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A Network Model of Immigration and Coexistence

By Yao-li Chuang, Tom Chou, and Maria R. D'Orsogna

In the summer of 2015, more than one million refugees from the Middle East, Central Asia, and Africa arrived in Europe via dangerous routes across the Mediterranean Sea and the Balkans [7]. German Chancellor Angela Merkel welcomed the newly arrived with an enthusiastic "Wir schaffen das"—"We can do this"—embodying the collective spirit of optimism that pervaded Europe at the time. The vast majority of migrants were fleeing civil wars, brutal dictatorships, or religious persecution; others were seeking better economic opportunities. Preferred destinations among the more prosperous nations included Germany, Sweden, and the U.K., whereas European law and geography placed most of the burden of processing asylum claims on border nations such as Italy, Greece, and Hungary, which were not prepared to cope with such unprecedented numbers of new arrivals.

Measures including the forced return of illegal migrants to Turkey in exchange for economic concessions attempted to stem the flow. Hungary closed its borders, and Italy eventually closed its ports. European lawmakers were unable to devise a clear burden-sharing system among member states; at the same time, refugees and smugglers quickly found and exploited new migrant routes as existing ones saw increased patrolling and border controls. Eventually, the perception of an unmanageable crisis touched the entire continent. Discontent among the general public grew, as did discussions on safety, integration, European identity, secularism, resource availability, and the role of non-governmental organizations. As a result, the issue of migration has dominated elections across Europe over the past few years, and nationalist parties have enjoyed large gains in many countries.

It is within this larger sociopolitical context that many migrants have settled into European cities, each with their own personal story of adaptation, hurdles, discoveries, kindness, and hostility from strangers. Outcomes have thus far been mixed; refugees have successfully integrated in many communities from Italy to Sweden, but in some cases there have been challenges and mistrust. A common observation is that newcomers who do not adapt well—either by circumstance, aversion from natives, lack of resources and/or motivation, etc.—tend to self-segregate and create insular communities [5]. While these enclaves provide immigrants with advantages and a sense of belonging, they may also prevent them from fully integrating into the larger society.

The fateful summer of 2015 presented a most daunting question: Is it possible to integrate vast numbers of asylum seekers in a way that is constructive for natives and migrants alike? This issue is also at the core of our recent mathematical modeling work, wherein we offer a quantitative setting for the study of immigration and coexistence [2]. We consider two communities—"hosts" (N_{h}) and "guests" (N_{g})—as nodes that interact on a social network, both seeking to improve their socioeconomic status. Each node i carries a time-dependent attitude x_i^t towards others and is assigned a utility function U_i^t that depends on its m_i^t connections. Over time, nodes adjust attitudes and reshape links to increase their utility; as a result, the network evolves towards either integration or segregation between hosts and guests. While the utility function follows game theoretic rules, attitudes are assumed to evolve

See **Immigration** *on page 2*

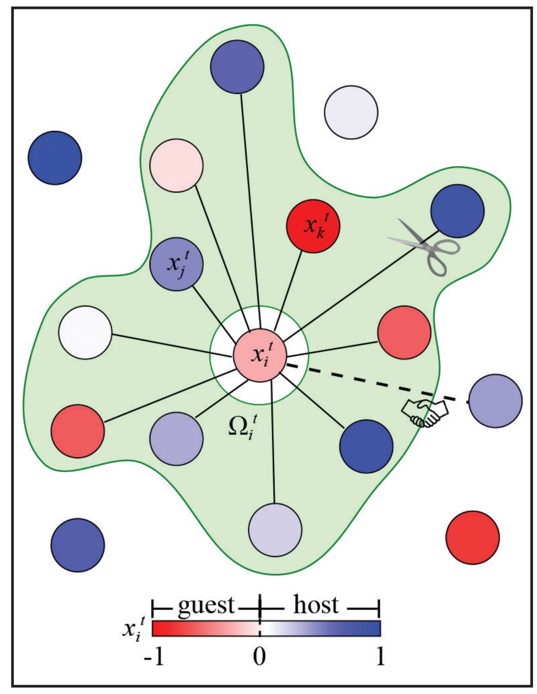

Figure 1. *Each node i is characterized by a variable attitude $-1 \leq x_i^t \leq 1$ at time t. Negative (red) values indicate guests and positive (blue) values represent hosts. The magnitude $|x_i^t|$ represents node i's degree of hostility towards members of the other group. All nodes j,k that are linked to node i represent the green-shaded social circle Ω_i^t of node i at time t. The utility U_i^t of node i depends on its attitude relative to that of its m_i^t connections. Nodes maximize their utility by adjusting their attitudes x_i^t and establishing or severing connections. Figure courtesy of Yao-li Chuang [2].*

The Mathematical Fight for Voting Rights

By Matthew R. Francis

State and local governments will redraw voting districts based on new information following completion of the 2020 U.S. Census. Ideally, this process ensures fair representation. In practice, however, districting often involves gerrymandering: the deliberate planning of districts to dilute the voting power of certain groups in favor of others, which violates the law.

Racial gerrymandering—drawing districts to limit the power of voters of color to select candidates they favor—is a particularly pernicious problem. Section 2 of the Voting Rights Act (VRA) of 1965 specifically prohibits this practice, but that has not stopped authorities from doing it anyway. "A number of court decisions have purposefully asked mathematicians, political scientists, and statisticians to use specific methods to try and understand racial gerrymandering," Matt Barreto, a professor of political science and Chicana/o studies at the University of California, Los Angeles, said.

Barreto and his colleagues employ powerful statistical methods and draw on census and other public data to identify gerrymandered districts. Utilizing these tools, mathematicians can test proposed district maps or draw their own, designing them from the ground up to prevent voter dilution.

Since gerrymanderers use the same data to intentionally disenfranchise voters, the question is whether mathematical approaches alone are enough to fight the problem. Just as machine learning algorithms can "learn" racism from their training data,[1] studies show that the results of algorithmic districting can be as bad as deliberate gerrymandering [2]. To put it another way, can math solve problems it did not create?

"Previous efforts that used mathematics were not as accurate, and they did whitewash over some of the black and brown voters living in communities," Barreto said. "By going that extra step and purposefully trying to bring in accurate data on racial and ethnic minorities, we can go back to our trusted mathematical and statistical methods to make sure we're getting accurate counts of people."

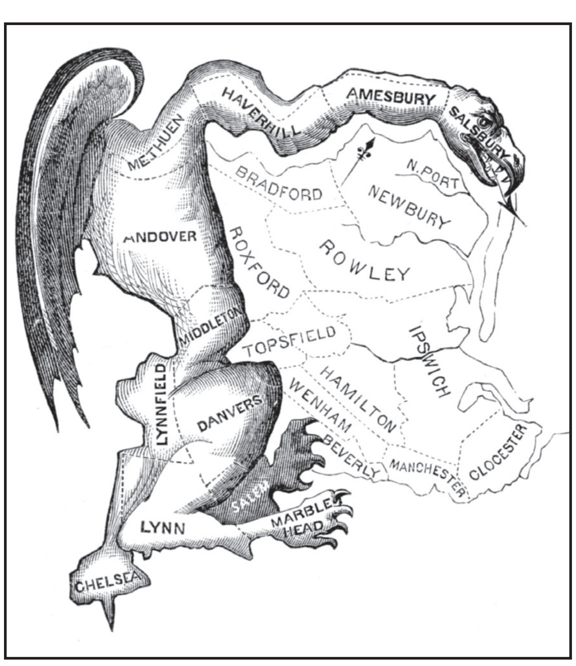

Figure 1. *Cartoonist Elkanah Tisdale's 1812 depiction of Massachusetts Governor Elbridge Gerry's partisan gerrymandering in favor of the Democratic-Republican Party. Public domain image.*

Racial Polarization, Racial Gerrymandering

In 1812, cartoonist Elkanah Tisdale noticed that one of the districts created under Massachusetts Governor Elbridge Gerry looked like the mythical fire-monster salamander, so he dubbed it the "Gerry-Mander" (that arguably makes "gerrymandering" the most important legal term ever coined in a cartoon, which pleases me as a frequent comics writer). This original gerrymander is a prime example of partisan gerrymandering because it was created to favor the Democratic-Republican Party over the Federalists (see Figure 1).

Racial gerrymandering has garnered less attention than its partisan counterpart, though the two often go hand in hand. However, racial gerrymandering also happens in effective one-party regions, such as cities where the Democratic Party dominates local politics. In practice, testing for unethical districting involves looking for racially polarized voting patterns — places in which minority voters strongly prefer one candidate over another, but districts are drawn to favor white voter preferences. Chicago—with a history of just two elected African American mayors despite its large black population—is a classic example of this form of gerrymandering.

Consider an imaginary mayoral election with two candidates: Smith, who is preferred by white/Anglo voters, and Herrera, who is preferred by Latinx voters. The city is divided in a such way that Latinx voters never amount to more than 40 percent of the total population in any district, while white voters never comprise fewer than 50 percent — regardless of the city's total racial and ethnic makeup. Racial gerrymandering ensures that Smith always wins over Herrera and Latinx preferences are never represented, which is a violation of the VRA. Perhaps the districting scheme splits apart Latinx-majority neighborhoods and lumps the fragments with white-majority areas; a more equitable and representative division would keep those neighborhoods whole, possibly even allowing for Latinx-plurality districts.

The challenge for mathematicians involves reconstructing racial voting patterns without violating voter privacy, which is protected by law. Barreto and his collaborators use ecological inference (EI), a technique that infers individual behaviors from population-level datasets. Their EI methods involve an iterative Bayesian approach, utilizing publicly available data from petitions, voter records (which merely tabulate if a registered voter casts a ballot), and the census.

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[1] https://sinews.siam.org/Details-Page/the-threat-of-ai-comes-from-inside-the-house

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Immigration

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through opinion dynamics; the two inform each other in a synergistic way.

Attitudes x_i^t vary between $-1 \le x_i^t \le 0$ for guests and $0 \le x_i^t \le 1$ for hosts; the magnitude $|x_i^t|$ indicates the degree of hostility towards the other group. Thus, $x_i^t \to 0^{\pm}$ characterizes most receptive guests or most hospitable hosts, while $x_i^t = \pm 1$ represents the highest level of xenophobia (see Figure 1, on page 1). The utility U_i^t is given by a pairwise reward—to which each node j linked to i contributes—and by a cost function for maintaining m_i^t connections, such that

$$U_i^t = \sum_{j \in \Omega_i^t} A_{ij} \exp\Big(-\frac{(x_i^t - x_j^t)^2}{2\sigma}\Big) - \exp\Big(\frac{m_i^t}{\alpha}\Big).$$

Here, Ω_i^t is the set of nodes linked to i at time t, so that m_i^t is given by its cardinality: $m_i^t = |\Omega_i^t|$. The pairwise reward depends on the attitude difference $|x_i^t - x_j^t|$ between nodes i and j; a diminishing attitude difference correlates with an increasingly high reward. Therefore, if both i and j are hosts or immigrants, the reward is maximized for $x_i^t = x_j^t$, leading to consensus within the group. But if i and j are from different groups, the reward is optimized only if both nodes adopt more cooperative attitudes: $x_i^t \to 0^-$ and $x_j^t \to 0^+$. The parameter σ controls the reward's sensitivity to attitude differences, the amplitude A_{ij} specifies the maximum possible reward, and the scaling coefficient α governs the cost of maintaining active links. Other models have considered residential segregation between two ethnic groups, with nodes seeking "friendly" neighbors with whom to connect. The most famous of these is the seminal Schelling model of segregation [3, 4, 6]. Our utility function U_i^t adds socioeconomic status as a decision-making factor in the link establishment process.

The dynamics unfold so that connectivities are modified at each time step to maximize utility. Attitudes are changed by imitation, so that

$$x_i^{t+1} = \begin{cases} \min\left(0, x_i^t + \dfrac{x_j^t - x_i^t}{\kappa}\right) & \text{for guests,} \\[2ex] \max\left(0, x_i^t + \dfrac{x_j^t - x_i^t}{\kappa}\right) & \text{for hosts,} \end{cases}$$

where κ governs attitude adjustment. Specifically, the timescale for guest cultural adjustment τ_g is given by κ and scaled by the probability of a guest being paired with a host N_h/N, so that $\tau_g \sim \kappa N/N_h$. Similarly, the host cultural adjustment timescale $\tau_h \sim \kappa N/N_g$. Since $N_h \gg N_g$, also $\tau_h \gg \tau_g$; adjustment times for hosts are longer than for guests. These cultural adjustment timescales are compared with the unitary timescale for social link remodeling. Finally, initial conditions represent the way in which guests are originally settled in the community. One extreme case involves a perfectly executed welcoming program that provides refugees with sufficient social ties to hosts, and where all nodes are randomly connected — regardless of attitudes and utilities. The other extreme case is that of guests who arrive in a completely foreign environment with nonexistent initial resources. Hosts are naturally connected to one another in their own state of equilibrium, and guests are introduced without any links to hosts or each other.

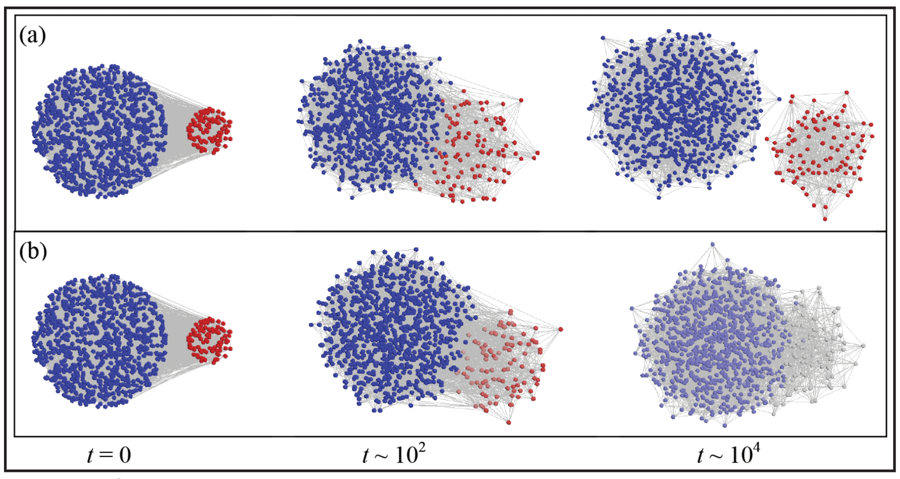

Figure 2. *Simulated dynamics leading to complete segregation (2a) and integration (2b) between guest (red) and host (blue) populations. Initial conditions are randomly connected guest and host nodes with attitudes $x^0_{i,\text{guest}} = -1$ and $x^0_{i,\text{host}} = 1$. Panels 2a and 2b differ only for κ, the attitude adjustment timescale, with $\kappa = 1000$ in 2a, where segregated clusters emerge, and $\kappa = 100$ in 2b, where a connected host-guest cluster arises over time. Figure courtesy of Yao-li Chuang [2].*

Figure 2 depicts two representative steady-state outcomes. In Figure 2a, hosts and guests segregate and maintain highly hostile attitudes. Any initial cross-group utilities yield low rewards that do not increase over time, so that all ties between hosts and guests are eventually severed. Enclaves emerge when the two separate communities adopt uniform but differing attitudes x_i. In Figure 2b, all nodes develop more cooperative attitudes that increase cross-group rewards, so that hosts and guests remain mixed. Eventually, $x_i^t \to 0$ on all nodes. For both scenarios, $|x_i^t - x_j^t| \to 0$ at steady state, but to which configuration society converges depends on parameter choices and initial conditions.

We find that the main predictor of integration versus segregation is the magnitude of the τ_g, τ_h timescales relative to the unitary network remodeling time. In the case of slow cultural adjustment, immigrant and host communities tend to segregate as accumulation of socioeconomic wealth occurs more efficiently through insular, in-group connections. Conversely, fast cultural adjustment enables the establishment and sustenance of cross-cultural bridges, allowing different groups to reach consensus and maintain active cooperation. This is shown in Figures 2a and 2b, where the only difference is the κ parameter that drives τ_g, τ_h. We also find that a high guest-to-host ratio N_g/N_h increases the likelihood of in-group connections and reduces communication between immigrant and host populations.

One possible approach to avoid segregation is the promotion of cross-group interactions via government incentives, or if newcomers carry or acquire desired skill sets, for example. Note that cultural adjustment does not necessarily mean that either side must abandon their identity; rather, we find that different groups must adopt tolerant attitudes towards one another, engaging in rapport building and acceptance to bridge differences and promote integration [1]. This is the long-term challenge for the future.

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Yao-li Chuang is a former mathematics researcher at the California State University, Northridge (CSUN) and a computational medicine postdoctoral researcher at the University of California, Los Angeles (UCLA). He specializes in dynamical systems, cancer research, system biology, and social sciences. Tom Chou is a professor in the Departments of Computational Medicine and Mathematics at UCLA. His scientific interests include statistical physics, soft matter physics, immunology, hematopoiesis, and computational psychiatry. Maria R. D'Orsogna is a professor of mathematics at CSUN and associate director of the Institute for Pure and Applied Mathematics at UCLA. She works on mathematical modeling of biological, behavioral, and social systems.

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SIAM News Transition

Karthika Swamy Cohen, who oversaw *SIAM News* as managing editor since July 2015, left SIAM last month for a new position. We are grateful for her contributions to SIAM and wish her the best of luck in her future endeavors.

Lina Sorg, who served as the associate editor of *SIAM News* since October 2015, has taken over as managing editor.