MATH 450 **Advanced Mathematics for Engineers I** **Fall 2006**

Review Sheet #2 for Final Exam

Problem 1. Determine whether the following statements are True or False. Justify your answer.

(a) If $F\{f(x)\} = \hat{f}(\omega)$ and $F\{g(x)\} = \hat{g}(\omega)$, then $F\{f(x) + g(x)\} = \hat{f}(\omega) + \hat{g}(\omega)$.

(b) If $F^{-1}\{\hat{f}(\omega)\} = f(x)$ and $F^{-1}\{\hat{g}(\omega)\} = g(x)$, then $F^{-1}\{\hat{f}(\omega)\,\hat{g}(\omega)\} = f(x)\,g(x)$.

(c) If $\mathcal{C}_1$ and $\mathcal{C}_2$ are two different contours with same starting and ending points (*i.e.*, $\mathcal{C}_1$ and $\mathcal{C}_2$ both start at z_1 and end at z_2), then $\oint_{\mathcal{C}_1} \bar{z}\, dz = \oint_{\mathcal{C}_2} \bar{z}\, dz$.

(d) The complex valued function $f(z) = z^2$ is differentiable everywhere in $\mathbb{C}$.

(e) $\cosh(z) = \frac{e^z + e^{-z}}{2}$.

Problem 2. Find the image of the domains described below under the given transformation:

(a) $D = \{z \in \mathbb{C} \;:\; |z| < 1\}$, $w(z) = \frac{\sqrt{2}}{2}(1+i)\frac{z+1}{z-1}$.

(b) $D = \{z \in \mathbb{C} \;:\; |z-1| < 1\}$, $w(z) = \frac{1}{z}$.

(c) $D = \{z \in \mathbb{C} \;:\; -\frac{\pi}{4} < \text{Arg}z < \frac{\pi}{4}\}$, $w(z) = \frac{z}{z-1}$.

Problem 3. Solve Laplace equation, $\nabla^2\Phi = 0$, over the domain inside the circle $x^2 + y^2 = 4$ and outside the circle $(x-1)^2 + y^2 = 1$, with boundary conditions $\Phi = 10$ on the outer circle, and $\Phi = 30$ on the inner circle, give your answer in terms of x and y. [Hint: use a bi-linear transformation to map the given domain into an infinite strip.]

Problem 4. Solve Laplace equation, $\nabla^2\Phi = 0$, over the domain outside the circle through the points 2, -2, and $4i$, and the circle through the points 2, -2, and $-4i$, with boundary conditions $\Phi = 20$ on the upper circle, and $\Phi = 0$ on the lower circle, give your answer in terms of x and y. [Hint: use a bi-linear transformation to map the given domain into an infinite wedge, then use polar coordinates to solve the Laplace equation.]

Problem 5. Find bi-linear transformations that map:

(a) The lower half plane to the disk $|w+1| < 1$ [Hint: do it as a composition of rotations, scalings, translations, and/or inversions.]

(b) The unit disk $|z| < 1$ onto the right half plane and taking $z = -i$ to the origin.

Problem 6. Evaluate the following integrals where $\mathcal{C}$ is the contour displayed below.

(a) $\oint_{\mathcal{C}} \frac{e^z}{(z^2+1)(z-i)(z-4)}\, dz$

(b) $\oint_{\mathcal{C}} \frac{e^{-z}}{z\,(z-3)^2(2z+5)}\, dz$

(c) $\oint_{\mathcal{C}} \frac{e^z}{z^2\,(z^2-9)}\, dz$

(d) $\oint_{\mathcal{C}} \frac{\cos z}{z\,(z+i)(z-i)}\, dz$

(e) $\oint_{\mathcal{C}} \frac{\sin z}{z\,(2z+1)(z-1)}\, dz$

(f) $\oint_{\mathcal{C}} \frac{e^z}{(2z-3)(z-i)(z+2)}\, dz$