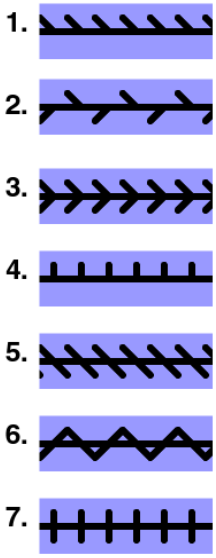
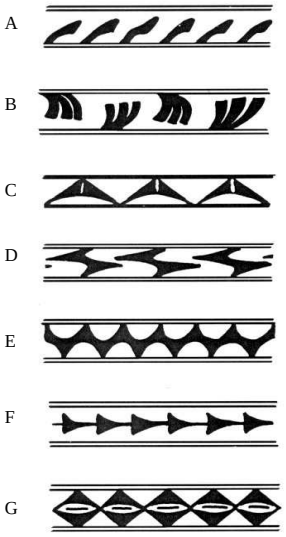
¶ 1. Frieze Patterns. Frieze patterns, or strip patters, are formed by repetition of a motif along a line. You encounter examples of these patterns in ribbons, needlework, pottery, rubber stamps, animal tracks, and multitude of other places.

These patterns then have symmetry in one direction. In fact, any rigid motion that preserves such pattern must preserve the direction line of the pattern.





For simplicity, we always position the frieze pattern so that its repetition runs horizontally. It may be that the pattern has no other rigid motion that preserve it other than translation along its line of repetition, like Pattern (1) above.

¶ 2. What are the possible mirror lines of a frieze pattern?

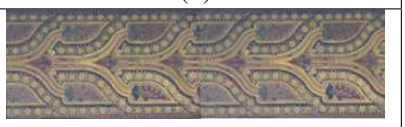
¶ 3. What kind of rotational symmetries can a frieze pattern have?

¶ 4. What are the rigid motions that preserve this pattern?

¶ 5. What are the rigid motions that preserve this pattern?

¶ 6. What are the rigid motions that preserve this pattern?

¶ 7. What are the rigid motions that preserve the patterns below?

 (A)	 (B)	 (C)
 (D)	 (E)	 (F)
 (G)	 (H)	 (J)
 (K)	 (L)	 (M)
 (N)	 (P)	 (Q)

¶ 8. Conway (a mathematician at Princeton) devised the following names for the different frieze groups. Determine the rigid motions of each of the 7 patterns below:

(a) Hop

(b) SpinHop

(c) Jump

(d) Sidle

(e) Step

(f) SpinJump

(g) SpinSidle

Crystallographic notation

The crystallographic format for describing the rigid motions of a pattern is `p---`

(a) The first symbol is always `p`, for "periodic."

(b) The second symbol is `m` is there is a vertical mirror line, `1` if not.

(c) The third symbol is `m` if there is a horizontal mirror line, `a` if there is a glide reflection (but not horizontal mirror), and `1` if there is neither horizontal mirror nor glide reflection.

(d) The fourth symbol is `2` if there is a half-turn, `1` if there is no half-turn.

¶ 9. What is the crystallographic symbold that corresponds to each of the terms in Conways's notation for the frieze patterns?

Conway	Cristallographic
Hop	
SpinHop	
Jump	
Sidle	
Step	
SpinJump	
SpinSidle	

¶ 10. Using the pattern  as basic pattern, construct frieze patterns with the following crystallographic symbols

(a) `pmm2`

(b) `pma2`

(c) `pm11`

(d) `p1m1`

(e) `p1a1`

(f) `p112`

(g) `p111`

¶ 11. Besides translations, there are four other kind of symmetries that a frieze pattern can have

(H) (horizontal) reflection in a mirror line along the direction of the frieze.

(V) (vertical) reflection on a mirror line perpendicular to the direction of the frieze.

(R) rotation by 180 degrees.

(G) glide reflection.

Frieze patterns are classified according to whether each of the four rigid motions appears. We can construct a flowchart to do the classification (repretition of the motif in the bottom row along one direction will give you the appropriate frieze pattern):

Horizontal mirror line?															
YES								NO							
Glide reflections?															
YES				NO				YES				NO			
Half turns?															
YES		NO		YES		NO		YES		NO		YES		NO	
Vertical mirror line?															
YES	NO	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO
H							D	~			bp		S	A	F

¶ 12. Using the letter b as your basic pattern, construct frieze patterns that have

(NNNN) Translations only

(YNNN) Translations and horizontal reflections

(NYNN) Translations and vertical reflections

(NNYN) Translations and half-turns

(NNNY) Translations and glide reflections

¶ 13. While there are a total of 16 possible combinations ($2\times2\times2\times2$), it turns out that there are only 7 pattern types. Some theoretical combinations are not realizable.

The above accounts for 5 of the 16 combinations. It is when you try to have 2 of the 4 rigid motions (2 Yes and 2 No) that you find that it becomes impossible, for that introduces another rigid motion. For example, if you choose 2 reflections (one vertical and one horizontal) then you get automatically a half turn (accounting for two combinations YYNN and YYYN), and if you chose either reflection but not the other, together with a half turn, then you automatically get the other reflection (accounting for two more combinations YNYN and NYYN).

If you choose a vertical reflection and a glide, then you get a half turn (accounting for two more NYNY and NYYY).

¶ 14. Conway et al. devised numerical versions of frieze patterns that can be used for practicing with addition and multiplication. The additive version requires you to first fill each rhombus in the border with 0's, and then fill in all the ramining rhombi with numbers so that each little diamond formed by four rhombi like

$$\begin{array}{ccc} & B & \\ A & & D \\ & C & \end{array}$$

satisfies $A + D = B + C + 1$. The multiplicative version requires you to fill the border with 1's, and then fill in the remaining rhombi with A, B, C, D so that $AD = BC + 1$.