

Attention! Please, note that this is the closed book test. You are not allowed to use graphing calculator. Simple calculators are allowed. Please, show all important steps in you solution but do not make your solution excessively long.

1. (15pt) Write the Lagrange equations, DO NOT SOLVE, for the problem of finding the closest point to the origin on the surface

$$xyz + xy + yz + xz + 6 = 0.$$

2. (20pt) Change integration order in the iterated integral. DO NOT EVALUATE:

$$\int_{-2}^{1} \int_{x^2}^{2-x} \tan(x + \sin(1 - y^2)) \,\mathrm{d}y \,\mathrm{d}x.$$

3. (15pt) Evaluate the double integral

$$\int \int_S x \, \mathrm{e}^y \, \mathrm{d}A$$

where S is the domain bounded by y = 4 and $y = x^2$

4. The lamina in the XY-plane has the form of the triangle with vertices (0,0), (1,0), (0,1). The density of the lamina is $\delta(x,y) = x^2 + y^2$. (10pt) Find center of mass of the lamina.

(10pt) Find moment of inertia of the lamina with respect to Z-axis.

5. (20pt) Evaluate by using polar coordinates

$$\int_0^{\sqrt{\pi}} \int_0^{\sqrt{\pi} - x^2} \cos(x^2 + y^2) \, \mathrm{d}y \, \mathrm{d}x$$

6. (15pt) Evaluate the area or the part of the cone

$$z^2 = x^2 + y^2$$

that is directly above the triangle in XY-plane with vertices (1,0), (0,-1), (0,1).