Sophisticated Graphing

sketching function's graph

by studying derivatives

Sketching Function By Information on its Derivatives

summary of the method

A Quick Motivation...

By looking at function's graph you can tell where the function is increasing, decreasing, concave up, down, has max, min, inflection points.

Conversely, by information on where the function is increasing, decreasing, concave up, down, has max, min, inflection points, you can imagine a possible curve that sketches function's graph up to some accuracy.

Summary of the method.

1. Precalculus analysis: a) Domain and range; b) Symmetry; c) X and Y-intercepts

2. Calculus analysis: a) Study $f'(x)$ for increasing, decreasing, min, max of $f(x)$; b) Study $f''(x)$ for concavity and inflection points of $f(x)$; c) Check for asymptotes.

3. Plot a few points: X, Y-intercepts, points of min, max, inflection points.

4. Sketch the graph: Connect the selected points with a smooth curve to maintain all the properties found in **2**.

Example 1. Sketch the graph of $f(x)$ if

a) $f(-4) = 0$, $f(-2) = -2$, $f(-0.5) = 0$, $f(3) = -1$;

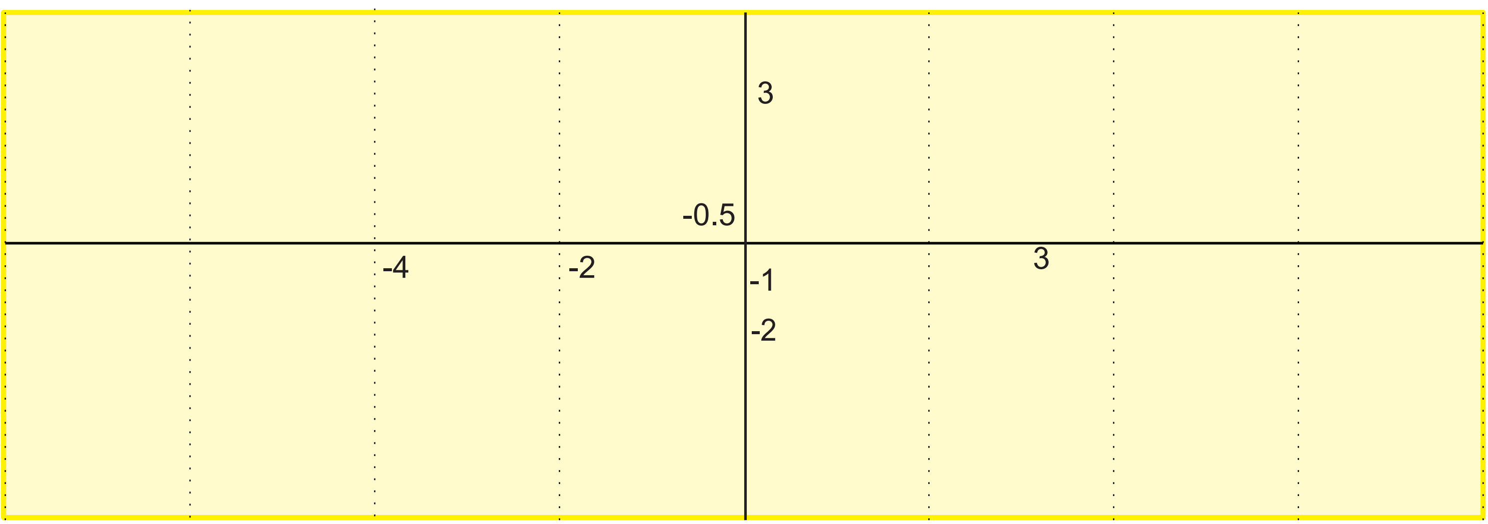
b) $f'(x) < 0$ for $x < -2$, $f'(x) = 0$ at $x = -2$, $f'(x) > 0$ for $-2 < x < 0$, $f'(x)$ is undefined at $x = 0$, $f'(x) > 0$ for $0 < x < 3$, $f'(x) = 0$ at $x = 3$, $f'(x) < 0$ for $x > 3$;

c) $f''(x) < 0$ for $x < -4$, $f''(x) = 0$ at $x = -4$, $f''(x) > 0$ for $-4 < x < 0$, $f''(x) < 0$ for $x > 0$;

d) $f(x)$ has horizontal asymptote $y = 3$ $(x \to -\infty)$; oblique asymptote $y = 3 - x$ $(x \to \infty)$; vertical asymptote $x = 0$.

a) We plot the given points on the graph:

$f(-4) = 0,\ f(-2) = -2,\ f(-0.5) = 0,$
$f(3) = -1;$



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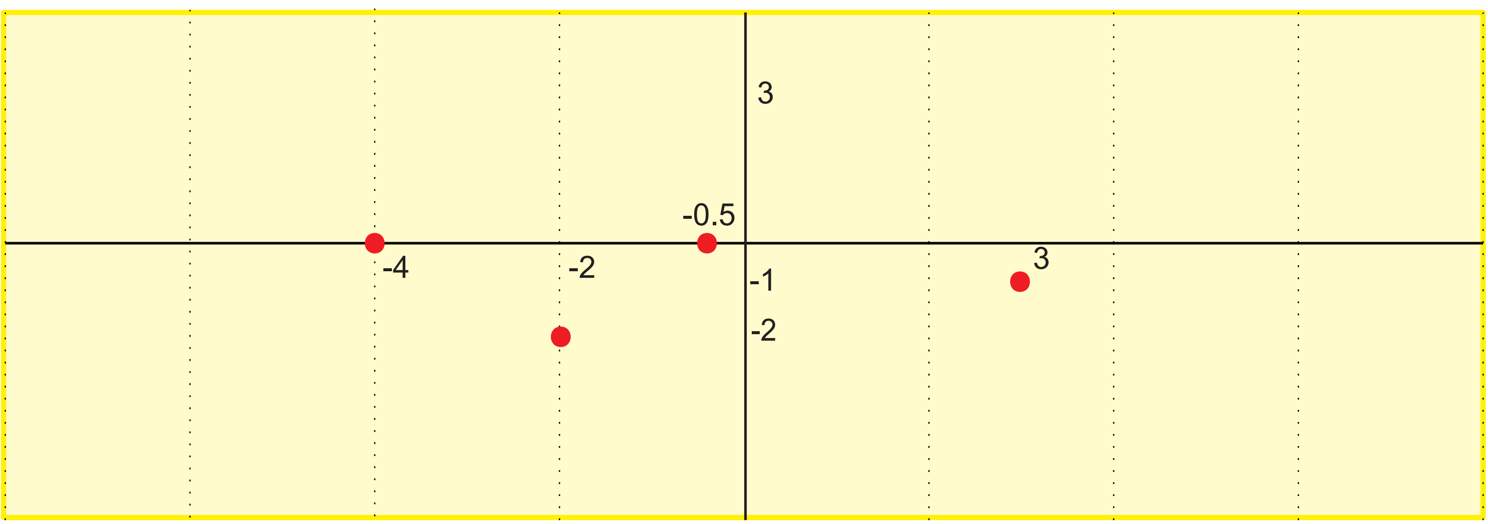
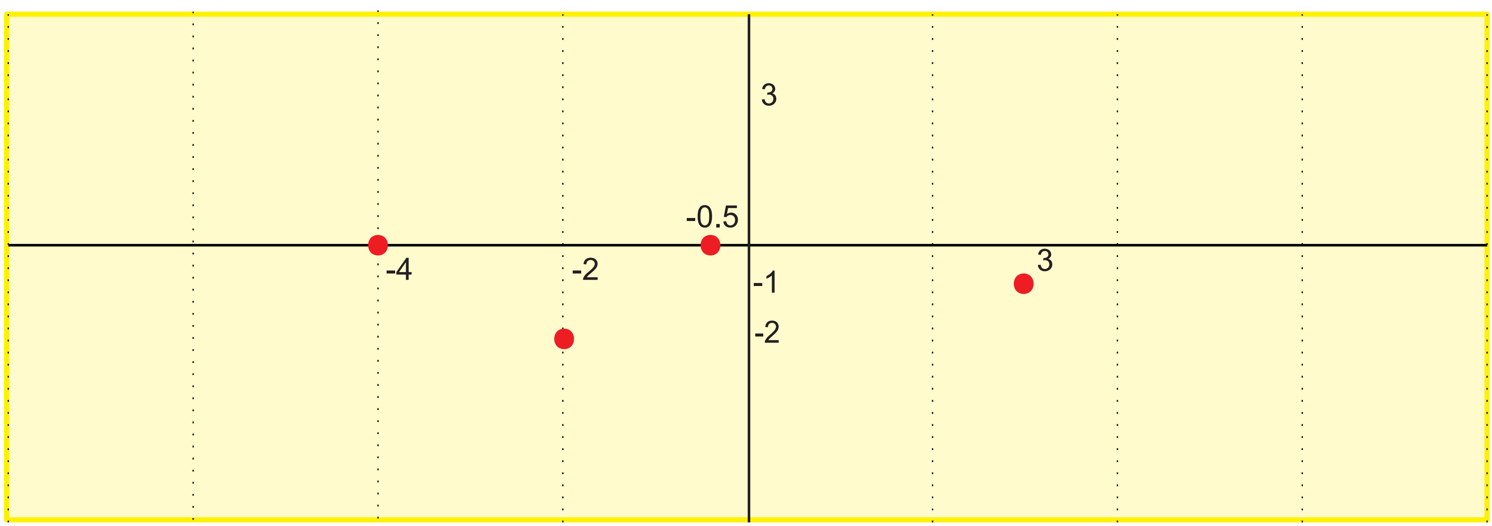


b) We plot information about the first derivative: $f'(x) < 0$ for $x < -2$, $f'(x) = 0$ at $x = -2$, $f'(x) > 0$ for $-2 < x < 0$, $f'(x)$ is undefined at $x = 0$, $f'(x) > 0$ for $0 < x < 3$, $f'(x) = 0$ at $x = 3$, $f'(x) < 0$ for $x > 3$;

(Use $f'(x) > 0$ on $I \Rightarrow f(x)$ increases on I; $f'(x) < 0$ on $I \Rightarrow f(x)$ decreases on I; and the first derivative test)



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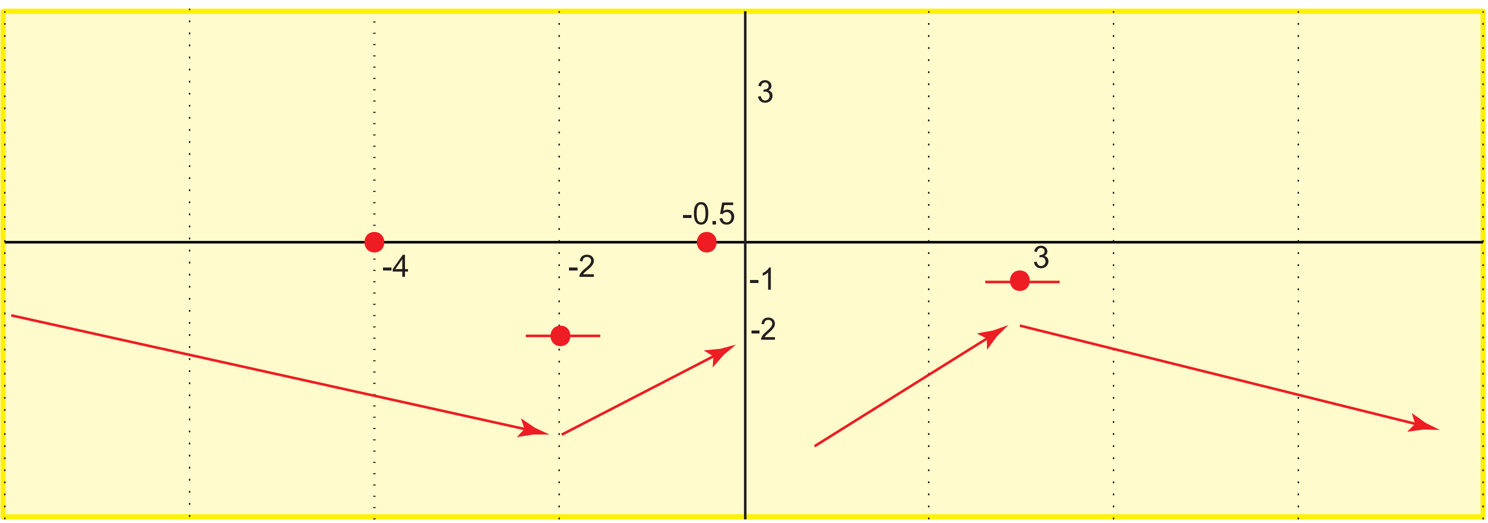

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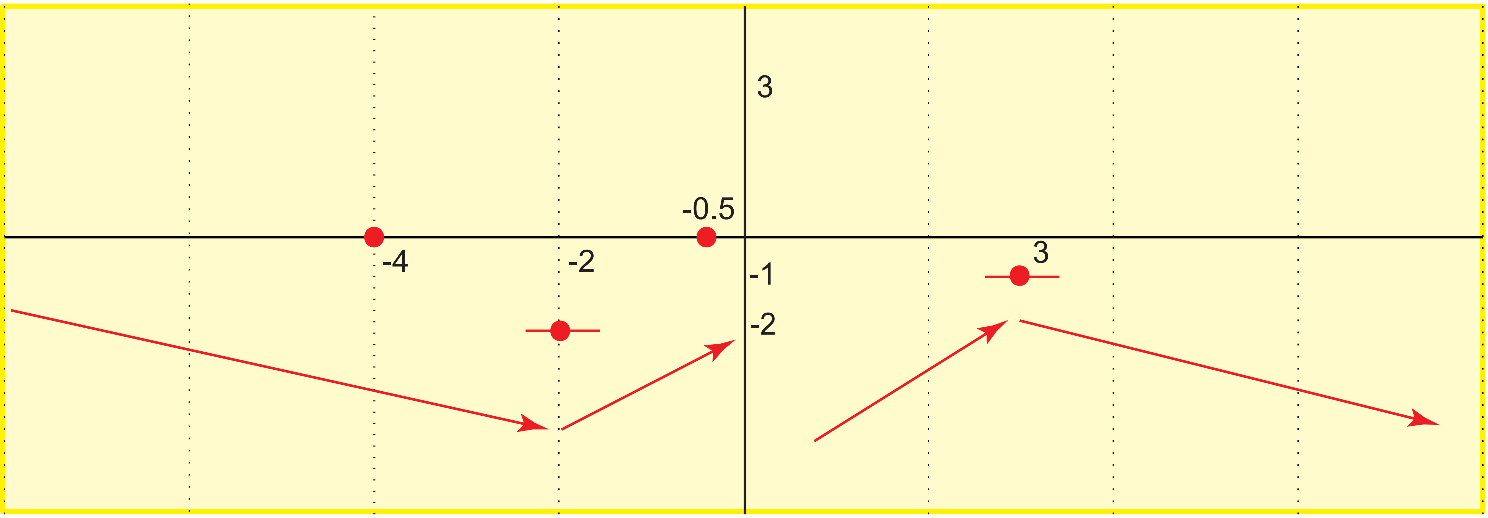
$f''(x) < 0$ for $x < -4$, $f''(x) = 0$ at $x = -4$, $f''(x) > 0$ for $-4 < x < 0$, $f''(x) < 0$ for $x > 0$;

(Use $f''(x) > 0$ on $I \Rightarrow f(x)$ is concave up on I; $f''(x) < 0$ on $I \Rightarrow f(x)$ is concave down on I; and the first derivative test)



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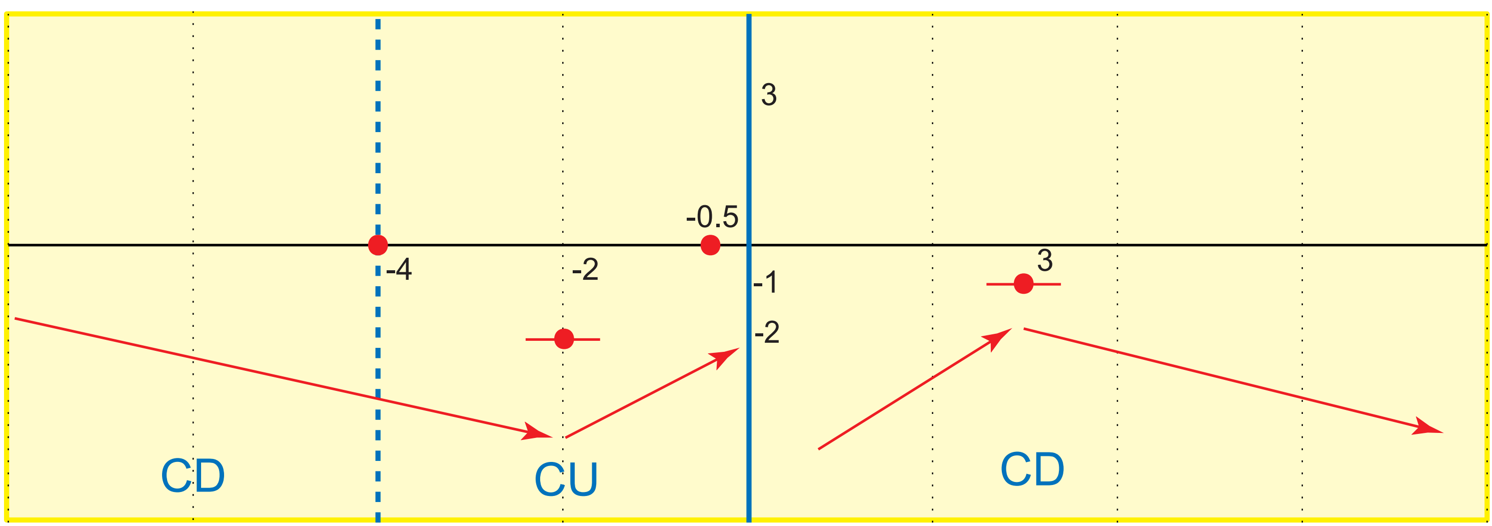
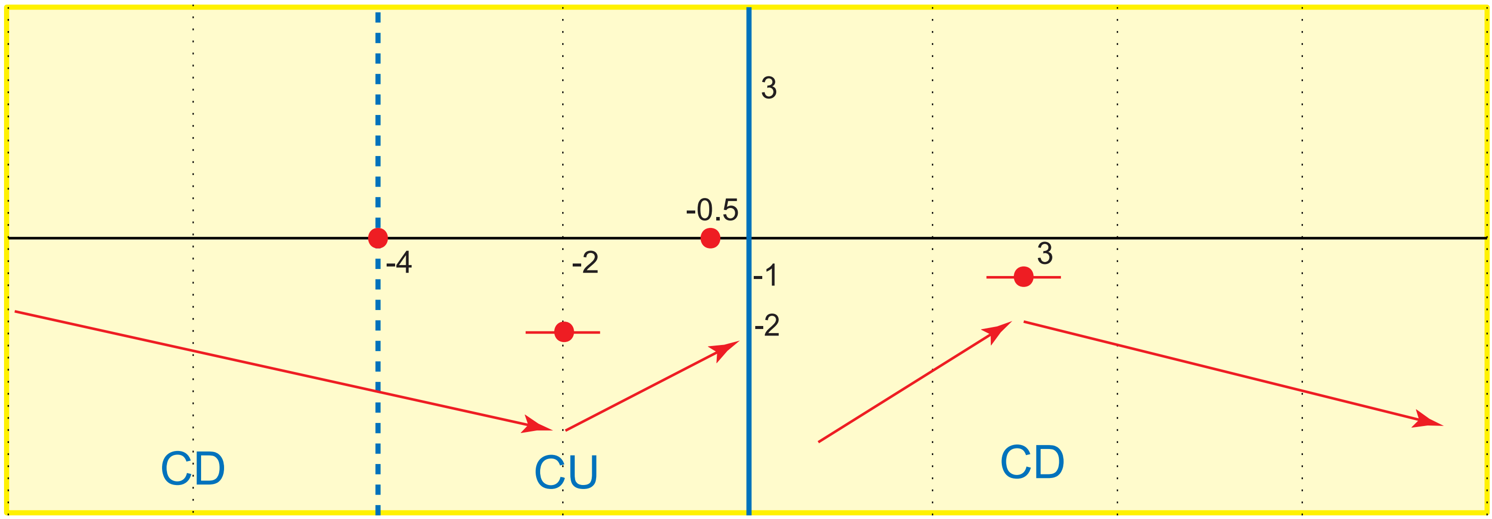


d) We plot information about the asymptotes: $f(x)$ has horizontal asymptote $y = 3$ $(x \to -\infty)$; oblique asymptote $y = 3 - x$ $(x \to \infty)$; vertical asymptote $x = 0$.



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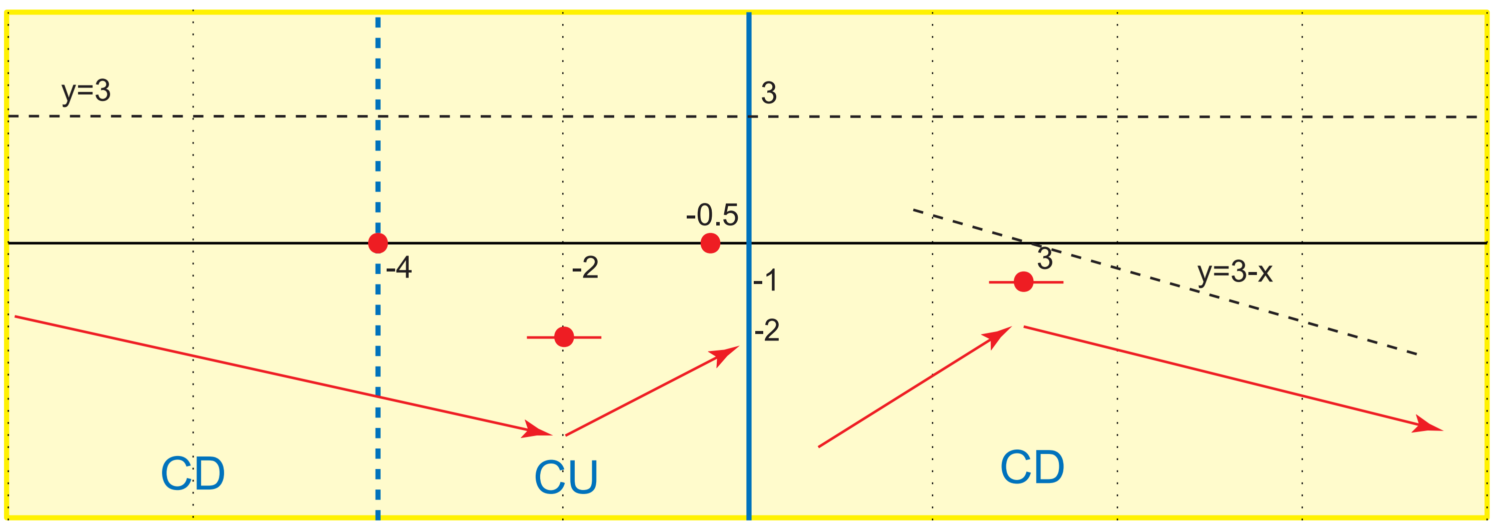
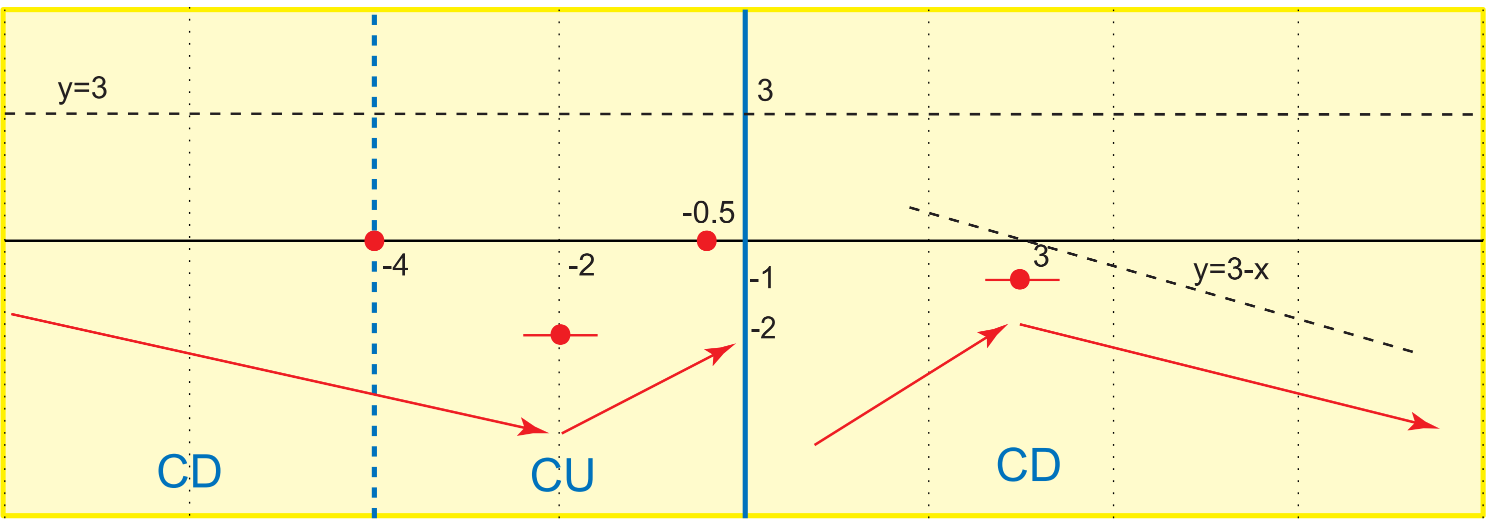


d) Finally, we sketch the graph:



d) Finally, we sketch the graph: