Math 581, Fall 2006

Project 1. Calculating the number π.

Due November 16, 2005

Topics covered. Cancellation. Running error analysis.

Problem 1. Consider the sequence of numbers defined by the following recurrence relation:

$$x_0 = 2,$$

$$x_i = \sqrt{2 - \sqrt{4 - (x_{i-1})^2}}, \qquad i = 1, 2, 3, \ldots \tag{1}$$

It can be verified by a geometrical argument that

$$\pi = \lim_{i \to \infty} x_i 2^i$$

- Write a code implementing algorithm (1).
- Use this code to calculate π approximately

  $$\pi \approx x_n 2^n$$

  for values of $n = 5, 10, 15, 20, 25, 30$. Explain what you observe.
- Modify (1) to avoid cancellation. Implement the new algorithm in the computer code. Check that the new code computes value of π better.

Problem 2 A rather fast approximation of π can be obtained by using the running fraction representation of π:

$$\frac{4}{\pi} = 1 + \cfrac{1}{3 + \cfrac{2^2}{5 + \cfrac{3^2}{7 + \cfrac{4^2}{9 + \cdots}}}}$$

An approximation to this continuous fraction can be computed using the following algorithm.

$a_{n+1} = 2(n+1) + 1;$

for $i = n : -1 : 0$

$\quad a_i = (2i + 1) + \dfrac{(i+1)^2}{a_{i+1}}$

end

- Implement the algorithm for approximating the running fraction. Use the code to calculate an approximation to $4/\pi$ for $n = 1, 2, 4, 6, 8$. Explain what you observe.
- Do running error analysis for the algorithm. Predict whether the error will increase or decrease with the growth of n.
- Implement running error analysis in the code. Computer running error for n=10.

Write a report. Include results of the calculations done in Problems 1 and 2. Add comments explaining the results. Attach the code.