

Attention! Please, note that this is the closed book test. You are not allowed to use graphing calculator. Simple calculators are allowed. Please, show all important steps in you solution but do not make your solution excessively long.

1. (15pt) The equation of a surface Cartesian coordinates is

$$xy - \frac{1}{\sqrt{x^2 + y^2 + z^2}} = 1 - \sqrt{x^2 + y^2}.$$

Change this equation into Spherical coordinates.

2. Evaluate the limits of functions of two variables: a) (10pt)

$$\lim_{(x,y)\to(0,0)}\frac{x^4-y^4}{x^2-y^2}$$

b) (10pt)

$$\lim_{(x,y)\to(0,1)}\frac{\sin(xy)}{xy}$$

3. (15pt) Find the <u>unit vector</u> in the direction of the fastest growth of

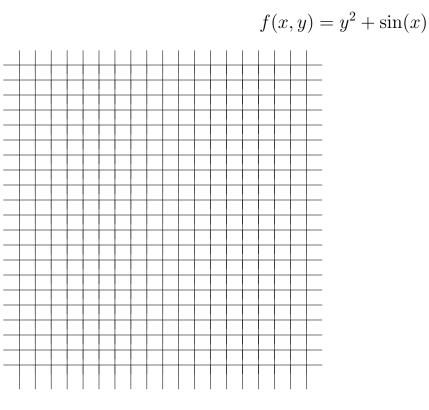
$$f(x,y) = \sqrt{x^2 - y^2}$$

at (1, 0)

4. (20pt) Let f(x, y), x(s, t, r), and y(s, t, r) be three differentiable functions. Use chain rule to find ∂ $\frac{\partial}{\partial}$

$$\frac{\partial}{\partial s}f(x(s,t,r),y(s,t,s))$$

5. (15pt) Sketch the graph of the function of two variables



6. a) (10pt) Write the differential of the function

 $f(x,y) = xy + \ln(xy) = 0$

b) (15pt) Use part a) to evaluate approximately change in function at (x, y) = (2, 3), corresponding to displacement (dx, dy) = (0.01, 0.03).