the theory of numbers were seen as two areas closely related to the main concerns of algebra. Although in 1895 the Strasbourg mathematician Heinrich Weber devoted the first volume of his vast survey of algebra to complex analysis, he emphasised that group theory was essential for the development of algebra, and number theory offered the best application for algebraic modes of thought.[89] To a young physics student who sought powerful mathematical syntheses that also held the promise of being solutions to practical problems, algebra might have seemed attractive. It would have been particularly appealing when taught by Minkowski, a professor who sought to relate the theory of numbers to geometry—a field in applied mathematics that required physical, spatial intuition.

Registering for courses carrying no examinations, as was the case at the Polytechnic, implies little about the familiarity that a student might acquire with the course material. If the instructor were unprepared or gave otherwise unexciting lectures, fewer than a third of his students might be expected to attend class. Such was the case with Minkowski, notarious among the students for his difficult and poorly delivered lectures.[90] Einstein later wrote that although he successfully received a diploma in 1900, he did not regularly attend most of his courses. As an aid in memorising the many unrelated facts required for his examinations, in March and April of 1900 Einstein pored over lecture notes meticulously compiled by his friend Grossmann.[91] One record of Einstein's examinations, a note written by Minkowski, indicates that in complex analysis Einstein received a grade of 5½ out of a possible 6, the same grade as that obtained by Grossmann and Jakob Ehrat but half a point lower than Kollros' perfect score.[92] (The registers at the Polytechnic are more obliging. There Einstein is recorded as receiving a 10 in complex analysis, a 10 in the physics laboratory, a 5 in astronomy, and an 18 for a special project. These grades added together in a complicated way to produce an average of 4.9, ranking below Kollros' 5.45, Grossmann's 5.24, and Ehrat's 5.14. Einstein's grades did not augur a brilliant career.[93])

According to the exposition in Grossmann's notebooks, the algebraic courses of Minkowski were one-semester introductions.[94] Just enough of the underlying theory was presented to arrive at some basic results. Then special topics were considered. No allusion to physics was made, and little emphasis was given to the general solution of polynomial equations by geometrically inspired techniques. Someone expecting a unified theoretical treatment that led to applications, probably what Einstein was waiting for, would have been disappointed.

Where one might have expected most attention to the introduction of

geometrical intuition was in Minkowski's course on the geometry of numbers. The format of the course was much less analytical and abstract than was the book that he had published in 1896, which defined the field for his mathematician colleagues.[95] To judge from Grossmann's and Minkowski's notes[96], the course was an earlier version of the lectures that Minkowski later gave at Göttingen, lectures recorded by A Axer and published in 1907 as a text on diophantine approximations.[97] Presuming no familiarity with number theory, algebra, or geometry, Minkowski's Zurich lectures introduced one new topic each session and frequently illustrated general points with specific examples. Minkowski's introduction would have appealed to a pure mathematician, but a physicist like young Einstein might have been less impressed. Minkowski began by restating Leopold Kronecker's mathematical programme: 'Everything leads back to the whole numbers.' Rational and irrational numbers and all algebraic quantities, in his view, were created from the whole numbers. 'In the last instance all of mathematics is a science of the whole numbers, and there are only true laws where the whole numbers appear.' The study of the whole numbers belonged to the theory of numbers. One could say, in Minkowski's view, that pure number theory treated how whole numbers related to arbitrary magnitudes, specifically real numbers. Minkowski emphasised that he would treat applied number theory, especially how geometrical intuition could help find mathematical laws more easily.[98]

In the first section of his lectures Minkowski introduced the number lattice, a central concept around which his lectures would turn.[99] The number lattice in two dimensions was defined by an orthogonal coordinate system (later the orthogonal restriction was relaxed), interest focusing only on the planar coordinates composed of two integers. The power of Minkowski's number lattice was illustrated by a basic theorem relating the area of convex bodies in a lattice defined by nonorthogonal axes to the number of lattice points contained within the perimeter of the convex bodies. Minkowski's theoretical development of the many-dimensional number lattice proved fruitful in finding integer or rational solutions to diophantine equations. In the light of the critical role played by frames of reference in Einstein's special theory of relativity, it is noteworthy that in his lectures Minkowski appealed to coordinate transformations in deriving several of his results. At one point he introduced multiplication of two-dimensional transformation matrices; Grossman found the calculation so unfamiliar that in his notes he was at first not careful to maintain the order of multiplication.

In both Grossmann's and Minkowski's records of the course there is little advanced machinery for other areas of mathematics, and Minkowski's exposition was strikingly less elegant than treatments by other mathematicians who also wrote on number theory from a non-arithmetical point of view, for example those of Felix Klein.[100] Einstein's experience with

Minkowski's exposition of pure mathematics would have reinforced his opinion that it was of little use for the physicist. Although he gave a passing mention to group theory in his first paper on special relativity in 1905, this subject and the geometry of many-dimensional vector spaces did not become a regular part of Einstein's mathematical arsenal until around 1911, when he realised that they were indispensable for constructing general relativity.

If Einstein's disappointment with the limited utility of pure mathematics for understanding physical laws was an expected outcome, his rejection of Minkowski's applied mathematics requires further explanation. For a physics student around 1900, one of the most important subjects in applied mathematics was theoretical mechanics. Generally taught by mathematicians, mechanics established the physical foundations for more advanced work in thermodynamics and electromagnetism, as well as for that in the neighbouring fields of geophysics and astrophysics. Einstein registered for the two principal introductions to mechanics that were given in the teachers' section at the Polytechnic. One, taught by his former patron Albin Herzog, began with the concept of a continuous distribution of matter and investigated problems in elasticity, strength of materials, and hydrostatics.[101] Herzog assumed only that the students had a working knowledge of elementary differential and integral calculus; he introduced all the physics required, appealing frequently to empirical laws. Complementing Herzog's treatment, Minkowski gave a course on the mechanics of rigid-body motion. He carefully formulated the laws of transformations between coordinate systems and proceeded to rigid-body kinematics.[102]

For his lectures Minkowski drew heavily on the recently published first volume of Felix Klein and Arnold Sommerfeld's major statement on mechanics, the *Theory of Tops*.[103] Klein and Sommerfeld's text was a revisionist treatment based on intuitive, non-formalist mathematics. It drew strength from Edward John Routh's classic text on dynamics, the sixth English edition of which was then being translated into German with Klein's and Sommerfeld's help.[104] In their own text, Klein and Sommerfeld emphasised the importance of arriving at a geometrical, intuitive grasp of mechanics: 'We will only require that our knowledge of mechanics is not based on formalism, but rather the reverse, that the analytical formulation appears as the final consequence of a basic understanding of the mechanical relationships themselves.'[105]

While he accepted the pedagogical charm of Klein and Sommerfeld's text, Minkowski was unable or unwilling to infuse his course with an appeal to intuitive, physical examples and motivations. His lectures were dull, formal recitations. In his opening remarks Minkowski paraphrased the third sentence of Klein and Sommerfeld's first chapter on the kinematics of tops: 'One speaks of kinematics', Minkowski began, 'when one investigates only the geometrical possibilities of motion, where only the concepts of space and time are considered; one speaks of kinetics or dynamics when one treats the mechanical possibilities of motion, where the concepts mass and force are taken up and the effect of the laws in nature, the influence on masses by forces, is taken into consideration.'[106] What was meant by a rigid body? Minkowski offered no physical insight. All motion was referred to three mutually perpendicular spatial axes. A mass distribution over a three-dimensional region constituted a body. To describe the orientation of the body, Minkowski continued, it was necessary to affix three internal perpendicular axes to the body. When the body was subjected to motion, the first time derivatives of a unit vector on each of these axes became the velocity components of the net speed. Next Minkowski asked his listeners to consider a body of infinitely many mass points, with two points having specified axes. A body was said to be rigid when for any two points there was no net change in motion over time. Unlike Minkowski, at this point in their text Klein and Sommerfeld provided a long, intuitive, 'geometrical' treatment of kinematics, presenting no equations of any sort.

Minkowski was kinder to his old physics teacher Heinrich Hertz than were Klein and Sommerfeld. Previously, when he was ***Privatdozent*** at the University of Bonn, Minkowski had been an enthusiastic follower of Hertz's approach to physics.[107] At the time Hertz was working towards eliminating the concept of force from physics. In 1898, although he repeated Klein and Sommerfeld's strictures against Hertz's approach, Minkowski did not take them to heart. He merely cautioned that mechanics ultimately rested on experience: 'Of course we must make use of the equations for the motion of a body to arrive at true natural laws. Naturally we cannot advance here by mere speculation. Our formulae should convey motion as it is actually seen in nature, and for this we have to calculate with the precise facts of practical experience.'[108] Hertz's electrodynamics had made a profound impression on Einstein in 1895. One wonders how receptive Einstein would have been several years later to Minkowski's unconvincing criticisms of Hertz's excessive abstraction.

Minkowski's remarks on practical experience are the more surprising because they constitute one of the very few passages in his lectures where he appealed to empirical reality. He generally omitted providing an intuitive discussion of force, work, impulse, and momentum, all of which were emphasised by Klein and Sommerfeld. Neither did he make use of many physical concepts. One rare case occurred in a discussion of impulse and momentum, where he mentioned that integrating the impulse imparted by gas molecules to the walls of a container would give the gas pressure; there he inappropriately referred his listeners to Klein and Sommerfeld's text. Minkowski's notes represent only a preliminary version of his lectures on mechaniés, but they indicate his general approach to physics. Natural laws, and mechanics in particular, had to be based in the last instance on observation. In practice, however, the development of mathematical theories of reality depended hardly at all on experimentally verifiable propositions. Minkowski's method of classical mechanics assumed space, time, coordinate systems, and the measurement process as operational concepts. Physical results were obtained by invoking new and independent mechanical idealisations and deducing mathematical relations among all these quantities.

Though Minkowski emphasised abstract development to a greater extent than did Klein and Sommerfeld, there is little reason to suppose that Einstein would have found the parent text more congenial. Not opposed in principle to mathematics, Einstein sought to understand fundamental problems of physics. In his early published work he investigated the meaning of the statistical mechanics of molecules, not the classical mechanics of points, bodies, and continua. Even more than statistical mechanics, electromagnetism was a subject in which the young Einstein expressed a strong interest. Though included in a mechanical picture of the world, it was not part of Minkowski's mechanics curriculum. Statistical mechanics and electrodynamics made use of many physical propositions that, by 1900, were not easily reduced to standard mechanical explanations. A course treating these problems might have appealed to Einstein. Minkowski's course on mathematical methods, based on assumptions concerning mechanical reality but not on investigations of the validity of these assumptions, served to reinforce Einstein's view that mathematics would be of little use to a physicist.

Einstein's classmate Louis Kollros has recalled that, in their last semester at the Polytechnic, Minkowski gave a lecture on capillarity. It consisted of Minkowski's earliest thoughts about a review of the subject which Felix Klein had asked him to write for the new encyclopaedia of mathematical sciences. After the lecture Einstein noted 'enthusiastically and melancholically', Kollros continued, '"that is the first lecture on mathematical physics which we have heard at the Poly".'[109] Einstein may, indeed, have seen something in Minkowski's lecture, for several months later he finished his first scientific paper on the consequences of the phenomenon of capillarity.[110] To judge from Einstein's remarks, however, Minkowski's lecture on capillarity was a welcome relief from the usual way that he presented mechanics as a series of mathematical formalisms.

Conclusion

For the young Einstein mathematical formalism was only a tool in the service of that which he and others called 'physical reasoning'. Einstein believed that the fundamental physical laws were arrived at by close comparison with experimental phenomena. These are precisely the values

that Einstein's secondary-school physics and mathematics teachers—Sickenberger, Tuchschmid, and Ganter—transmitted to him. The values were not emphasised in Minkowski's dry, formal procedures, and Einstein found his lectures uninspiring.

In view of Einstein's extreme independence of character one may ask whether the affinities between his thought and that of several of his early teachers indicate causal relationships or contingent ones. In presenting some circumstances of Einstein's youth I have described the textbooks and lectures that came to his hands as well as the styles and thought of some of the instructors who stood before him. How these men and materials struck Einstein is what appears most difficult for a historian to answer. I do not seek to claim exclusivity for these texts and teachers. Many other events happening outside school produced a strong impression on Einstein. Future scholars with access to new archival material may revise our understanding of the origins of Einstein's revolutionary ideas of 1905. They will have to take into account, nonetheless, that in his secondary schooling Einstein was taught a great deal of mathematics and physics at the hands of sympathetic teachers. By 1900 he knew about many branches of mathematical thought, and his rejection of advanced mathematics was based on familiarity rather than on ignorance.

The formative educational experiences of a scientific genius like Einstein may well lie not in advanced training but in events at the time of secondary schooling. His passage through Aarau was a significant one for Einstein, quite possibly far more influential than his years of education in Zurich. Einstein had left Munich as an inexperienced, precocious schoolboy of fifteen. He came to the Polytechnic in 1896 as an unusually mature and clear-minded young man with many of his basic sensibilities already fixed. He readily absorbed the scientific values of his teachers at the cantonal school, but he was considerably less impressed with a different—one might say an opposing—approach to natural knowledge in the lectures of Minkowski at Zurich.

The evidence presented here supports the view that Einstein's experience with secondary education infused him with a prejudice for approaching physical reality as a whole subject and for viewing mathematics as incapable in itself of providing a formulation for nature's laws. Other mathematicians and physicists educated in German-speaking Europe soon after Einstein—Max Born and Hermann Weyl, to mention just two—did adopt the belief elaborated by Minkowski and his mathematician colleagues that pure mathematics would be able to resolve long-standing problems in physical theory. The advocates of pure mathematics won many supporters among twentieth-century physicists. Though he came to have great respect for mathematics, Einstein could not accept mathematical manipulation as a substitute for physical reasoning. Einstein's suspicion of mathematics surely contributed to his rejection of quantum mechanics during the 1920s, when many physicists sought to articulate an indeterminist epistemology to justify the success of formal methods.[111] From this point of view Einstein was not the first of a new wave of twentieth-century physicists but rather the last great thinker in a nineteenth-century tradition that included Hendrik Antoon Lorentz, Heinrich Hertz, and Max Planck. None in this company shrank from mathematical calculations, but each invoked mathematics as nothing more than an aid to physical reasoning.

Einstein maintained his views on the appropriate place of mathematics in physical theory until the end of his life. A close physician friend of Einstein's in the 1920s clarified how Einstein remained suspicious of mathematical formalism. Einstein remarked on a problem that he had come up against: 'I'm afraid I'm wrong again. I can't put my theory into words. I can only formulate it mathematically, and that's suspicious.'[112] Leopold Infeld, the most perceptive writer among all the physicists who worked with Einstein, has relayed how in the 1930s Einstein believed that mathematical formalism was something to be mastered and then transcended before truth could be fathomed. Infeld reports Einstein's observation: 'God does not care about our mathematical difficulties. He integrates empirically.'[113]

Notes and references

1 The standard story is given in, among other sources, Friedrich Paulsen's ***German Education: Past and Present*** translated by T Lorenz (London 1908) pp 205 ff. A recent re-evaluation of events unrelated to mathematics and science instruction during the period 1880–95 is James C Albisetti's ***Kaiser, Classicists, and Moderns: Secondary School Reform in Imperial Germany*** (dissertation, Yale University 1976). Albisetti makes much use of August Messer's ***Die Reformbewegung auf dem Gebiete des preussischen Gymnasialwesens von 1882 bis 1901*** (Leipzig 1901). Polemical accounts are provided in Felix Klein's ***Vorträge über den mathematischen Unterricht an den höheren Schulen, I: Von der Organisation des mathematischen Unterrichts,*** recorded by Rudolf Schimmack (Leipzig 1907), and Schimmack's Göttingen ***Habilitationsschrift, Die Entwicklung der mathematischen Unterrichts-Reform in Deutschland*** (Leipzig 1911). Of great value is Herbert Göllnitz's ***Beiträge zur Geschichte des physikalisch–chemischen Unterrichts an den höheren Schulen Deutschlands seit der Mitte des 19. Jahrhunderts*** (Leipzig 1920) pp 68–163. A general picture based on statistics taken from published sources is painted in Fritz K Ringer's ***Education and Society in Modern Europe*** (Bloomington 1979) pp 32–112.

2 Ernst Mach, ***Der relative Bildungswert der philologischen und der mathematisch–naturwissenschaftlichen Unterrichtsfächer der höheren Schulen*** (Leipzig 1886) pp 11, 19.

3 Quotations from Paul Cauer's sympathetic review of Uhlig's ***Heidelberger Erklärung,*** appearing in the ***Berliner philologische Wochenschrift*** **9** (1889) 541–5.

4 'Education in Central Europe', ***Report of the Commissioner of Education for the Year 1896/97*** **1** (Washington, DC 1898) 177, 299. Information was drawn from official German sources.

5 ***75 Jahre Luitpold Gymnasium München*** (Munich 1967) p 6.

6 The Luitpold Gymnasium that Einstein attended was completely destroyed during the Second World War. A replacement of sorts is found in the present Albert-Einstein Gymnasium. The present Luitpold Gymnasium was the former Luitpold-Kreis Realschule, founded in 1891 and elevated to ***Gymnasium*** status well into the twentieth century. ***Ibid*** p 11.

7 K Luitpold-Gymnasium in München, ***Jahresbericht für das Studienjahr 1888189*** (Munich 1889) p 15; ***Jahresbericht für 1894195*** (Munich 1895) p 23.

8 Abraham A Fraenkel estimated that around 1900 there were 8700 Jews in Munich. Fraenkel, ***Lebenskreise: Aus den Erinnerungen eines jüdischen Mathematikers*** (Stuttgart 1967) p 36.

9 ***Ibid*** pp 60–1.

10 ***Jahresbericht 1894195*** (ref 7) p 12.

11 Quotations from the prefatory passages reprinted, in Josef Krist's ***Anfangsgründe der Naturlehre für die unteren Klassen der Mittelschulen*** (Vienna 1877). Krist long wanted to provide first-hand experience with apparatus for his pupils. An extensive exposition of pedagogical apparatus is given by Josef Krist in 'Regnaults Apparate zur Untersuchung der Compressibilität, Ausdehnung, spezifischen Wärme, u.s.w. in der ihnen von S. Silbermann gegebenen Einrichtung' ***Repertorium für physikalische Technik*** **2** (1867) 65–105.

12 ***Jahresbericht 1894195*** (ref 7) p 3.

13 ***75 Jahre*** (ref 5) pp 10–11.

14 Adolf Sickenberger, ***Leitfaden der elementaren Mathematik*** (Munich 1894). Citations from the preface to the first edition of 1888, reprinted in the 1894 edition. The first edition contained a part on algebra and a part on 'Planimetrie'. The second and third editions constituted a revision of the algebraic part of the first edition.

15 Anton Reiser [pseudonym of Rudolf Kayser], ***Albert Einstein: A Biographical Portrait*** (London 1931) pp 36–7.

16 A Einstein, 'Autobiographical Notes', in ***Albert Einstein – Philosopher-Scientist*** ed P A Schilpp (La Salle, Ill 1949) pp 9–11, 15.

17 Haig Gordon Garbedian, ***Albert Einstein: Maker of Universes*** (New York 1939) p 13. Max Talmey, ***The Relativity Theory Simplified and the Formative Period of Its Inventor*** (New York 1932) pp 161–4. Anton Reiser [Kayser], ***Einstein*** (ref 15) pp 37–9, mentions Spieker's book as different from a first, unnamed geometry book. Cf Ronald W Clark's ***Einstein: The Life and Times*** (London 1973) p 31. My identification of the geometry book is supported by discussion in Maja Winteler-Einstein's 'Albert Einstein: Beitrag für sein Lebensbild' (Florence 1924) pp 13–14. The text is a 39-page typescript available in the Einstein Archives, Princeton.

18 According to correspondence from Miss Dukas, the four books are, respectively: Georg Freiherr von Vega, ***Logarithmisch–Trigonometrisches Handbuch***

ed C Bremiker (Berlin 1869); H B Liibsen, ***Ausführliches Lehrbuch der Analysis zum Selbstunterricht mit Rücksicht auf die Zwecke des praktischen Lebens*** (Leipzig 1868); H **B** Liibsen, ***Einleitung in die Infinitesimal-Rechnung (Differential und Integral-Rechnung) zum Selbstunterricht . . .*** (Leipzig 1869); Heis and Eschweiler, ***Lehrbuch der Geometrie zum Gebrauch an höheren Lehranstalten* 2*: Stereometrie*** (Cologne 1881). Miss Dukas declined to let me examine the books, which until her recent death were located at the Institute for Advanced Study, Princeton.

19 Banesh Hoffmann, with the assistance of Helen Dukas, ***Albert Einstein: Creator and Rebel*** (New York 1972) pp 22–3.

20 In 1929 a former teacher of Einstein's looked through his records at the Luitpold Gymnasium. He found that Einstein consistently received very good grades in all. subjects, including Latin and Greek. H Wieleitner, 'Albert Einstein am Münchner Luitpold-Gymnasium', ***Münchener Neuste Nachrichten*** 14 March 1929.

21 A Einstein, 'Autobiographische Skizze', in ***Helle Zeit—Dunkle Zeit*** ed Carl Seelig (Zurich 1956) pp 9–10.

22 ***Jahresbericht 1894195*** (ref 7) p 11.

23 Philipp Frank, ***Einstein: His Life and Times*** transl George Rosen, ed Shuichi Kusaka (New York 1947) p 17.

24 A Reiser [Kayser], ***Einstein*** (ref 15) pp 37, 41-4.

25 Brunella Malvicino, Pier Enzo Peirano ***et al,*** 'La Bassa'Valle di Susa industriale, 1870–1918: Lineamenti storici per l'analisi di un territorio', in ***Patrimonio edilizio essistente: un passato e un futuro*** (Turin 1980) pp 43–99. The story of Swiss manufacturers in northern Italy is recounted in Hans Rudolf Schmid's ***Die Familie Abegg von Zurich und ihre Unternehmungen*** (Zurich 1972). See also Pietro Regoliosi's 'Elettrotecnica', in the collection ***Storia di Milano* 16*: Principio di secolo (1901–1915)*** (Milan 1962) pp 879–902.

26 Elena Sanesi, 'Einstein e Pavia', ***Settanta* 3** no 29 (October 1972) 33-41.

27 H G Garbedian, ***Einstein*** (ref 17) p 19. On 26 January 1896, Einstein's father, on his behalf, petitioned the state of Württemburg to discontinue Einstein's citizenship. Zentrales Staatsarchiv Merseburg, Rep. 76, Vc, Sekt. 2, Tit. 23, Litt. F, Nr. 2, p. 118: Württemburg Interior Ministry to Prussian Ministry for Learning, Art, and Public Instruction, 27 July 1923. Compare Clark's ***Einstein*** (ref 17) p 41.

28 Paul Forman and Paul Hanle, ***Einstein: A Centenary Exhibition*** (Washington, DC 1979) p 14. If he had successfully completed the lower classes at his ***Gymnasium—as*** his father had done at the Stuttgart Realschule—Einstein would have qualified to spend only one year in the army as a so-called 'volunteer'. M Winteler-Einstein, 'Albert Einstein' (ref 17) pp 5, 16.

29 This is how.I interpret the remark of John Plesch in ***Janos: The Story of a Doctor*** transl Edward Fitzgerald (London 1947) p 219.

30 A Einstein, 'Autobiographische Skizze' (ref 21) p 9. Carl Seelig claims that a friend of the Einsteins then living in Zurich, Gustav Meier, advised Albert to go to Aarau. Seelig, ***Albert Einstein: Leben und Werk eines Genies unserer Zeit*** (Zurich 1960) p 6.

31 Jagdish Mehra, 'Albert Einsteins erste wissenschaftliche Arbeit', ***Physikalische Blätter* 27** (1971) 386–91. A photocopy of the essay may be found at Princeton University, Firestone Library, Einstein microfilms, IA9.

32 Information taken from ***Encyclopaedia Britannica,*** eleventh edition (1911). The discussion in this and the following section departs from the remarks by Gerald Holton on Aarau in 'On Trying to Understand Scientific Genius', ***Thematic Origins of Scientific Thought*** (Cambridge, Mass 1973) pp 372–3.

33 Augusta Wedler-Steinberg, ***Geschichte der Juden in der Schweiz vom 16. Jahrhundert bis nach der Emanzipation*** ed Florence Guggenheim-Griinberg, 2 vols (Goldach, Switzerland 1966–1970) **1** 87, 100, 165, 168, 223; **2** 46–7, 112, 133.

34 T Müller-Wolfer, ***Die Aargauische Kantonsschule in den vergangenen 150 Jahren*** (Aarau 1952) pp 14–63.

35 August Tuchschmid, 'Die Entwicklung der Aargauischen Kantonsschule von 1802 bis 1902', in ***Jubiläum der Aargauischen Kantonsschule am 6. Januar 1902: Vorträge und Reden*** (Aarau 1902) pp 13–63, on p 27.

36 Christa Jungnickel, 'Teaching and Research in the Physical Sciences and Mathematics in Saxony, 1820–1850', ***Historical Studies in the Physical Sciences* 10** (1979) 3-47, on p 26.

37 Wilhelm Oechsli, ***Geschichte der Gründung des Eidg. Polytechnikums mit einer Uebersicht seiner Entwicklung 1855–1905.*** Vol I of ***Eidg. Polytechnikum Festschrift*** (Frauenfeld 1905) p 281.

38 Albert Barth, ***Die Reform der höheren Schulen in der Schweiz*** (Basle 1919) pp 15, 21.

39 A Tuchschmid, 'Entwicklung' (ref 35) pp 47–54.

40 ***Ibid*** pp 61–2.

41 ***Programm der Aargauischen Kantonsschule: Schuljahr 1896197*** (Aarau 1897) pp 12–16.

42 Max Flückiger, ***Albert Einstein in Bern*** (Berne 1974) p 26.

43 Carl Seelig, ***Albert Einstein: A Documentary Biography*** transl M Savill (London 1956) pp 19–21. Flückiger ***(Einstein*** [ref 42] pp 24–6) cites relevant manuscript material on this subject.

44 From Einstein's ***Personalakte*** at the cantonal school. I am grateful to Professor Dr Hans Troxler-Keller, Bibliothek der Aargauischen Kantonsschule, for having kindly made this material available to me.

45 ***Programm der Aarg. Kantonsschule*** (ref 41) p 17; Seelig, ***Einstein*** (ref 43) p 17.

46 Staatsarchiv Aarau, Departementsakten Erziehungsdirektion, Mappe Ks/ Kantonsschule, 1896, 'Abgangsnoten für die Schüler der 4. techn. Klasse (Herbst 1896)', dated 5 September 1896. Paul Forman kindly provided me with a copy of this and all other material cited subsequently as belonging in the Staatsarchiv Aarau.

47 Gustav Uhlig, in ***Zur Erinnerung an die Einweihung des neuen Kantonsschulgebäudes in Aarau am 26. April 1896*** (Aarau 1897) p 35; Friedrich Mühlberg in ***Jubiläum 1902*** (ref 35) pp 74–5.

48 T Müller-Wolfer, ***Die Aargauische Kantonsschule*** (ref 34) pp 120–3.

49 ***Programm der Aarg. Kantonsschule*** (ref 41) p 56. The natural history museum of the Aargau Scientific Society was also installed in the new building, but its collection seems not to have been unusual for the time. Friedrich Mühlberg,

'Das Aargauische Naturhistorische Museum', in *Festschrift* ***zur*** *Feier des hundertjährigen Bestandes der Aargauischen Naturforschenden Gesellschaft* ed F Mühlberg (Aarau 1911) pp cxi–cxxviii, cxix.

50 On physical laboratories in German secondary schools at this time see Paul Bode's *Die Reform des mathematischen und naturwissenschaftlichen Unterrichts an höheren Schulen in der Gegenwart* [dissertation, University of Leipzig 19111; E Schneider, *Reformbestrebungen im Bereich des Physikunterrichts der höheren Schule* (dissertation, University of Kiel 1953) pp 35–87. A carbon copy of Schneider's dissertation is located in the Universitätsbibliothek, Kiel. Jürgen Sievert *(Zur Geschichte des Physikunterrichts* (dissertation, University of Bonn 1967) p 218) lists only eleven German secondary schools where laboratory instruction was provided in 1896. See also Göllnitz, *Beiträge* (ref 1) pp 140–8.

51 A Hartmann, in *Jubiläum Prof. Dr. August Tuchschmid* [Aarau 19221 p ***5.***

52 ***Zur*** *Erinnerung* (ref 47) p ***5.***

53 *Zbid* pp 22–6.

54 A Hartmann, in *Jubiläum Prof. Dr. August Tuchschmid* (ref 51) pp 4–9.

55 *Zbid.*

56 *Prof. Dr. Friedrich Mühlberg: Separatabdruck aus dem Jahresbericht der Aargauischen Kantonsschule* ***für*** *1915116* (Aarau 1916).

57 C Seelig, *Einstein* (ref 43) p 19.

58 According to Tuchschmid's report on the excursion in *Programm der Aarg. Kantonsschule* (ref 41) pp 19–21.

59 C Seelig, *Einstein* (ref 43) p 13.

60 *Prof. Dr. Heinrich Ganter: Separatabdruck aus dem Jahresbericht der Aargauischen Kantonsschule* ***für*** *1915116* (Aarau 1916).

61 Cited in *ibid* p **8.**

62 Heinrich Ganter and Ferdinand Rudio, *Die Elemente der analytischen Geometrie der Ebene,* ***zum*** *Gebrauch an höheren Lehranstalten sowie* ***zum*** *Selbststudium:* **1** *Die analytische Geometrie der Ebene* (2nd ed, Leipzig 1894).

63 *Prof. Dr. Heinrich Ganter* (ref 60) p 7.

64 *Zbid.*

65 C Seelig, *Einstein* (ref 43) p 21.

66 Staatsarchiv Aarau. Einstein to Titl. Erziehungsdirektion des Kantons Aargau, 7 September 1896.

67 Aargauische Kantonsschule, *Programm* ***für*** *die Maturitätsprüfung an der technischen Abteilung im Herbst 1896* [Aarau 18961.

68 *Programm der Aarg. Kantonsschule* (ref 41) p 16.

69 Staatsarchiv Aarau, 'Maturitätsprüfung der Gewerbeschule im Herbst 1896'.

70 Surveys of questions asked on *Maturitätsprüfungen* in Prussia, Hamburg, and the rest of Germany during this time are given in Walter Lietzmann, *Die Organisation des mathematischen Unterrichts an den höheren Knabenschulen in Preussen* (Leipzig 1910) pp 42–93; John William Albert Young, *The Teaching of Mathematics in the Higher Schools of Prussia* (London 1900) pp 89–92; Francisque Marotte, *L'Enseignement des sciences mathématiques et physiques dans l'enseignement secondaire des garçons en Allemagne* (Paris 1905) pp 53–60. Marotte provides samples of questions in mathematics, physics, and chemistry.

71 Staatsarchiv Aarau. Examination paper headed: 'Aufgabe **I.** Albert Einstein'.

72 *Ibid.*

73 Staatsarchiv Aarau. Examination paper headed: 'Albert Einstein'.

74 *Ibid.* 'Tangentenbussole und Galvanometer. Albert Einstein',.

75 *Ibid.* 'Aufgabe. Albert Einstein'.

76 *Ibid.* 'Nachweis der früheren Vergletscherung **. . .** Albert Einstein'.

77 *Ibid.* 'Inhaltsangabe von Goethes Götz von Berlichingen. Albert Einstein'.

78 Bibliothek der Aargauischen Kantonsschule, Einstein's *Personalakte.*

79 Staatsarchiv Aarau. 'Mes projets d'avenir. Albert Einstein'.

80 The latest and best discussion is given in Russell McCormmach's 'Editor's Foreword', *Hist. Stud. Phys. Sci.* **7** (1976) xi–xxxv.

81 Jules T Muheim, 'Die ETH und ihre Physiker und Mathematiker. Eine Chronologie der Periode 1855–1955', *Neue Zürcher Zeitung: Beilage Forschung* ***und*** *Technik* 9 April 1975.

82 R McCormmach, 'Editor's Foreword' (ref 80).

83 A Einstein, 'Autobiographical Notes' (ref 16) p 17.

84 Maurice Solovine, 'Introduction', in Albert Einstein's *Lettres* à *Maurice Solovine* (Paris 1956) p vii.

85 Cited in McCormmach's 'Editor's Foreword' (ref 80) p xiv.

86 Copies of Einstein's, Grossmann's, and Kollros' matriculation records are available in the **Wissenschaftshistorischen** Sammlung of the ETH-Bibliothek, Zurich.

87 W F Meyer, *Arithmetik und Algebra* (Leipzig 1898–1904), vol **I** of *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen.*

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90 Hermann Minkowski to David Hilbert, 31 January 1897. Lili Rüdenburg and Hans Zassenhaus, eds, *Hermann Minkowski: Briefe an David Hilbert* (Berlin 1973) p 94, Minkowski commented that attendance in his course on analytical mechanics was declining drastically, even among the cleverest students. This he attributed to his demanding lecture style. Hermann Weyl contrasted Hilbert's 'fairly fluent' speech to Minkowski's 'hesitant' speech in 'David Hilbert and His Mathematical Work', *Bulletin of the American Mathematical Society* **50** (1944) 654.

91 A Einstein, 'Autobiographische Skizze' (ref 21) p **11.**

92 Minkowski papers, box V, folder 11, Niels Bohr Library, American Institute of Physics, New York (henceforth AIP).

93 Einstein Archives, box 38, Princeton University Library.

94 Grossmann's notebooks for five of Minkowski's courses survive in the library of the Federal Institute of Technology. Einstein registered for the same courses: 'Geometrie der Zahlen, WS 1897/98', vols **I** and **II;** 'Potentialtheorie, **SS** 1898', 2 vols; 'Funktionentheorie, **SS** 1898', **1** vol; 'Elliptische Funktionen, WS 1898/99', **1** vol; 'Algebra, **SS** 1899', **1** vol, Handschriftenabteilung, Bibliothek der ETH, Zurich, Hs 421:27–Hs 421:31.

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96 AIP. Minkowski papers, box V, folder 13. 'Wintersemester 1897198. Geometrie der Zahlen'.

97 H Minkowski, *Diophantische Approximationen* (Leipzig 1907). Minkowski

noted in the foreword that the text was based on lectures given at the University of Göttingen during the winter semester 1903/04.
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100 Felix Klein, 'Ausgewählte Kapitel der Zahlentheorie, **I**: Vorlesung, gehalten im Wintersemester 1895/96', mimeographed (Göttingen 1896) p 1.
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106 AIP. 'Wintersemester 1898/99. Mechanik'.
107 H Minkowski to D Hilbert, 22 December 1890, in ***Briefe*** (ref 90) pp 39-40.
108 AIP. 'Wintersemester 1898/99. Mechanik'.
109 **L** Kollros, 'Erinnerungen eines Kommilitonen', in Seelig, ***Helle Zeit*** (ref 21) pp 21, 23.
110 A Einstein, 'Folgerungen aus den **Kapillaritätserscheinungen'**, ***Annalen der Physik*** **4** (1901) 513–23, received 16 December 1900. By Einstein's account this paper must have been completed in the year of 'intellectual depression' that followed his having successfully passed the final examinations at the Polytechnic. Einstein, 'Autobiographische Skizze' (ref 21) p 12.
111 The standard reference is Paul Forman's 'Weimar Culture, Causality, and the Quantum Theory, 1918–1927: Adaptation by German Physicists and Mathematicians to a Hostile Intellectual Environment', ***Hist. Stud. Phys. Sci.*** **3** (1971) 1–115. Forman has elaborated his views in 'The Reception of an Acausal Quantum Mechanics in Germany and Britain', in ***The Reception of Unconventional Science*** ed Seymour H Mauskopf (Boulder, Colo 1978) pp 11–50.
112 J Plesch, ***Janos*** (ref 29) p 208.
113 Leopold Infeld, ***Quest: The Evolution of a Scientist*** (Garden City, NY 1941) p 279.