Applied Project: Kepler's Laws of Planetary Motion

Due: Wed. Mar. 9, 2005

You can add up to five extra points to your final score in the course by completing this project.

Preliminaries:

Before you start working on the problems described below, you should familiarize yourself with Kepler's laws of planetary motion. The information in the textbook (Section 14.4) should be enough to work out the problems listed/described below, but further reading is highly recommended (see references below).

The book proves Kepler's first law (pg. 912). It would be very difficult to complete this project without first reading and understanding that proof. Before attempting to solve the problems below, you should make sure that you've understood the coordinate system used to describe the orbits of planets, the notation (i.e., understand what every vector in Fig. 8, pg. 913 represent), and the notation of Newton's law of gravitation. To this end, I recommend you take some notes as you read the proof of the first law. You may want, for example, to write a list describing the variables and constants that are used in the proof, and, possibly, replicate the proof as you read it, adding comments that help you understand it.

Grading Policy:

You need to show all your work to receive credit (full or partial). Follow and label the steps suggested in the problems and organize your work clearly.

The maximum score you can get in this assignment is 30 points, 15 points per problem (I will divide it by 6 to figure out how many points to add to your final grade). Partial credit will be given for right ideas that may have lead you to the proof even if you couldn't complete it, but arriving at the right conclusions based on incorrect assumptions/statements will not earn you any credit.

Part 1: Problem 1 (prove Kepler's Second Law) **or** Problem 2 (prove Kepler's Third Law) on page 916 of Stewart. Your choice.

Part 2: Solve the following problem presented by N. Grossman in his book "The Sheer Joy of Celestial Mechanics" (Exercise V.2, pg. 91): In the movie "Journey to the Far Side of the Sun" [1969], an astronaut blasts off from Earth to begin a journey to Venus. An explosion occurs soon after liftoff, and the astronaut wakes up in a hospital. Soon he begins to notice disturbing differences in chirality[1]: most people are left-handed and wear wedding rings on the right hand, coats button on the 'wrong' side, and so on. He learns that he is on Anti-Earth, which is the same mass and shape as the Earth and which revolves around the Sun in the same orbit but always on the other side of the Earth's diameter. Anti-Earth cannot be seen through Earth-based telescopes. Anti-Earth does know about Earth and –naturally– is hostile to it. An annihilating invasion is about to be launched. The astronaut manages to escape with the help of a pretty nurse, and they get back to warn Earth.

Use the following plan to show that the existence of an Anti-Earth could have (and would have) been deduced long ago. [a] Suppose that the Earth is represented as a point of mass m revolving in a

[1] see reference [3] for a definition of chirality

circular orbit of radius a around a center of force (the Sun) of mass M. [b] Increase the usual Newtonian attraction of the Sun by the attraction of the hidden Anti-Earth and compute the change in the length of the Earth year under the new central force. (How many seconds longer or shorter will the year be under the Anti-Earth influence than the 'real' year is?)

[c, show that] The presence of the Anti-Earth would be seen from the Earth as an increase in the constant GM, and that changed constant would then change the periods of the other planets when calculated from Kepler's Third Law. [d] But those periods are observed to be in harmony with the period of the Earth when it is calculated from Kepler's Third Law. [Use this to conclude:] no Anti-Earth.

Additional references:

[1] Nathaniel Grossman. *The Sheer Joy of Celestial Mechanics.* Birkhäuser, Boston, 1996.

[2] Nasa. *Johannes Kepler: His Life, His Laws and Times.*
`http://www.kepler.arc.nasa.gov/johannes.html`

[3] `http://www2.math.uic.edu/~fields/anim/8-knot.html`