Quadratic Surfaces and Parametric Space Curves

Preliminaries

The objective of this lab is to familiarize yourself with the shapes and properties of cylinders, quadratic surfaces (section 13.6) and parametric space curves (section 14.1).

Start Maple by clicking on the icon that appears either on your desktop or the dock. If you want the plots displayed on separate windows, go to the **Maple** menu and select **Preferences**. A dialog window will pop up, click on **Display** and select **window** for **Plot display**, then click the **Apply to Session** button at the bottom of the box.

At the prompt, type:

```
> with(plots):
```

This will allow you to use different plotting commands available in Maple.

Don't forget to type a semicolon (;) (or a colon (:) if you don't want to see the output) at the end of each command!!!

Plotting Cylinders and Quadratic Surfaces

To plot cylinders and quadratic surfaces in Maple we can use the `implicitplot3d` command. The syntax for this command is (no need to type this):

> implicitplot3d(*expression*, x=a..b, y=c..d, z=e..f, *options*);

where *expression* is some expression involving `x, y,` and `z`, and `a, b, c, d, e,` and `f` real constants specifying the ranges of `x, y,` and `z`. For example, a cylinder can be plotted using:

```
> implicitplot3d(x^2 + y^2 = 1, x=-1..1, y=-1..1, z=-3..3);
> implicitplot3d(x^2 + z^2 = 1, x=-1..1, y=-3..3, z=-1..1);
```

Try now some of the quadratic surfaces described in the table at the end of section 13.6 (pg. 872). There are three types you should get familiar with and be able to recognize and sketch (roughly):

Ellipsoids:

```
> implicitplot3d((1/4)*x^2 + (1/9)*y^2 + (1/16)*z^2 = 1, x=-2..2, y=-3..3, z=-4..4);
```

Paraboloids:

```
> implicitplot3d(z=(1/4)*x^2 + (1/9)*y^2, x=-4..4, y=-6..6, z=0..6, scaling=constrained);
> implicitplot3d(y=(1/4)*x^2 + (1/9)*z^2, x=-4..4, y=0..6, z=-6..6, scaling=constrained);
> implicitplot3d(x=4*(y-2)^2 + (z-2)^2, x=0..10, y=-1..5, z=-2..6);
```

Cones:

```
> implicitplot3d(z^2=(1/4)*x^2 + (1/9)*y^2, x=-3..3, y=-4..4, z=-1..1);
> implicitplot3d(y^2=(1/4)*x^2 + (1/9)*z^2, x=-3..3, y=-1..1, z=-4..4);
```

Other quadratic surfaces that we'll come across are hyperboloids:

Of One Sheet:

```
> implicitplot3d((1/4)*x^2 + (1/9)*y^2 - z^2 =1, x=-5..5, y=-7..7, z=-2..2);
> implicitplot3d(-x^2 + (1/9)*y^2 + (1/4)*z^2 =1, x=-2..2, y=-7..7, z=-5..5);
```

Of Two Sheets:

```
> implicitplot3d((1/4)*x^2 - (1/9)*y^2 - z^2 =1, x=-4..4, y=-6..6, z=-2..2);
```

And **Hyperbolic Paraboloids:**

```
> implicitplot3d(z = - (1/4)*x^2 + (1/9)*y^2, x=-5..5, y=-6..6, z=-2..2);
```

Plotting in Cylindrical and Spherical Coordinates

Maple can also plot surfaces expressed in cylindrical and spherical coordinates using the `plot3d` command:

```
> plot3d(r(theta,z), theta=a..b, z=c..d, coords=cylindrical);
> plot3d(rho(theta,phi), theta=a..b, phi=c..d, coords=spherical);
```

Here are some examples:

Cylindrical Coordinates:

```
> plot3d(1, theta=0..2*Pi, z=-3..3, coords=cylindrical);
> plot3d(z, theta=0..2*Pi, z=0..3, coords=cylindrical);
```

Spherical Coordinates:

```
> plot3d(1, theta=0..2*Pi, phi=0..Pi, coords=spherical);
> plot3d(sin(theta)*cos(phi), theta=0..2*Pi, phi=0..Pi, coords=spherical);
```

Plotting Parametric Curves in 3d

The Maple command `spacecurve` allows you to plot parametric curves in space. Try, for instance, the parametric equations of a line:

```
> spacecurve([1+ t, 2 - 3*t, -1 - t, t=0..10]);
```

Here are some other examples:

```
> spacecurve([cos(5*t), sin(5*t), t, t=0..Pi]);
> spacecurve([cos(5*t), sin(5*t), log(t), t=0..Pi]);
> spacecurve([t, 1/(1+t^2), t^2, t=-10..10]);
> spacecurve([cos(t), sin(t), sin(5*t), t=0..2*Pi]);
```

That's all!