Wallpaper patterns

So far we have studied patterns that have no translation repetition (the rosette patters) and patters that have translation repetition in only one direction (frieze patterns). What happens with patterns that have translation repetitions in tow independent directions, so as to fill out the plane? These are the wallpaper patterns, and there are exactly 17 such patterns.

   Roughly speaking, such patterns are made out of a basic cell or motif which is then repeated by means of rigid motions so as to fill the plane. The wallpaper pattern refers not to the motif, but to the way in which is repetition is structured across the plane. It turns out that there are only 17 possible ways to do so.
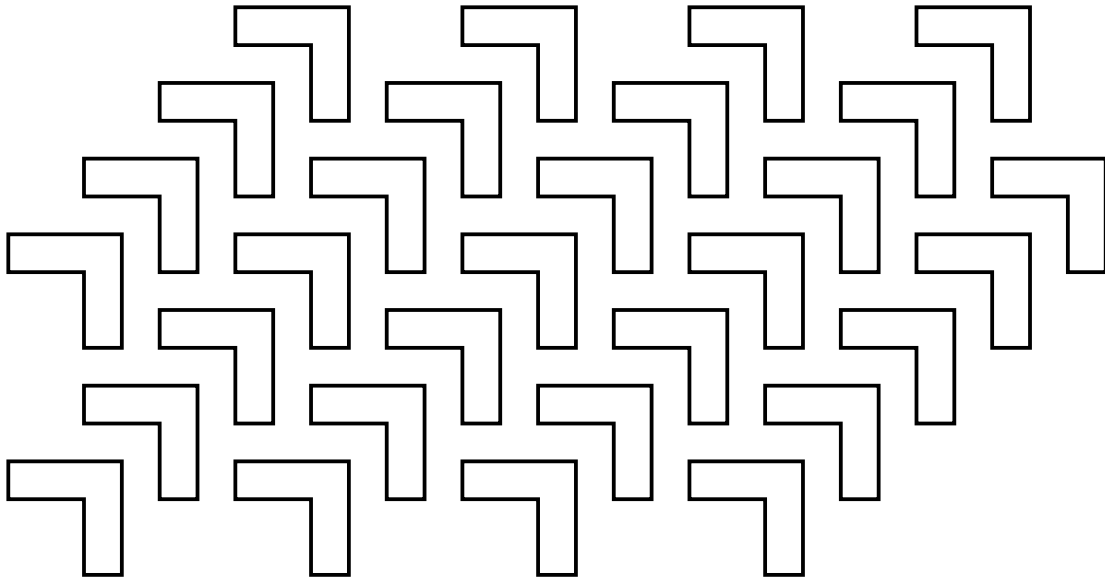
1. What isometries does this wallpaper pattern have?
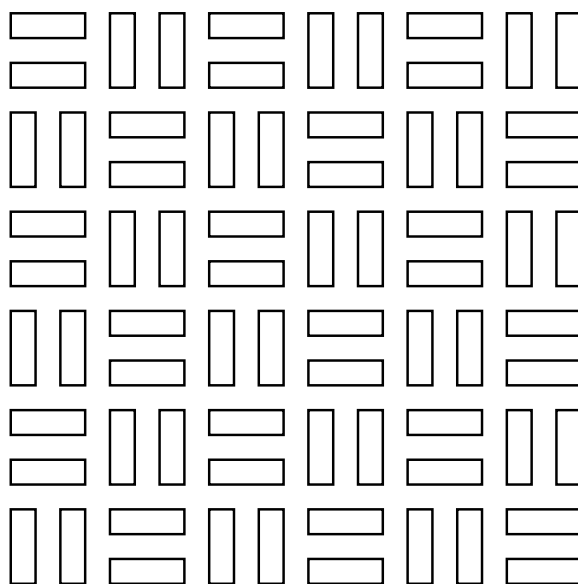
   (a) Translation and reflection only

   (b) Translation and rotation only

   (c) Translation, rotation, and reflection

   If there are reflections, indicate the reflection lines. If there are rotations, indicate the center of rotation.

2. What isometries does this wallpaper pattern have?


(a) Translation and reflection only

(b) Translation and rotation only

(c) Translation, rotation, and reflection

If there are reflections, indicate the reflection lines. If there are rotations, indicate the center of rotation.

Rotations in wallpaper patterns   Patterns may have rotations. We have seen that rosette patterns can have rotation of any order. On the other hand, frieze patterns can only have rotations of order 1 (full turn) or 2 (half turns). It turns out that wallpaper patterns can have rotations of order 1, 2, 3, 4, and 6 only.
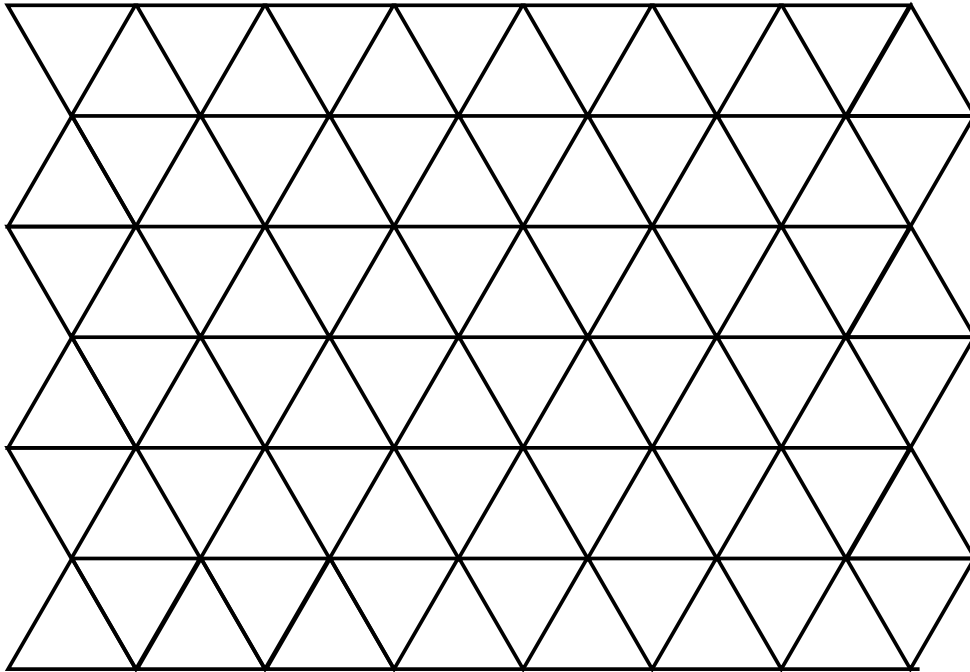
3. Using the grid and motif below, construct a wallpaper pattern with rotation of order 2 exactly.
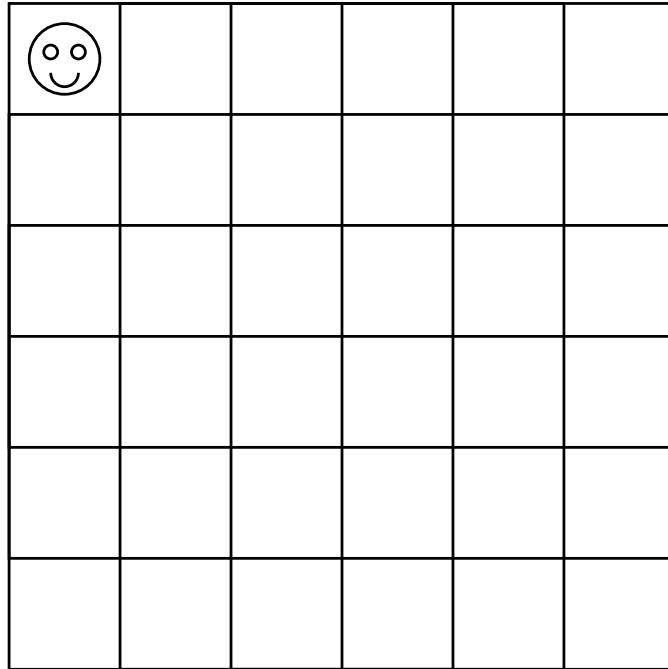
4. Using the grid below and motif of your choice, construct a wallpaper pattern with rotation of order 4 exactly.
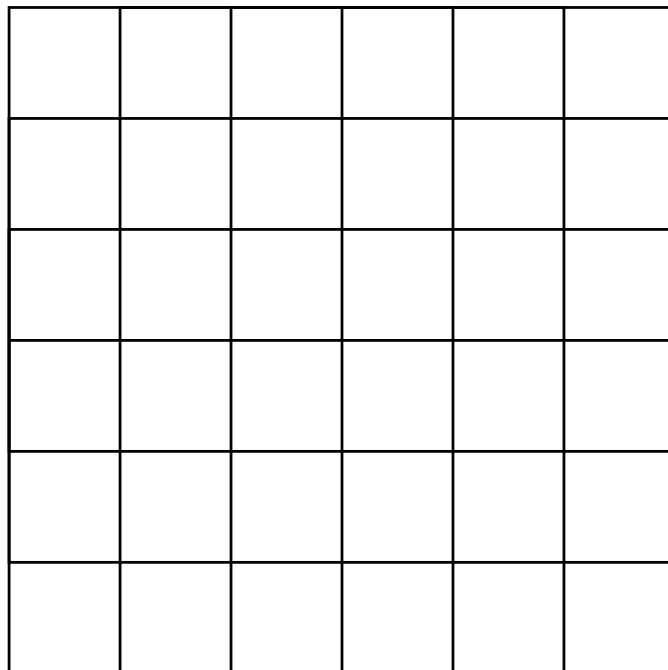
5. Using the grid below and motif of your choice, construct a wallpaper pattern with rotation of order 3 exactly.

6. Using the grid below and motif of your choice, construct a wallpaper pattern with rotation of order 6 exactly.