ANCOVA

Psy 420

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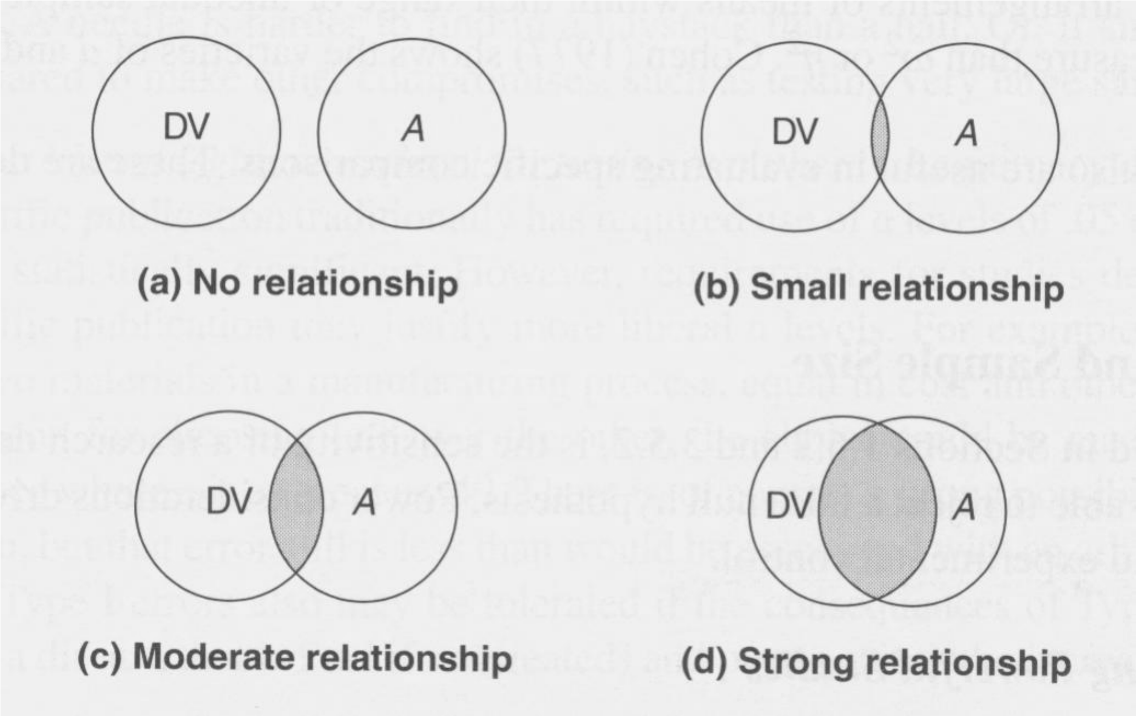
What is ANCOVA?

Analysis of covariance

- an extension of ANOVA in which main effects and interactions are assessed on DV scores after the DV has been adjusted for by the DV’s relationship with one or more Covariates (CVs)

Remember Effect Size?

- For basic ANOVA effect size is shown below



- What would it look like with a covariate?

Basic requirements

1 DV (I, R) – continuous

1 IV (N, O) – discrete

1 CV (I, R) – continuous

Basic requirements

- Minimum number of CVs that are uncorrelated with each other (Why would this be?)
- You want a lot of adjustment with minimum loss of degrees of freedom
- The change in sums of squares needs to greater than a change associated with a single degree of freedom lost for the CV

Basic requirements

- CVs should also be uncorrelated with the IVs (e.g. the CV should be collected before treatment is given) in order to avoid diminishing the relationship between the IV(s) and DV.
  - How would this affect the analysis?

Covariate

- A covariate is a variable that is related to the DV, which you can't manipulate, but you want to account for it's relationship with the DV

Applications

- Three major applications
  - Increase test sensitivity (main effects and interactions) by using the CV(s) to account for more of the error variance therefore making the error term smaller

Applications

- Adjust DV scores to what they would be if everyone scored the same on the CV(s)
  - This second application is used often in non-experimental situations where subjects cannot be randomly assigned

Applications

- Subjects cannot be made equal through random assignment so CVs are used to adjust scores and make subjects more similar than without the CV

- This second approach is often used as a way to improve on poor research designs.

- This should be seen as simple descriptive model building with no causality

Applications

- Realize that using CVs can adjust DV scores and show a larger effect or the CV can eliminate the effect

Applications

- The third application is addressed in 524 through MANOVA, but is the adjustment of a DV for other DVs taken as CVs.

Assumptions

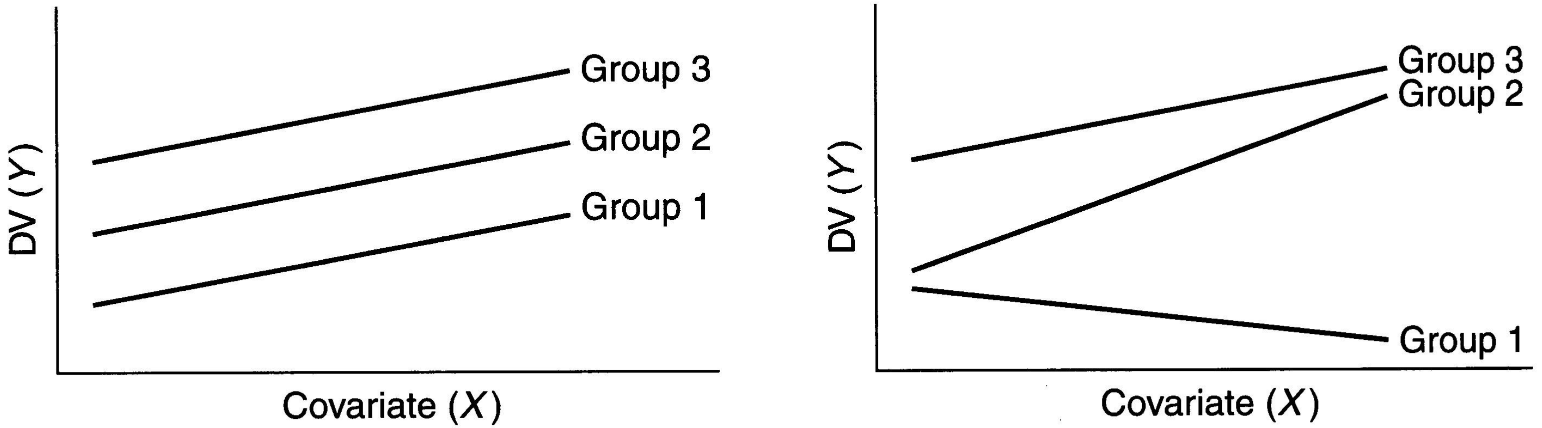
- Normality of sampling distributions of the DV and each CV
- Absence of outliers – on the DV and each CV
- Independence of errors
- Homogeneity of Variance
- Linearity – there needs to be a linear relationship between each CV and the DV and each pair of CVs

Assumptions

- Absence of Multicollinearity –
  - Multicollinearity is the presence of high correlations between the CVs.
  - If there are more than one CV and they are highly correlated they will cancel each other out of the equations
  - How would this work?
  - If the correlations nears 1, this is known as singularity
  - One of the CVs should be removed

Assumptions

- Homogeneity of Regression
  - The relationship between each CV and the DV should be the same for each level of the IV



(a) Homogeneity of regression (slopes) **(b) Heterogeneity of regression (slopes)**

Assumptions

- Reliability of Covariates
  - Since the covariates are used in a linear prediction of the DV no error is assessed or removed from the CV in the way it is for the DV
  - So it is assumed that the CVs are measured without any error

Fundamental Equations

- The variance for the DV is partitioned in the same way

$$\sum_i \sum_j \left(Y_{ij} - GM\right)^2 = n\sum_j \left(\bar{Y}_j - GM\right)^2 + \sum_i \sum_j \left(Y_{ij} - \bar{Y}_j\right)^2$$

$$SS_{total} = SS_A + SS_{S/A}$$

Fundamental Equations

- Two more partitions are required for ANCOVA, one for the CV

$$\sum_i \sum_j \left(X_{ij} - GM \right)^2 = n \sum_j \left(\bar{X}_j - GM_{(x)} \right)^2 + \sum_i \sum_j \left(X_{ij} - \bar{X}_j \right)^2$$

$$SS_{T(x)} = SS_{A(x)} + SS_{S/A(x)}$$

- And one for the CV-DV relationship

$$\sum_i \sum_j \left[\left(Y_{ij} - GM_{(y)} \right) \left(X_{ij} - GM_{(x)} \right) \right] = n \sum_j \left[\left(\bar{Y}_j - GM_{(y)} \right) \left(\bar{X}_j - GM_{(x)} \right) \right] + \sum_i \sum_j \left[\left(Y_{ij} - \bar{Y}_j \right) \left(X_{ij} - \bar{X}_j \right) \right]$$

$$SP_T = SP_A + SP_{S/A}$$

Fundamental Equations

- The partitions for the CV and the CV/DV relationship are used to adjust the partitions for the DV

$$SS'_A = SS_A - \left[\frac{(SP_T)^2}{SS_{T(x)}} - \frac{(SP_{S/A})^2}{SS_{S/A(x)}} \right]$$

$$SS'_{S/A} = SS_{S/A} - \frac{(SP_{S/A})^2}{SS_{S/A(x)}}$$

Fundamental Equations

- In other words, the adjustment of any subject's score $(Y - Y')$ is found by subtracting from the unadjusted deviation score $(Y - GM_y)$ the individuals deviation on the CV $(X - GM_x)$ weighted by the regression coefficient
- $(Y - Y') = (Y - GM_y) - B_{y.x}(X - GM_x)$

Fundamental Equations

- Degrees of Freedom
  - For each CV you are calculating a regression equation so you lose a df for each CV
  - $df'_T = N - 1 - \#CVs$
  - $df'_A =$ are the same
  - $df'_{S/A} = a(n - 1) - \#CVs = an - a - \#CVs$

Analysis

- Sums of squares for the DV are the same
- Sums of squares for the CV:

$$SS_{A(x)} = \frac{\sum\left(\sum a_{j(x)}\right)^2}{n} - \frac{T_x^2}{an}$$

$$SS_{S/A(x)} = \sum X^2 - \frac{\sum\left(\sum a_{j(x)}\right)^2}{n}$$

$$SS_{T(x)} = \sum X^2 - \frac{T_x^2}{an}$$

$$SP_{S/A} = \sum XY - \frac{\sum\left(\sum A_{j(x)}\right)\left(\sum A_{j(y)}\right)}{n}$$

$$SP_{Total} = \sum XY - \frac{T_x T_y}{an}$$

Analysis – Example

a_1 = Parking Lot View		a_2 = Ocean View		a_3 = Ski Slope View	
X	Y	X	Y	X	Y
2	1	2	3	3	6
1	1	4	4	4	8
4	3	4	3	2	6
4	5	4	5	5	7
1	2	3	4	3	5
12	12	17	19	17	32

$T_x = 46$ $SX^2 = 162$ $SXY = 215$

$T_y = 63$ $SY^2 = 325$

Analysis – Example

$$SS_A = \frac{12^2 + 19^2 + 32^2}{5} - \frac{63^2}{15} = 41.2$$

$$SS_{A(x)} = \frac{12^2 + 17^2 + 17^2}{5} - \frac{46^2}{15} = 3.3$$

$$SS_{S/A} = 325 - \frac{12^2 + 19^2 + 32^2}{5} = 19.2$$

$$SS_{S/A(x)} = 162 - \frac{12^2 + 17^2 + 17^2}{5} = 17.6$$

$$SS_{T(x)} = 162 - \frac{46^2}{15} = 20.9$$

$$SP_{S/A} = 215 - \frac{(12)(12) + (17)(19) + (17)(32)}{5} = 12.8$$

$$SP_T = 215 - \frac{(46)(63)}{15} = 21.8$$

Analysis – Example

$$SS'_A = 41.2 - \left[\frac{(21.8)^2}{20.9} - \frac{(12.8)^2}{17.6} \right] = 27.8$$

$$SS'_{S/A} = 19.2 - \left[\frac{(12.8)^2}{17.6} \right] = 9.9$$

Source	Adjusted SS	Adjusted df	Adjusted MS	F
A	27.8	2	13.9	15.44
S/A	9.9	11	0.9	

Adjusted means

- When using ANCOVA the means for each group get adjusted by the CV-DV relationship.
- If the Covariate has a significant relationship with the DV than any comparisons are made on the adjusted means.

Adjusted means

$$B_{S/A} = \frac{SP_{S/A}}{SS_{S/A(x)}}$$

$$\bar{Y}'_{A_i} = \bar{Y}_{A_i} - B_{S/A}(\bar{X}_{A_i} - \bar{X}_T)$$

$$B_{S/A} = \frac{12.8}{17.6} = .72727$$

$$\bar{Y}'_{A_1} = 2.40 - .72727(2.40 - 3.07) = 2.89$$

$$\bar{Y}'_{A_2} = 3.80 - .72727(3.40 - 3.07) = 3.56$$

$$\bar{Y}'_{A_3} = 6.40 - .72727(3.40 - 3.07) = 6.16$$

Adjusted Means

	ANOVA Raw Score Means			ANCOVA Adjusted Means		
	a_1	a_2	a_3	a_1	a_2	a_3
CV pattern	Low	Medium	High			
1st DV pattern	Low	Medium	High	Medium	Medium	Medium
2nd DV pattern	Medium	Medium	Medium	High	Medium	Low
3rd DV pattern	High	Medium	Low	Higher	Medium	Lower

Specific Comparisons

- For BG analyses Fcomp is used
- Comparisons are done on adjusted means

$$F_{A_{comp}} = \frac{SS'_{A_{comp}}}{MS'_{error(A_{comp})}}$$

$$SS'_{A_{comp}} = \frac{n(\sum w_j \bar{Y}'_j)^2}{\sum w_j^2}$$

$$MS'_{error(A_{comp})} = MS'_{S/A}\left(1 + \frac{SS_{A_{comp(x)}}}{SS_{S/A(x)}}\right)$$

$$SS'_{A_{comp(x)}} = \frac{n(\sum w_j \bar{X}'_j)^2}{\sum w_j^2}$$

Specific Comparisons

- Small sample example

$$SS'_{A_{comp1}} = \frac{5\left[(-2)(2.89)+(1)(3.56)+(1)(6.16)\right]^2}{(-2)^2+(1)^2+(1)^2} = 12.94$$

$$SS'_{A_{comp1(x)}} = \frac{5\left[(-2)(2.4)+(1)(3.4)+(1)(3.4)\right]^2}{(-2)^2+(1)^2+(1)^2} = 3.33$$

$$MS'_{error(A_{comp1})} = .90\left(1+\frac{3.33}{17.60}\right) = 1.07$$

$$F_{A_{comp1}} = \frac{12.94}{1.07} = 12.09$$

Effect Size

- Effect size measures are the same except that you calculate them based on the adjusted SSs for effect and error

Applications of ANCOVA

Types of designs

Repeated Measures with a single CV measured once

	Easy	9:00 PM	11:00 PM	1:00 AM	3:00 AM
Rock	6	8	7	5	3
	6	9	7	5	1
	4	8	5	4	1
	3	6	6	4	2
Classical	6	2	4	8	10
	7	6	5	9	10
	3	2	4	6	10
	5	3	5	6	9

General Linear Model

Within-Subjects Factors

Measure: MEASURE_1

TIME	Dependent Variable
1	T1
2	T2
3	T3
4	T4

Between-Subjects Factors

		Value Label	N
MUSIC	1	rock	4
	2	classical	4

Descriptive Statistics

	MUSIC	Mean	Std. Deviation	N
T1	rock	7.75	1.258	4
	classical	3.25	1.893	4
	Total	5.50	2.828	8
T2	rock	6.25	.957	4
	classical	4.50	.577	4
	Total	5.38	1.188	8
T3	rock	4.50	.577	4
	classical	7.25	1.500	4
	Total	5.88	1.808	8
T4	rock	1.75	.957	4
	classical	9.75	.500	4
	Total	5.75	4.334	8

Mauchly's Test of Sphericity[b]

Measure: MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[a]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
TIME	.362	3.785	5	.592	.663	1.000	.333

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

b.
Design: Intercept+EASY+MUSIC
Within Subjects Design: TIME

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
TIME	Sphericity Assumed	4.100	3	1.367	1.802	.190	.265
	Greenhouse-Geisser	4.100	1.990	2.060	1.802	.215	.265
	Huynh-Feldt	4.100	3.000	1.367	1.802	.190	.265
	Lower-bound	4.100	1.000	4.100	1.802	.237	.265
TIME * EASY	Sphericity Assumed	4.246	3	1.415	1.866	.179	.272
	Greenhouse-Geisser	4.246	1.990	2.134	1.866	.205	.272
	Huynh-Feldt	4.246	3.000	1.415	1.866	.179	.272
	Lower-bound	4.246	1.000	4.246	1.866	.230	.272
TIME * MUSIC	Sphericity Assumed	181.199	3	60.400	79.620	.000	.941
	Greenhouse-Geisser	181.199	1.990	91.050	79.620	.000	.941
	Huynh-Feldt	181.199	3.000	60.400	79.620	.000	.941
	Lower-bound	181.199	1.000	181.199	79.620	.000	.941
Error(TIME)	Sphericity Assumed	11.379	15	.759			
	Greenhouse-Geisser	11.379	9.951	1.144			
	Huynh-Feldt	11.379	15.000	.759			
	Lower-bound	11.379	5.000	2.276			

Tests of Within-Subjects Contrasts

Measure: MEASURE_1

Source	TIME	Type III Sum of Squares	df	Mean Square	F	Sig.
TIME	Linear	2.854	1	2.854	2.122	.205
	Quadratic	.034	1	.034	.126	.737
	Cubic	1.213	1	1.213	1.828	.234
TIME * EASY	Linear	2.352	1	2.352	1.749	.243
	Quadratic	.036	1	.036	.136	.728
	Cubic	1.858	1	1.858	2.801	.155
TIME * MUSIC	Linear	178.048	1	178.048	132.410	.000
	Quadratic	3.146	1	3.146	11.749	.019
	Cubic	.005	1	.005	.007	.935
Error(TIME)	Linear	6.723	5	1.345		
	Quadratic	1.339	5	.268		
	Cubic	3.317	5	.663		

Levene's Test of Equality of Error Variances[a]

	F	df1	df2	Sig.
T1	.753	1	6	.419
T2	.290	1	6	.609
T3	1.287	1	6	.300
T4	3.483	1	6	.111

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a.
Design: Intercept+EASY+MUSIC
Within Subjects Design: TIME

Tests of Between-Subjects Effects

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Intercept	28.000	1	28.000	39.455	.002	.888
EASY	11.327	1	11.327	15.960	.010	.761
MUSIC	6.436	1	6.436	9.069	.030	.645
Error	3.548	5	.710			

Estimated Marginal Means

1. Grand Mean

Measure: MEASURE_1

Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
5.625[a]	.149	5.242	6.008

a. Covariates appearing in the model are evaluated at the following values: EASY = 5.00.

2. MUSIC

Measure: MEASURE_1

MUSIC	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
rock	5.169[a]	.212	4.624	5.715
classical	6.081[a]	.212	5.535	6.626

a. Covariates appearing in the model are evaluated at the following values: EASY = 5.00.

3. TIME

Measure: MEASURE_1

TIME	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1	5.500[a]	.417	4.427	6.573
2	5.375[a]	.239	4.759	5.991
3	5.875[a]	.236	5.268	6.482
4	5.750[a]	.293	4.997	6.503

a. Covariates appearing in the model are evaluated at the following values: EASY = 5.00.

4. MUSIC * TIME

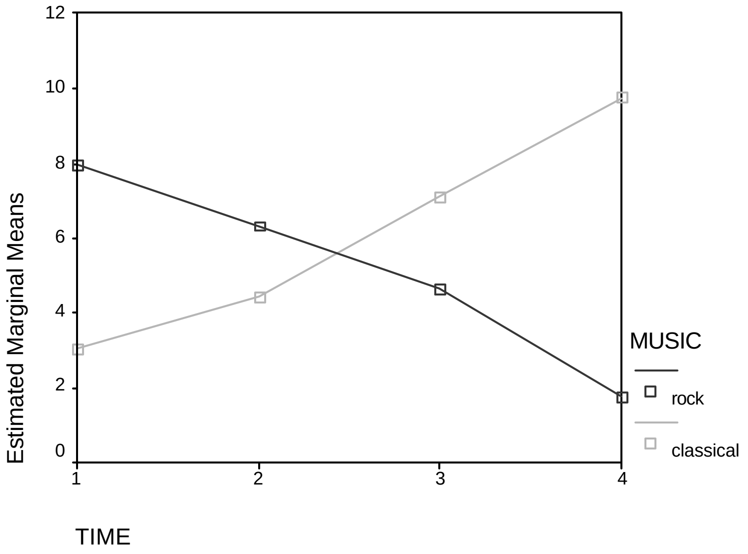
Measure: MEASURE_1

MUSIC	TIME	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
rock	1	7.935[a]	.595	6.406	9.465
	2	6.327[a]	.341	5.449	7.204
	3	4.649[a]	.337	3.784	5.514
	4	1.766[a]	.418	.692	2.840
classical	1	3.065[a]	.595	1.535	4.594
	2	4.423[a]	.341	3.546	5.301
	3	7.101[a]	.337	6.236	7.966
	4	9.734[a]	.418	8.660	10.808

a. Covariates appearing in the model are evaluated at the following values: EASY = 5.00.

Profile Plots

Estimated Marginal Means of MEASURE_1



Repeated Measures with a single CV measured at each time point

Case	T1_X	T1_Y	T2_X	T2_Y
1	4	9	3	15
2	8	10	6	16
3	13	14	10	20
4	1	6	3	9
5	8	11	9	15
6	10	10	9	9
7	5	7	8	12
8	9	12	9	20
9	11	14	10	20

MANOVA syntax

```
MANOVA
 t1_y t2_y with t1_x t2_x
 /WSFACTOR = trials(2)
 /PRINT = SIGNIF(EFSIZE),
   CELLIFO(MEANS)
 /WSDESIGN trials
 /DESIGN.
```

- Note: there are 2 levels for the TRIALS effect. Average tests are identical to the univariate tests of significance.
- The default error term in MANOVA has been changed from WITHIN CELLS to
- WITHIN+RESIDUAL. Note that these are the same for all full factorial
- designs.
- * * * * * * A n a l y s i s   o f   V a r i a n c e * * * * * *
- 9 cases accepted.
- 0 cases rejected because of out-of-range factor values.
- 0 cases rejected because of missing data.
- 1 non-empty cell.
- 1 design will be processed.
- -
- Cell Means and Standard Deviations
- Variable .. T1_Y

	Mean	Std. Dev.	N
For entire sample	10.333	2.784	9

- -
- Variable .. T2_Y

	Mean	Std. Dev.	N
For entire sample	15.111	4.428	9

- -
- Variable .. T1_X

	Mean	Std. Dev.	N
For entire sample	7.667	3.742	9

- -
- Variable .. T2_X

	Mean	Std. Dev.	N
For entire sample	7.444	2.789	9

- -

- * * * * * * A n a l y s i s   o f   V a r i a n c e -- design   1 * * *
- Tests of Between-Subjects Effects.
- Tests of Significance for T1 using UNIQUE sums of squares
- Source of Variation SS DF MS F Sig of F
- WITHIN+RESIDUAL 91.31 7 13.04
- REGRESSION 100.80 1 100.80 7.73 .027
- CONSTANT 109.01 1 109.01 8.36 .023
- -
- Effect Size Measures
- Partial
- Source of Variation ETA Sqd
- Regression .525
- -
- Regression analysis for WITHIN+RESIDUAL error term
- --- Individual Univariate .9500 confidence intervals
- Dependent variable .. T1
- COVARIATE B Beta Std. Err. t-Value Sig. of t
- T3 .79512 .72437 .286 2.780 .027
- COVARIATE Lower -95% CL- Upper ETA Sq.
- T3 .119 1.471 .525
- -

- * * * * * * A n a l y s i s o f V a r i a n c e -- design 1 * * *
- Tests involving 'TRIALS' Within-Subject Effect.
- Tests of Significance for T2 using UNIQUE sums of squares
- Source of Variation SS DF MS F Sig of F
- WITHIN+RESIDUAL 26.08 7 3.73
- REGRESSION .70 1 .70 .19 .677
- TRIALS 99.16 1 99.16 26.62 .001
- -
- Effect Size Measures
- Partial
- Source of Variation ETA Sqd
- Regression .026
- TRIALS .792
- -
- Regression analysis for WITHIN+RESIDUAL error term
- --- Individual Univariate .9500 confidence intervals
- Dependent variable .. T2
- COVARIATE B Beta Std. Err. t-Value Sig. of t
- T4 -.21805 -.16198 .502 -.434 .677
- COVARIATE Lower -95% CL- Upper ETA Sq.
- T4 -1.405 .969 .026
- -

BG ANCOVA with 2 CVs

	Spouse Libido	Libido	Rating
Viagra	7	7	11
	7	6	10
	6	4	8
	7	8	10
	7	6	10
Levitra	6	6	8
	6	5	8
	6	9	9
	5	8	8
	7	9	10
Cialis	4	6	4
	7	6	7
	6	5	5
	5	6	5
	7	6	6

Univariate Analysis of Variance

Between-Subjects Factors

		Value Label	N
Drug Typr	1	Viagra	5
	2	Levitra	5
	3	Cialis	5

Descriptive Statistics

Dependent Variable: Rating of Effectiveness

Drug Typr	Mean	Std. Deviation	N
Viagra	9.80	1.095	5
Levitra	8.60	.894	5
Cialis	5.40	1.140	5
Total	7.93	2.154	15

Levene's Test of Equality of Error Variances[a]

Dependent Variable: Rating of Effectiveness

F	df1	df2	Sig.
2.340	2	12	.139

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+SLIBIDO+LIBIDO+DRUG

Tests of Between-Subjects Effects

Dependent Variable: Rating of Effectiveness

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	63.140[a]	4	15.785	88.024	.000	.972
Intercept	.037	1	.037	.208	.658	.020
SLIBIDO	6.663	1	6.663	37.155	.000	.788
LIBIDO	2.590	1	2.590	14.443	.003	.591
DRUG	25.243	2	12.621	70.382	.000	.934
Error	1.793	10	.179			
Total	1009.000	15				
Corrected Total	64.933	14				

a. R Squared = .972 (Adjusted R Squared = .961)

Estimated Marginal Means

1. Grand Mean

Dependent Variable: Rating of Effectiveness

Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
7.933[a]	.109	7.690	8.177

a. Covariates appearing in the model are evaluated at the following values: Spouse's Libido = 6.20, Own Libido = 6.47.

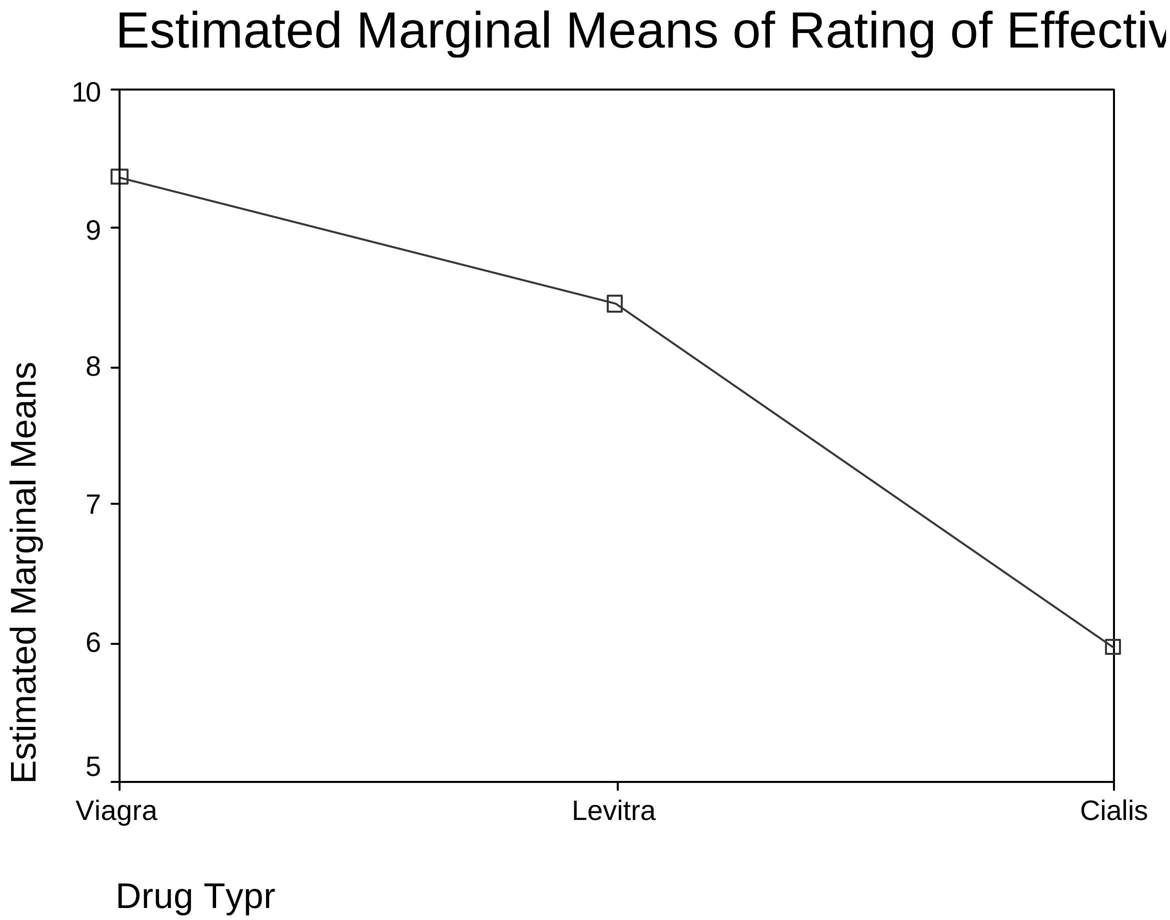
2. Drug Typr

Dependent Variable: Rating of Effectiveness

Drug Typr	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Viagra	9.381[a]	.210	8.912	9.850
Levitra	8.449[a]	.212	7.978	8.920
Cialis	5.970[a]	.203	5.517	6.422

a. Covariates appearing in the model are evaluated at the following values: Spouse's Libido = 6.20, Own Libido = 6.47.

Profile Plots



Correlations among variables

Correlations

Correlations

		Spouse's Libido	Own Libido	Rating of Effectiveness
Spouse's Libido	Pearson Correlation	1	.135	.677**
	Sig. (2-tailed)	.	.630	.006
	N	15	15	15
Own Libido	Pearson Correlation	.135	1	.443
	Sig. (2-tailed)	.630	.	.098
	N	15	15	15
Rating of Effectiveness	Pearson Correlation	.677**	.443	1
	Sig. (2-tailed)	.006	.098	.
	N	15	15	15

**. Correlation is significant at the 0.01 level (2-tailed).

Alternatives to ANCOVA

- When CV and DV are measured on the same scale
  - ANOVA on the difference scores (e.g. DV-CV)
  - Turn the CV and DV into two levels of a within subjects IV in a mixed design

Alternatives to ANCOVA

- When CV and DV measured on different scales
  - Use CV to match cases in a matched randomized design
  - Use CV to group similar participants together into blocks. Each block is then used as levels of a BG IV that is crossed with the other BG IV that you are interested in.

Alternatives to ANCOVA

- Blocking – may be the best alternative
  - Because it doesn't have the special assumptions of ANCOVA or repeated measures ANOVA
  - Because it can capture non-linear relationships between CV and DV where ANCOVA only deals with linear relationships.