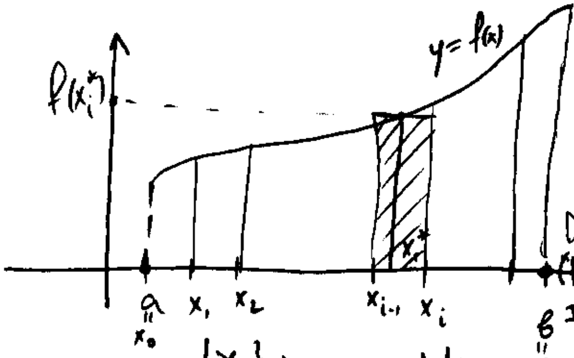
The Riemann Integral
(definite Integral)



Definite (Riemann) Integral

$$\int_a^b f(x)\,dx = \lim_{\Delta x_i \to 0} \underbrace{\sum_{i=1}^{n} f(x_i^*)\Delta x_i}_{\text{Riemann Sum}}$$

upper limit

lower limit

indicates integration variable

see (*)

$\{x_i\}$ is A partition of $[a,b]$

$\boxed{a<b}$

$\Delta x_i = x_i - x_{i-1}$

$x_i^* \in [x_{i-1}, x_i]$ any point inside $[x_{i-1}, x_i]$

(*) $\Delta x_i \to 0$ Really means $\max_i \{|\Delta x_i|\} \to 0$

THIS DEFINITION IS RESTRICTIVE, IT DOESN'T COVER CASES $a>b$, $a=b$

Define, in addition,

$$\int_a^a f(x)dx = 0 \ (a=b), \quad \int_a^b f(x)dx = -\int_b^a f(x)dx \ (a>b)$$

WHICH FUNCTIONS CAN BE USED IN INTEGRAL?

<u>Thm</u> If $f(x)$ is <u>bounded on $[a,b]$</u> and if <u>it is continuous</u> on $[a,b]$ except at a finite number of points, then f is integrable on $[a,b]$

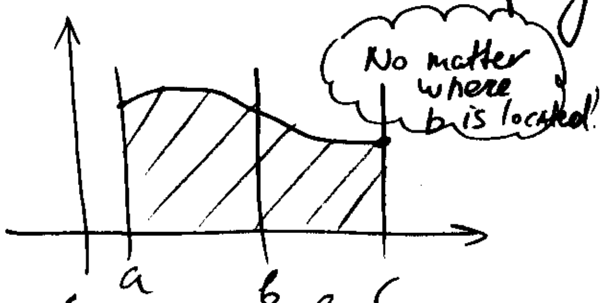
Properties:

$$\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$

$$\int_a^b c\,f(x)dx = c\int_a^b f(x)dx$$

Interval additive Property



$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$