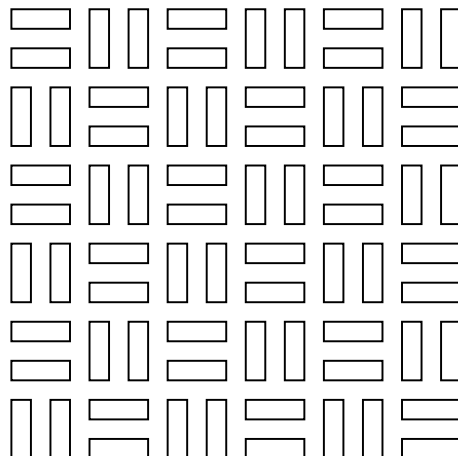


Name: _____

¶ 1. **Wallpaper patterns.** Wallpaper patterns are formed by repetition of a motif in such a way as to cover the plane. The wallpaper pattern refers not to the motif, but to the way in which is repetition is structured across the plane. As with rosette and frieze patterns, the group of a wallpaper pattern, that is, the group of rigid motions that take the pattern onto itself, is formed by translations, rotations, reflections, and glide reflections. Unlike the rosette and frieze patterns, the group of a wallpaper patterns includes translations in two independent directions. The mathematical theory establishes that there are exactly 17 different wallpaper groups.

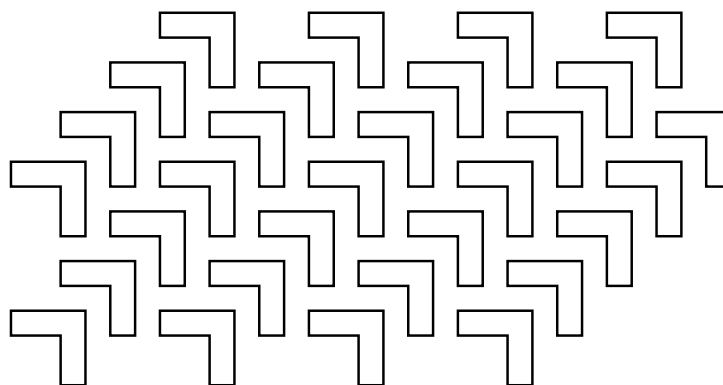
¶ 2. What isometries does this wallpaper pattern have?



- (a) Translation and reflection only
- (b) Translation and rotation only
- (c) Translation, rotation, and reflection

If there are reflections, indicate the reflection lines. If there are rotations, indicate the center of rotation.

¶ 3. What isometries does this wallpaper pattern have?



- (a) Translation and reflection only
- (b) Translation and rotation only
- (c) Translation, rotation, and reflection

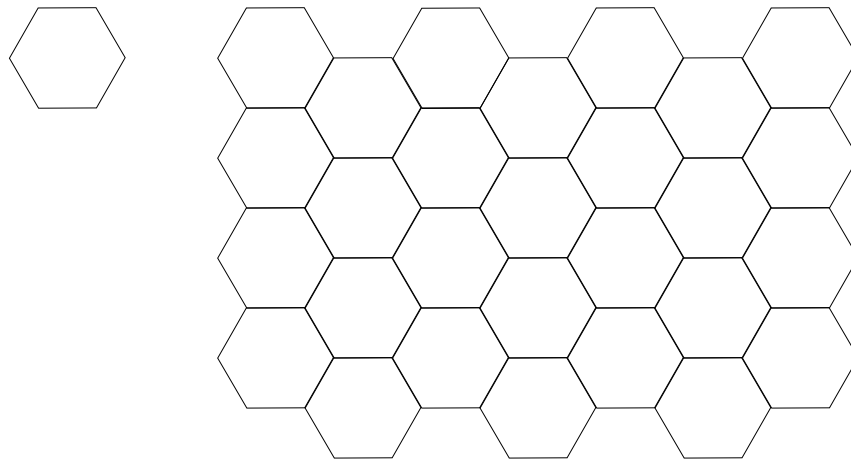
If there are reflections, indicate the reflection lines. If there are rotations, indicate the center of rotation.

Name: _____

¶ 4. The simplest wallpaper group is that which consists only of translations in two different directions. To design a patten with such group, start with a motif that has no symmetry (no mirror lines or rotations), and translate it along tow different directions.

¶ 5. In the next example we construct a wallpaper pattern whose group has lines of reflection and glide reflection meeting at right angles. The original motif has no symmetries, so we reflect it and glide reflect it so as to give those symmetries to the full wallpaper pattern.

¶ 6. The next example contains a rotation by 60 degrees. For that, we start with a motif that has rotation symmetry of order 6 (and no mirror lines, and the repeat that patten on a hexagonal grid.

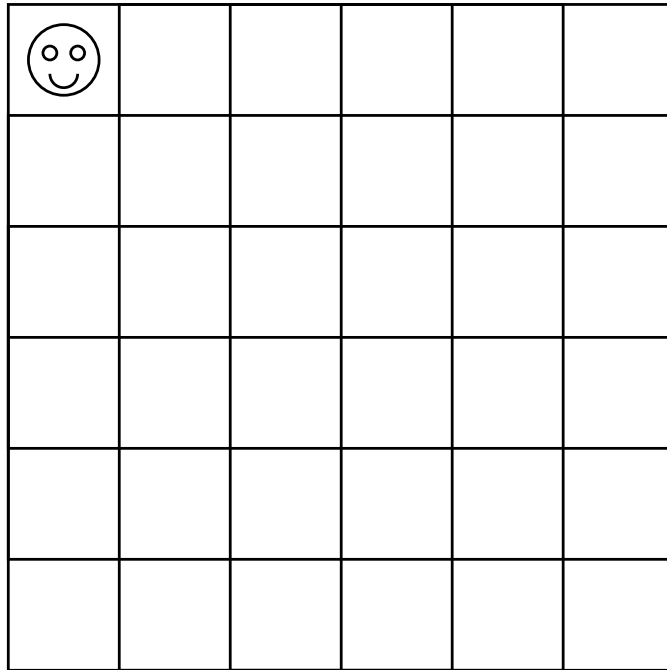


The center of each hexagon is a canter of rotational symmetry for the full pattern. But other centers of symmetry appears as well because the multiple combinations of motions within the group of the pattern. For example, the points where three hexagons meet are centers of rotations by 120 degrees, and the mid points of each of the sides of an hexagon are centers of rotation of 180 degrees.

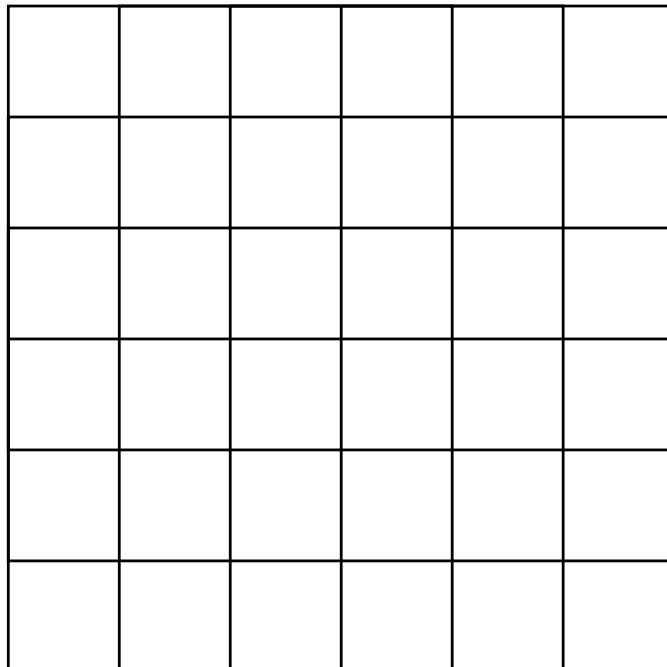
Note that each of the hexagon can be divided into 6 equilateral triangles, and that two such triangles combine to form a rhombus that can be used as basic motif or fundamental domain.

¶ 7. **Rotations in wallpaper patterns.** Patterns may have rotations. We have seen that rosette patterns can have rotation of any order. On the other hand, frieze patterns can only have rotations of order 1 (full turn) or 2 (half turns). It turns out that wallpaper patterns can have rotations of order 1, 2, 3, 4, and 6 only.

(a) Using the grid and motif below, construct a wallpaper pattern with 2-fold rotational symmetry.

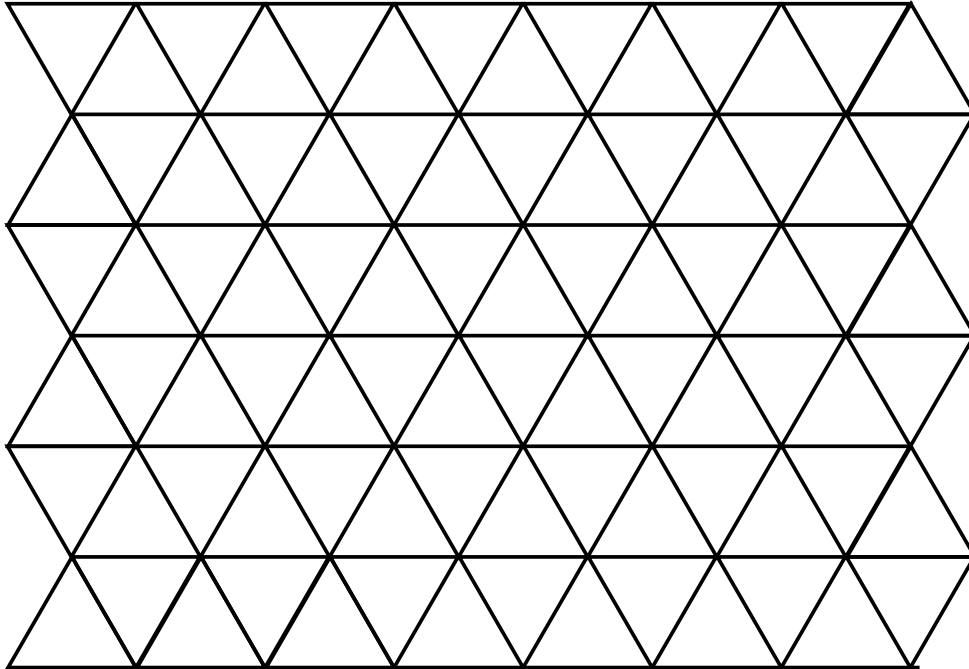


(b) Using the grid below and motif of your choice, construct a wallpaper pattern with 4-fold rotational symmetry.

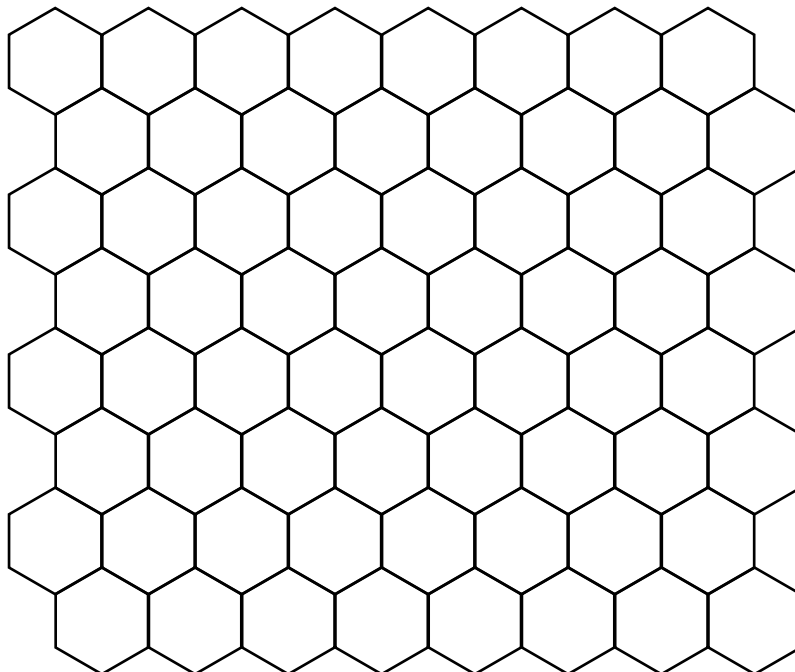


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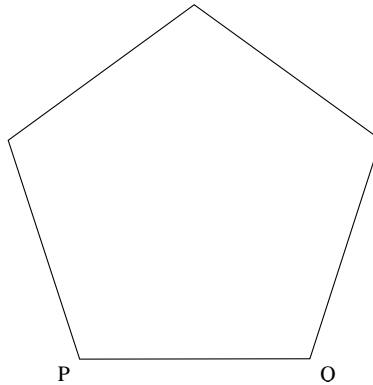
¶ 8. Using the grid below and motif of your choice, construct a wallpaper pattern with 3-fold rotational symmetry.



¶ 9. Using the grid below and motif of your choice, construct a wallpaper pattern with 6-fold rotational symmetry.



¶ 10. The basic cell (fundamental domain), and the pattern of mirror lines, glide reflection lines, and centers of rotation completely determine the structure of the whole pattern. One important feature of wallpaper groups is that only 2-, 3-, 4-, or 6-fold rotational symmetry (that is, rotations by 60, 90, 120 and 180 degrees) is possible. This fact is known as the crystallographic restriction. To see why 5 fold symmetry is not possible, we do the following. Among all center of 5-fold symmetry, chose two of them as closest as possible (the size of the basic motif imposes a limit as to how close such centers could be). Say that they are P and Q , as in the figure below.



Rotate P about Q by 72 degrees to obtain the point R , and then rotate Q about P by 72 degrees to obtain the point S . The points R and S are also center of 5-fold symmetry, but it is clear that $RS < PQ$.

Similar arguments can be used to exclude 7-fold rotational symmetries, and so on.

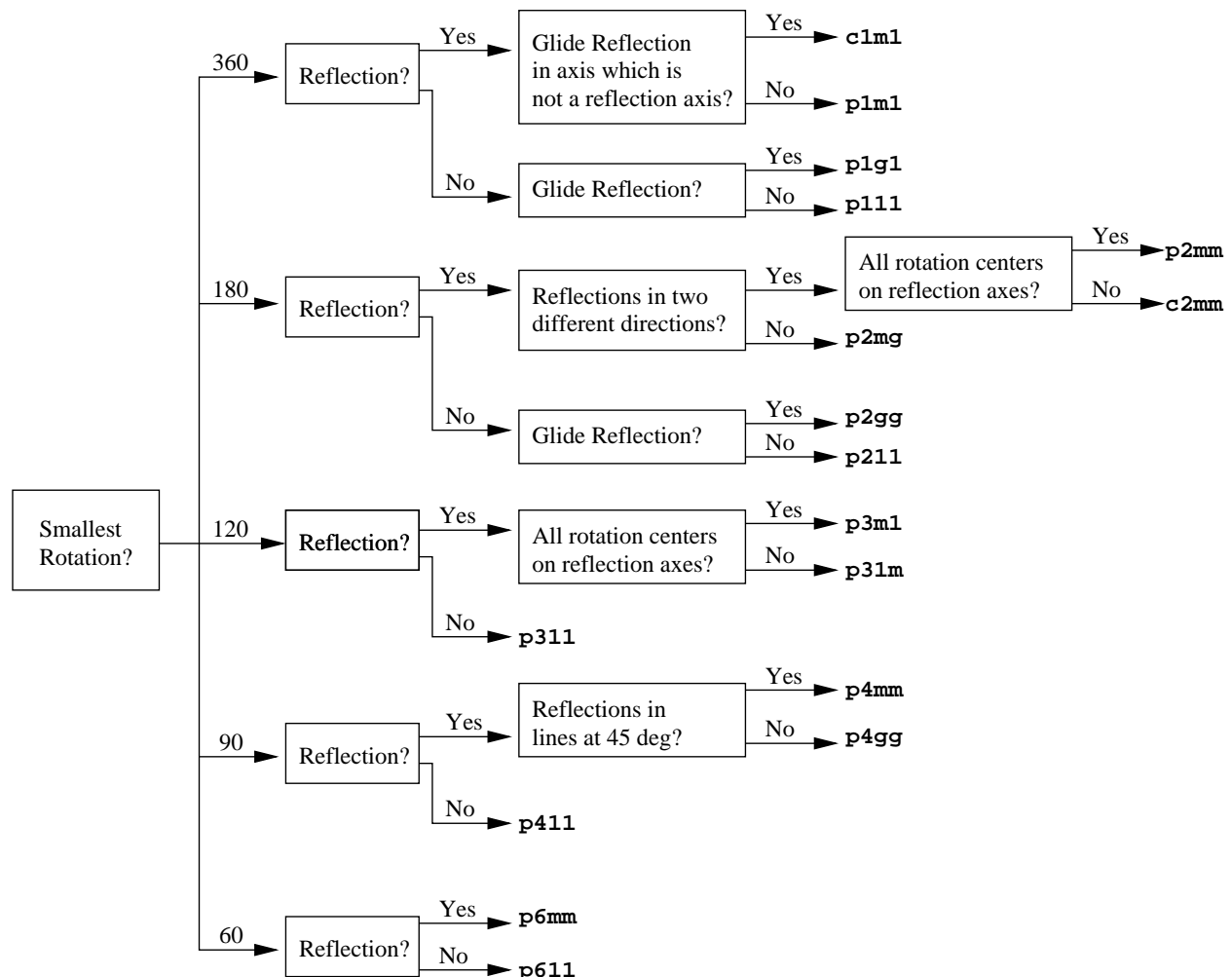
Name: _____

¶ 11. Wallpaper patterns: Crystallographic Notation.

Crystallographers have standard notation for the 17 different wallpaper patterns. The full notation consists of 4 symbols:

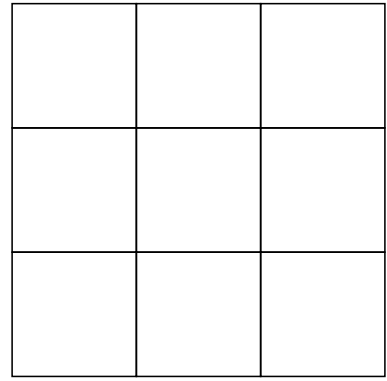
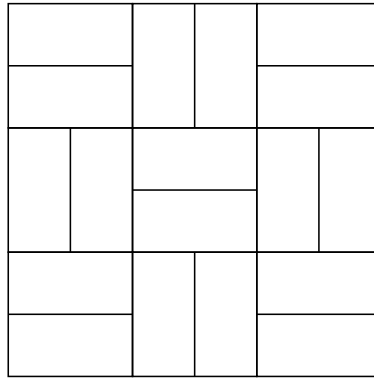
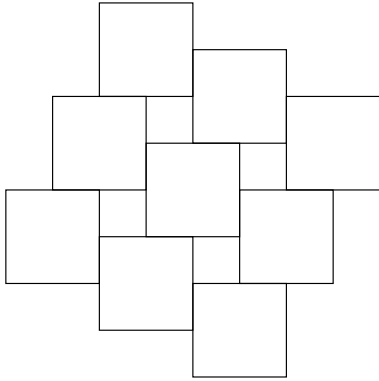
- (a) The first symbol is either a c (for *centered*) if all rotation centers lie on reflection lines, or a p (for *primitive*) otherwise
- (b) The second symbol indicates rotational symmetry. It is either of order 1, 2, 3, 4, or 6, corresponding to 360, 180, 120, 90, or 60 degrees. The symbol is the largest applicable number: for example, if symmetries of 360, 120 and 60 degrees are present, then the second symbol is 6.
- (c) The third symbol is either m, g, or 1, corresponding to the presence of *mirror*, *glide*, or no reflection symmetry.
- (d) The fourth symbol is either m, g, or 1, and describes the symmetry relative to an axis at an angle to the symmetry axis of the third symbol. That angle would be determine by the second symbol: if that second symbol is 1 or 2, then look for reflection or glide line at angle 180°; if the second symbol is 4, then look for lines at angle 45°, and if the second symbol is 3 or 6, then look for lines at angle 60°.

This notation is summarized in the following flowchart:

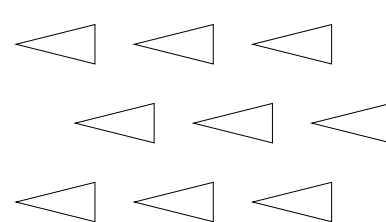
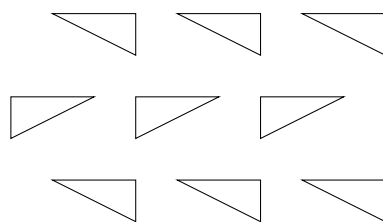
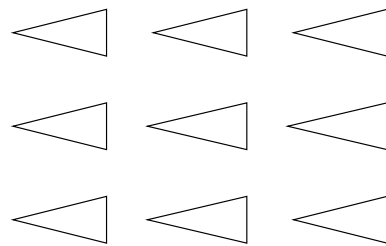
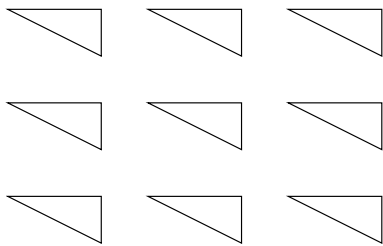


Name: _____

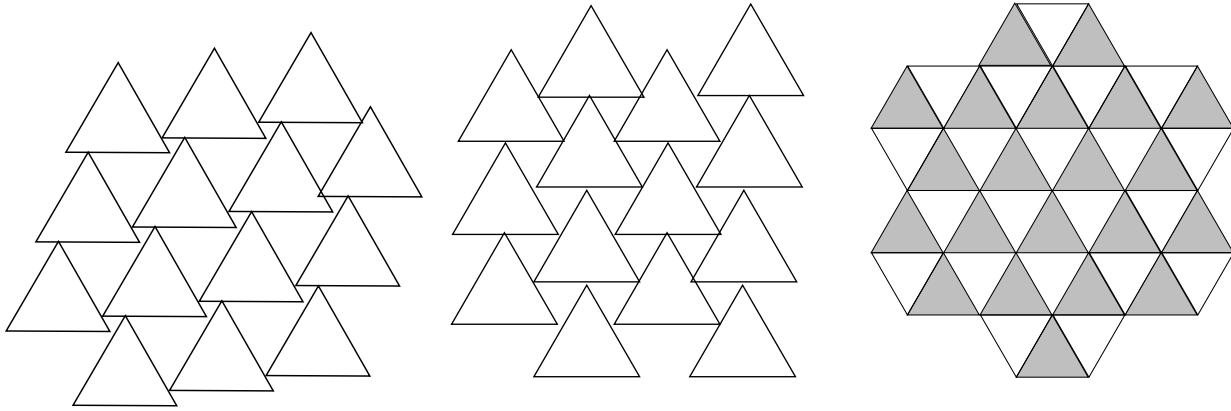
¶ 12. Determine the crystallographic notation for the wallpaper groups of the following patterns. Then find a fundamental region for each.



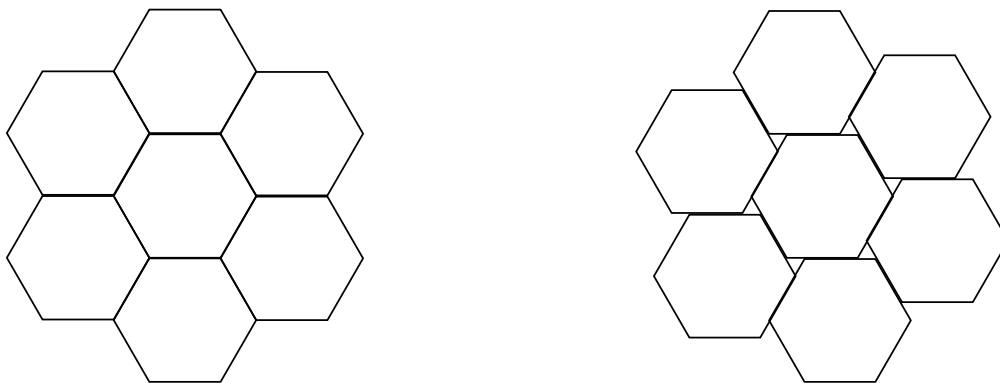
¶ 13. Determine the crystallographic notation for the wallpaper groups of the following patterns. Then find a fundamental region for each.



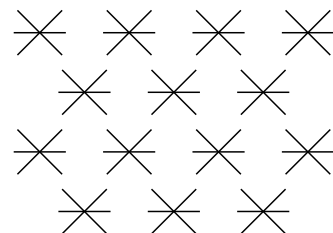
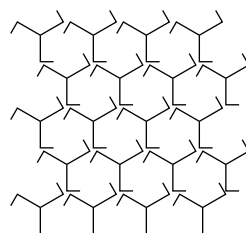
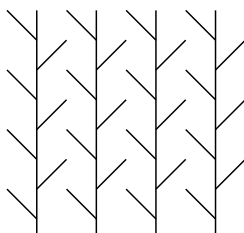
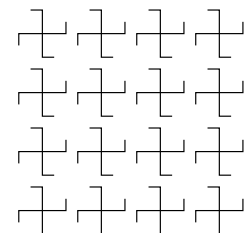
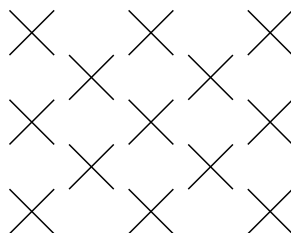
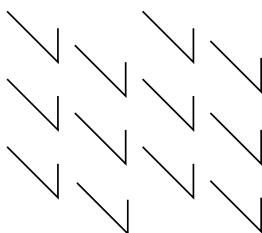
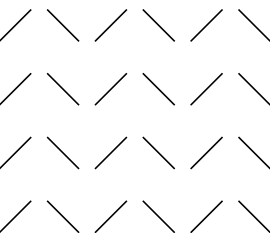
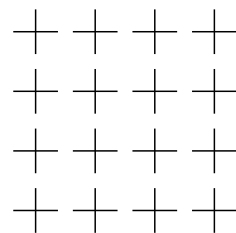
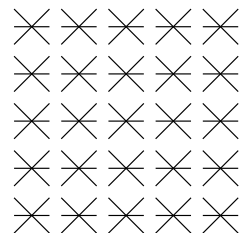
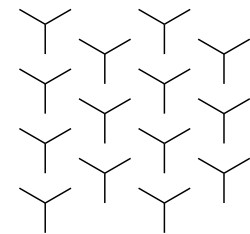
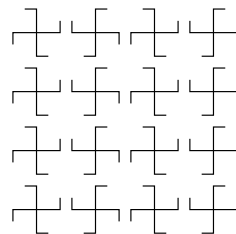
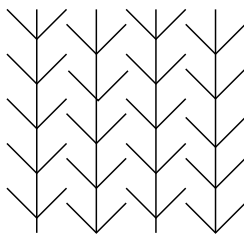
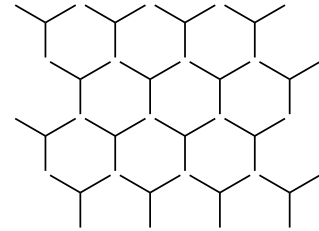
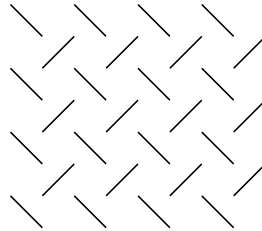
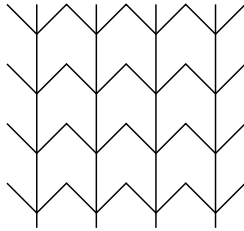
¶ 14. Determine the crystallographic notation for the wallpaper groups of the following patterns. Then find a fundamental region for each.



¶ 15. Determine the wallpaper group of the following patterns.



¶ 16. Use the flowchart on the previous page to identify the crystallographic notation of the following wallpaper patterns.



¶ 17. Out of the 17 possible wallpaper patterns, there are 2 missing. Can you identify them and draw a section of each of them?

¶ 18. **M.C. Escher** was a Dutch artist whose work illustrates in a dramatic way all the wallpaper patters. Use the crystallographic pattern flowchart to identify the following patterns from Escher's work.

