MATH 450 **Advanced Mathematics for Engineers I** **Fall 2006**

Review Sheet #1 for Midterm Exam

Problem 1. Determine whether the following statements are True or False. Justify your answer.

(a) If $y_1(x)$ and $y_2(x)$ are two solutions of

$$y'' + 2y' + y = 0, \qquad x \in [0, L],$$

then $y(x) = \alpha y_1(x) + \beta y_2(x)$, $\alpha, \beta \in \mathbb{R}$ is the general solution of the equation.

(b) $\sin ix = \sinh x$

(c) $\cosh ix = \cos x$

(d) For any complex valued function $w(z)$, $w(\bar{z}) = \overline{w(z)}$

(e) The power series $\sum_{n=0}^{\infty} \frac{-1}{n+1} x^{n+1}$ converges to $\ln(1-x)$, for $x \in [-1, 1)$

(f) $|e^{ix}| = 1$ for all $x \in \mathbb{R}$

(g) If $z = x + iy$, then $|e^z| = e^x$

(h) Let

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if} \quad -\pi < x < 2 \\ 2 & \text{if} \quad 2 < x \le \pi. \end{cases}$$

The Fourier series $a_0 + \sum_{n=0}^{\infty}(a_n \cos nx + b_n \sin nx)$, with

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\cos nx\,dx, \text{ and } b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\sin nx\,dx,$$

converges to $f(x)$ for all $x \in [-\pi, \pi]$.

(i) The function $f(x) = x^4 + \cos 3\pi x$ can be written as $f(x) = f_e(x) + f_o(x)$, where $f_e(x)$ is even, and $f_o(x)$ is odd.

(j) If $f(x)$ is odd, then $f'(x)$ is also odd

Problem 2. Use the method of undetermined coefficients to obtain the general solution of

(a) $y'' + y = 3\sin 2x - 5 + 2x^2$

(b) $y'' - 2y' + y = x^2\, e^x$

Problem 3. In section 4.2 we saw how to find a power series solution for second order linear ODEs of the form $y'' + p(x)y' + q(x)y = 0$. Consider the equation

$$x^2 y'' - y = 0, \qquad x_0 = 2.$$

(a) Identify $p(x)$ and $q(x)$. Explain.

(b) Seek a power series solution around x_0.

(c) Write your solution as the linear combination of two solutions $y_1(x)$ and $y_2(x)$.

(d) Show that $y_1(x)$ and $y_2(x)$ are linearly independent.

Problem 4. Find the Fourier series of $f(x) = |x|$, $x \in [-\pi, \pi]$.

Problem 5. Consider the function

$$f(x) = \begin{cases} -1 & \text{if} \quad x \in [-\pi, 0] \\ 1 & \text{if} \quad x \in (0, \pi]. \end{cases}$$

(a) Find its Fourier series.

(b) To what value does the series converge at $x = 0$?

Problem 6. Consider the function $f(x) = 2 + x$ on the interval $x \in [0, 3]$.

(a) Find an odd periodic extension, $f_{ext}(x) = f_{ext}(x + L)$, with period $L = 6$.

(b) Draw the graph of $f_{ext}(x)$ and label the lines of (anti)symmetry of the graph.

(c) Find the coefficients of a Fourier series that converges to $f_{ext}(x)$.

Problem 7. Consider the Stum-Liouville problem

$$y'' + \lambda y = 0, \;\; y'(-1) = 0, \;\; y'(1) = 0.$$

(a) Identify $p(x)$, $q(x)$, $w(x)$, α, β, γ, and δ.

(b) Find its eigen pairs $(\lambda_n, \phi_n(x))$.

(c) Find the coefficients a_n of the expansion $f(x) = \sum_{n=1}^{\infty} a_n \phi_n(x)$ for the function

$$f(x) = \begin{cases} 0 & \text{if} \quad x \in [-1, 0] \\ 1 & \text{if} \quad x \in (0, 1]. \end{cases}$$

Problem 8. Use the method of separation of variables to solve the heat equation, $u_t = \alpha^2 u_{xx}$, with boundary conditions $u(0, t) = 25$ and $u_x(4, t) = 0$, and initial conditions $u(x, 0) = f(x) = 25$.