Sophisticated Graphing.
Part B

more examples on
sketching function's graph

Summary of the Method

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1. Precalculus analysis: a) Domain and range; b) Symmetry; c) X and Y-intercepts

2. Calculus analysis: a) Study $f'(x)$ for increasing, decreasing, min, max of $f(x)$; b) Study $f''(x)$ for concavity and inflection points of $f(x)$; c) Check for asymptotes.

3. Plot a few points: X, Y-intercepts, points of min, max, inflection points.

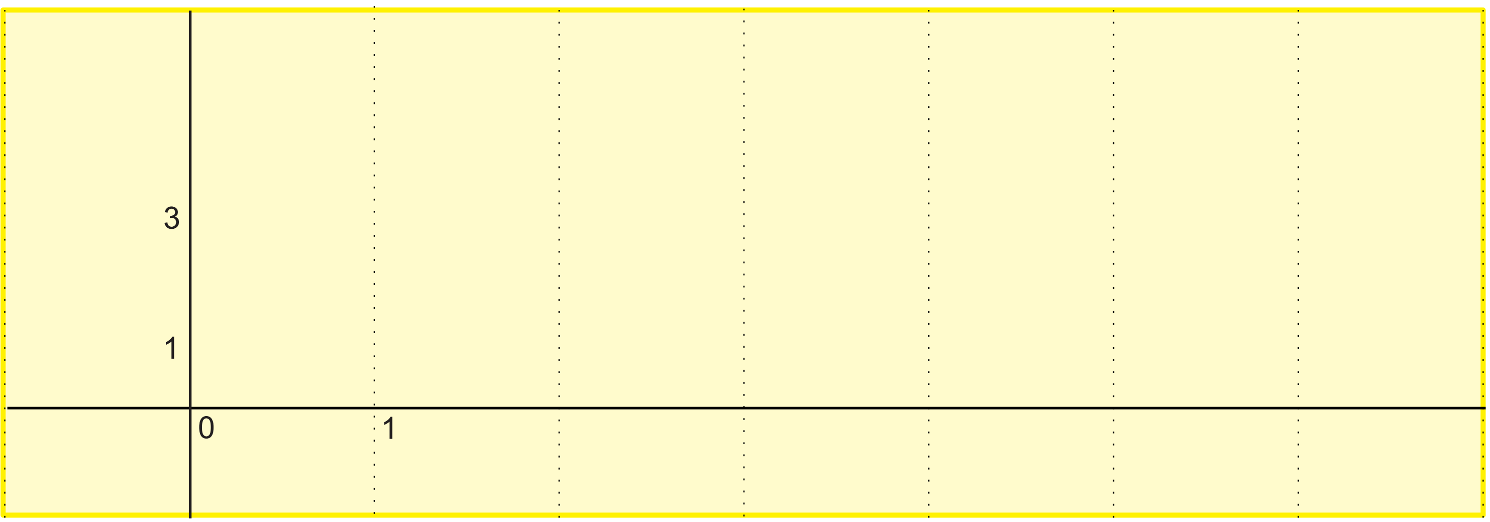
4. Sketch the graph: Connect the selected points with a smooth curve to maintain all the properties found in **2**.

Example 2. Sketch the graph of the function

$$f(x) = \frac{2}{\sqrt{x}} + x.$$

1. Precalculus analysis:

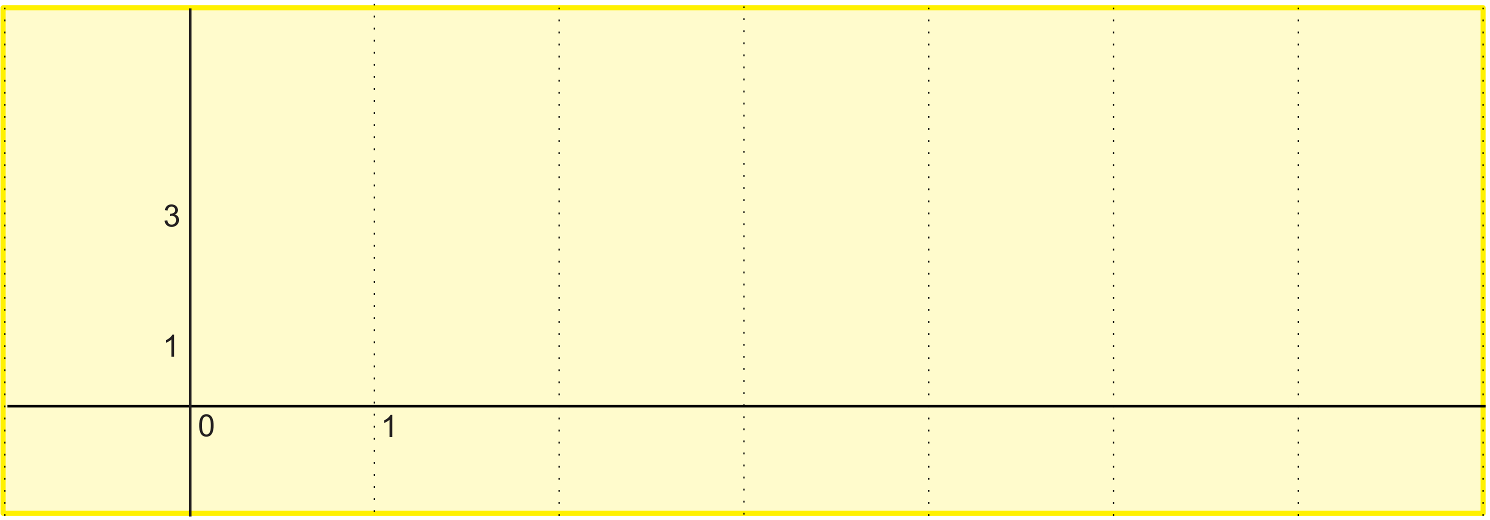
a) Domain is $x > 0$, range is $y > 0$; b) Neither even or odd; c) No X or Y-intercepts



2. Calculus analysis: a) $f'(x)$:

$$f'(x) = -\frac{1}{2}2x^{-3/2} + 1 = -\frac{1}{x^{3/2}} + 1 = \frac{x^{3/2} - 1}{x^{3/2}}$$

$f'(x) < 0$ for $0 < x < 1$; $f'(x) = 0$ at $x = 1$;
$f'(x) > 0$ for $x > 1$;



Analysis by the first derivative

$f'(x) > 0$ on $I \Rightarrow f(x)$ increases on I;

$f'(x) < 0$ on $I \Rightarrow f(x)$ decreases on I;

First Derivative Test for min and max:

$$\searrow\nearrow \quad \left.\begin{array}{l} f(x) < 0 \text{ for } x < c \\ f'(c) = 0, \\ f(x) > 0 \text{ for } x > c \end{array}\right\} \Rightarrow \text{MIN at } x = c$$

$$\nearrow\searrow \quad \left.\begin{array}{l} f(x) > 0 \text{ for } x < c \\ f'(c) = 0, \\ f(x) < 0 \text{ for } x > c \end{array}\right\} \Rightarrow \text{MAX at } x = c$$

2. Calculus analysis: a) $f'(x)$:

$$f'(x) = -\frac{1}{2}2x^{-3/2} + 1 = -\frac{1}{x^{3/2}} + 1 = \frac{x^{3/2} - 1}{x^{3/2}}$$

$f'(x) < 0$ for $0 < x < 1$; $f'(x) = 0$ at $x = 1$;
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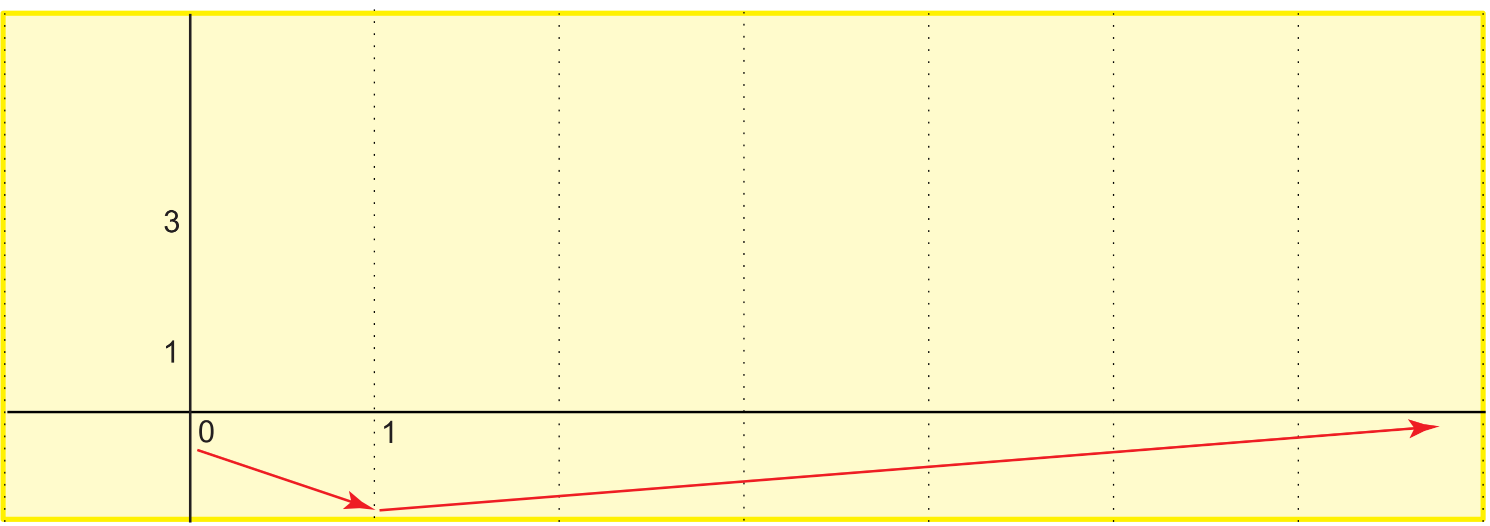
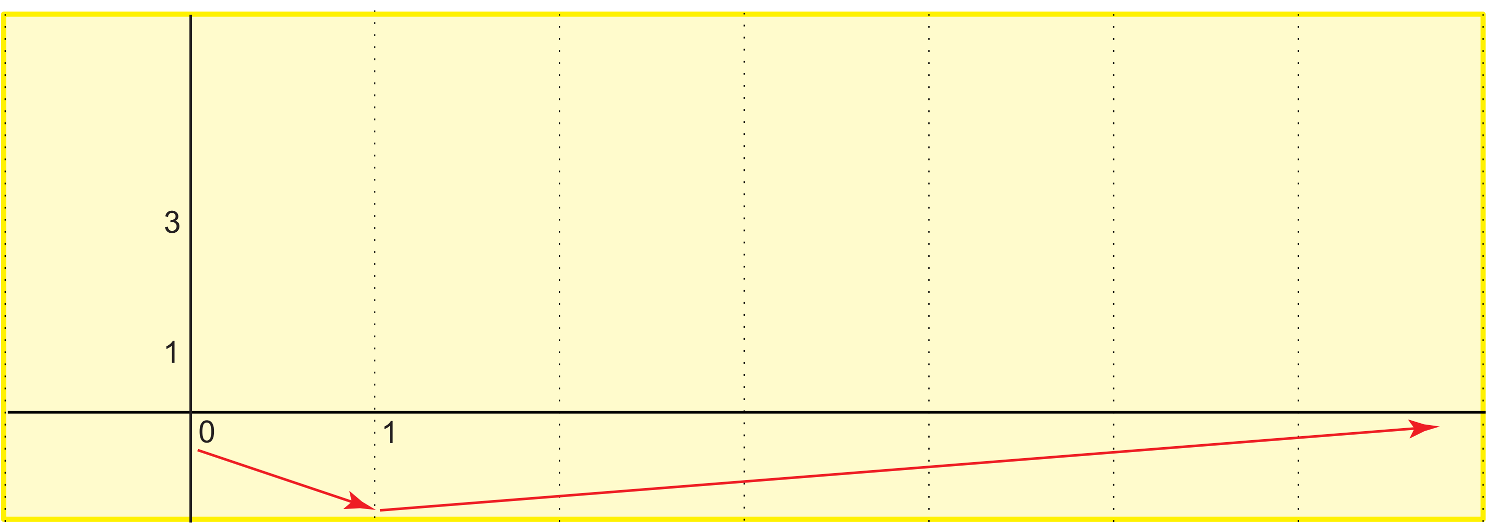


2. Calculus analysis: b) $f''(x)$:

$$f''(x) = \frac{3}{2}x^{-5/2} = \frac{3}{2\sqrt{x^5}}$$

$f''(x) > 0$ for $x > 0$;



Analysis by the second derivative

$f''(x) > 0$ on $I \Rightarrow f(x)$ is concave up on I;

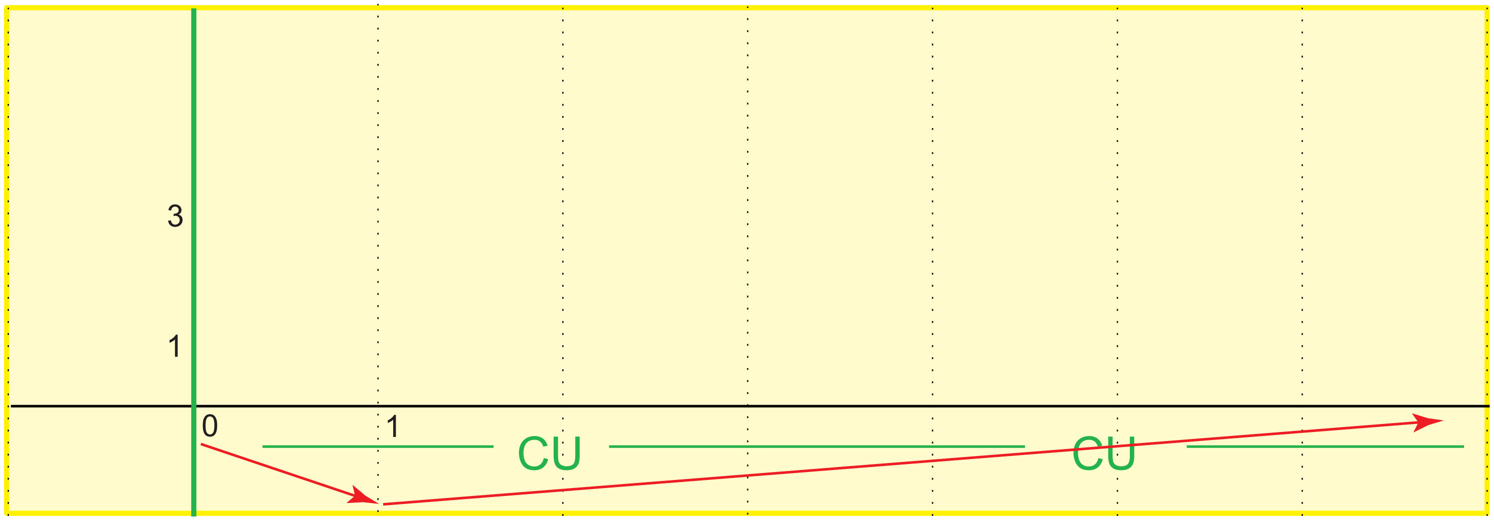
$f''(x) < 0$ on $I \Rightarrow f(x)$ is concave down on I;

change in concavity $=$ inflection point

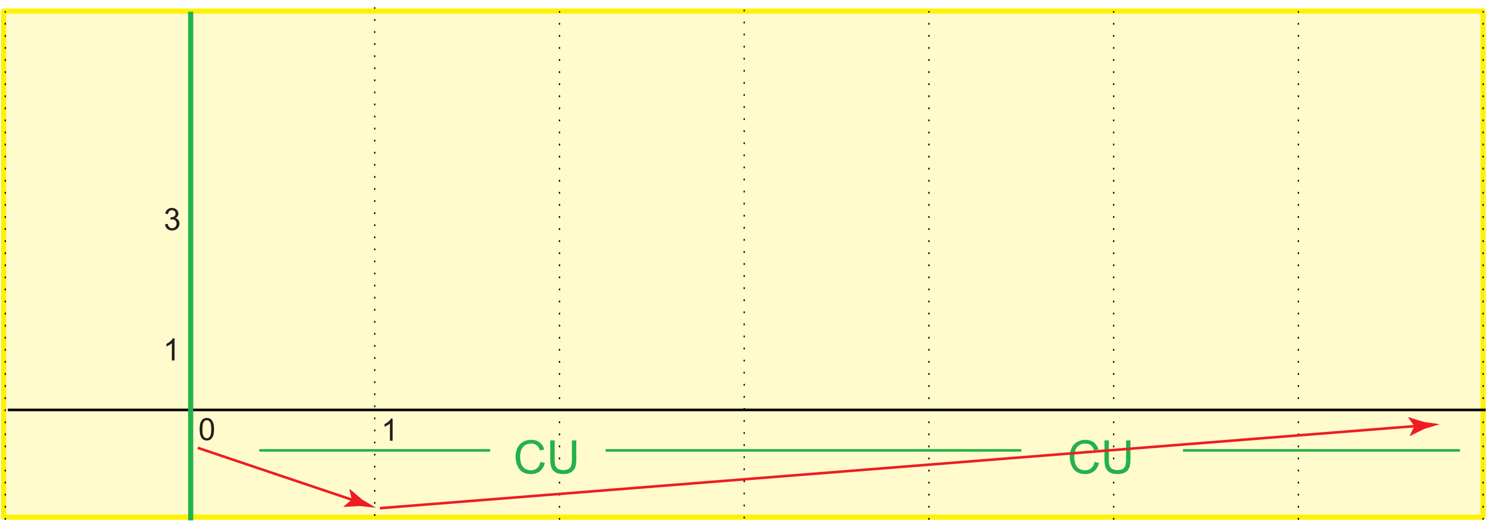
2. Calculus analysis: b) $f''(x)$:

$$f''(x) = \frac{3}{2}x^{-5/2} = \frac{3}{2\sqrt{x^5}}$$

$f''(x) > 0$ for $x > 0$; no inflection points.



2. Calculus analysis: c) Vertical, Horizontal, and Oblique Asymptotes



Asymptotes of $f(x)$

Vertical: $x = c$; (Division by 0, raising to ∞)

Horizontal:

$y = L$ as $x \to \infty$ if ; $L = \lim_{x \to \infty} f(x)$;

$y = L$ as $x \to -\infty$ if ; $L = \lim_{x \to -\infty} f(x)$;

Oblique: $f(x) \approx mx + b$ as $x \to \infty$ if !!EMPIRIC!!

$$m = \lim_{x \to \infty} \frac{f(x)}{x}, \qquad \text{or} \qquad m = \lim_{x \to \infty} f'(x),$$

$$b = \lim_{x \to \infty} [f(x) - mx]$$

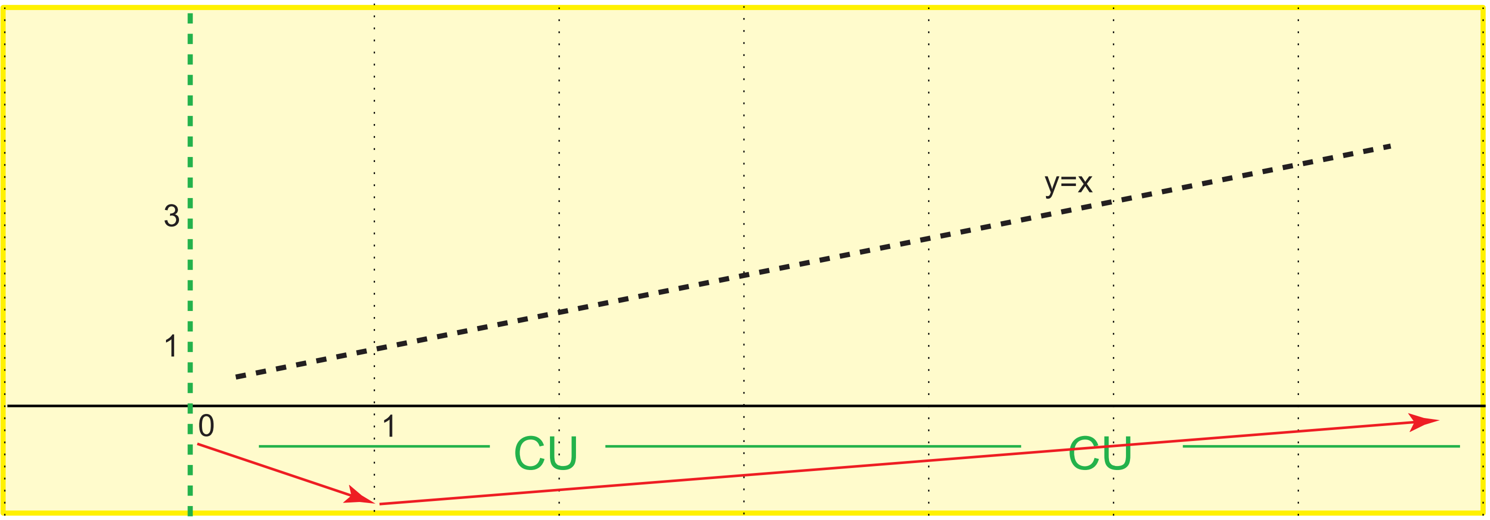
Similarly for $x \to -\infty$

2. Calculus analysis: c) Asymptotes

Vertical: $x = 0$; Oblique: $y = x$;

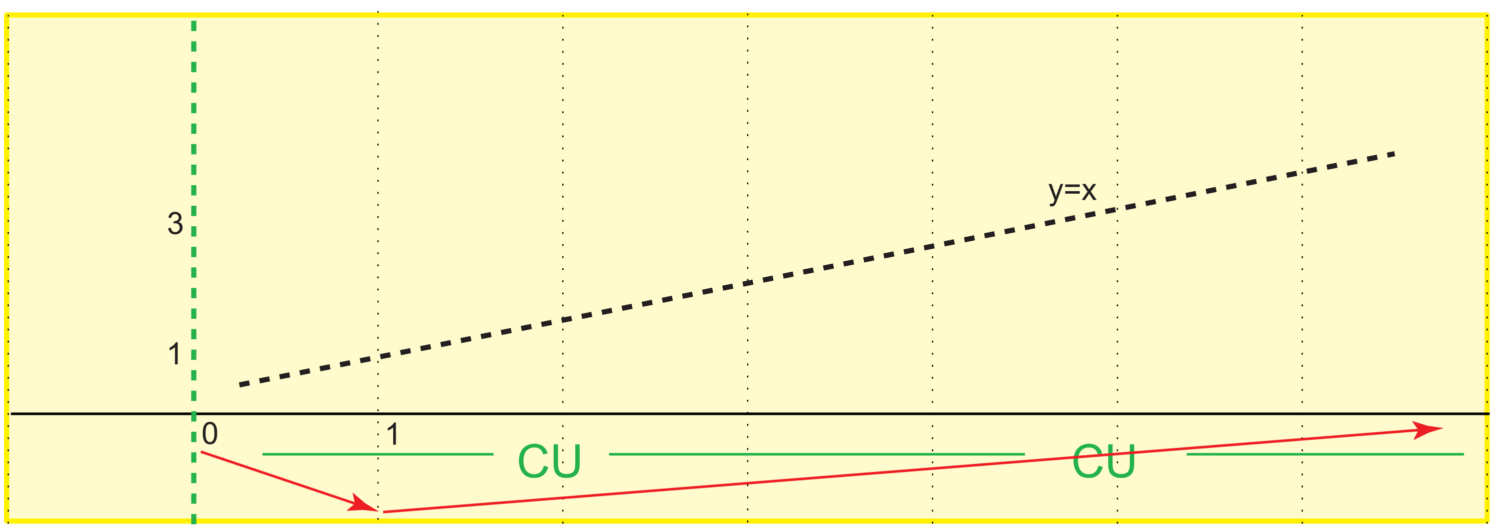
$$m = \lim_{x\to\infty} f'(x) = \lim_{x\to\infty}\left[-\frac{1}{x^{3/2}} + 1\right] = 1$$

$$b = \lim_{x\to\infty}(f(x) - mx) = \lim_{x\to\infty}\left[\frac{2}{\sqrt{x}} + x - 1\cdot x\right] = 0$$



3. Plot a few points: Min, Max, inflection, intercepts

Min at $x = 1,\ f(1) = 3$



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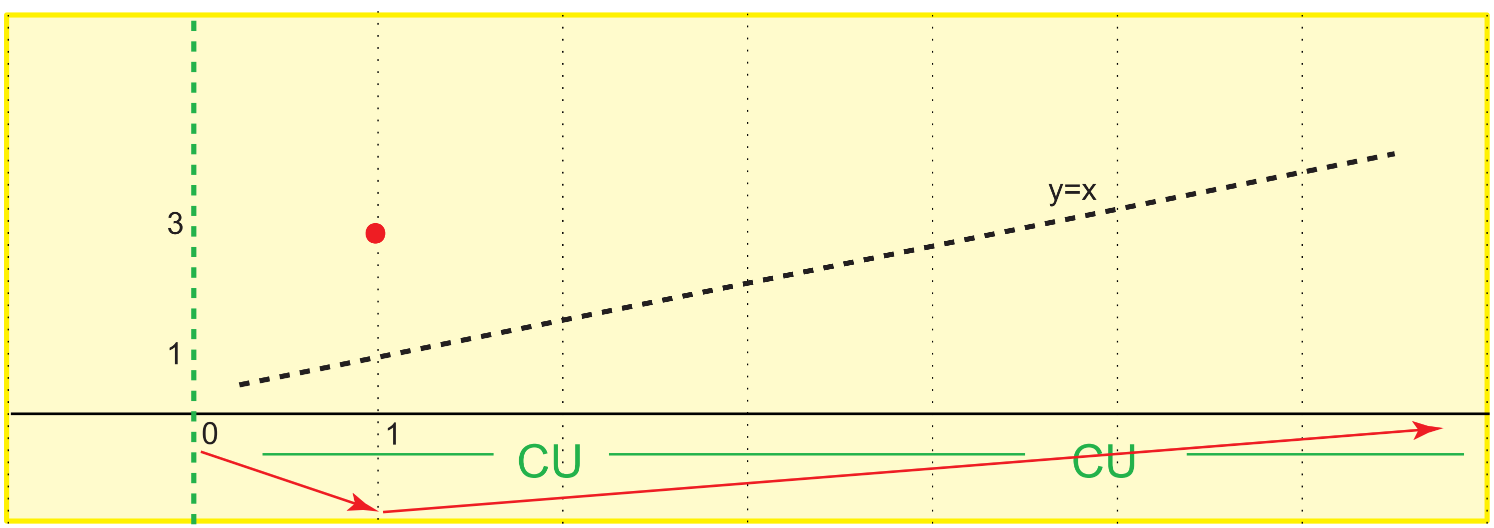
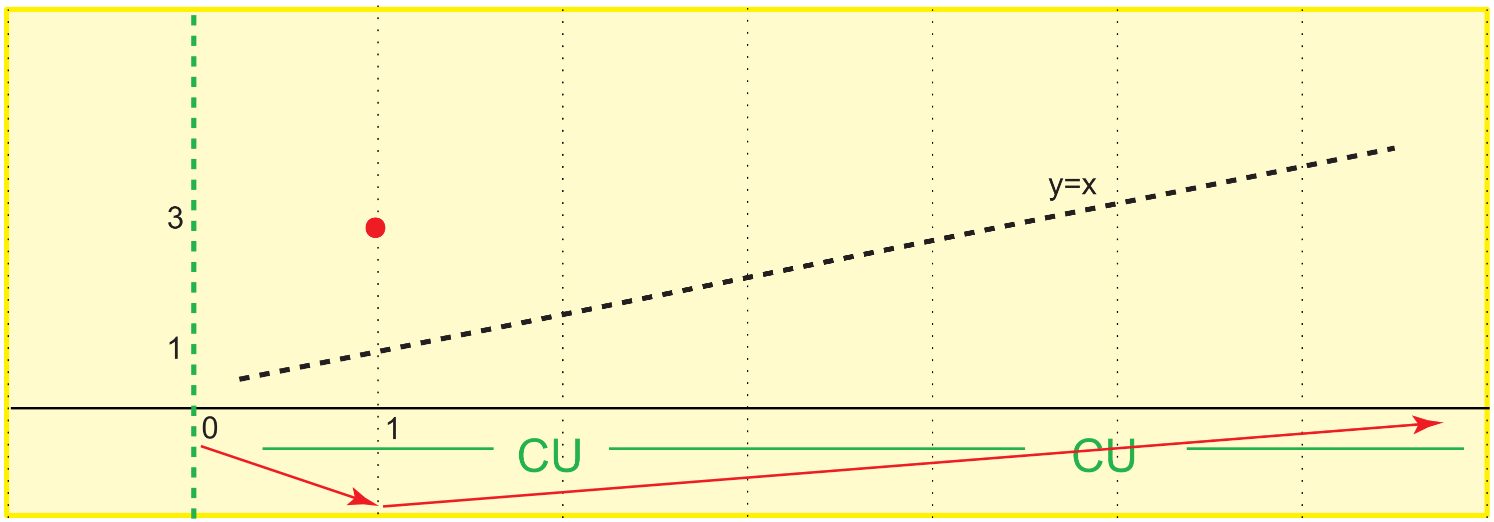
Min at $x = 1,\ f(1) = 3$



4. Sketch the graph: Connect points with a smooth curve to maintain all properties



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